NAG Fortran Library Routine Document

S21DAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21DAF returns the value of the general elliptic integral of the second kind $F(z, k', a, b)$ for a complex argument z, via the routine name.

2 Specification

```
complex FUNCTION S21DAF(Z, AKP, A, B, IFAIL)
INTEGER                     IFAIL
real                        AKP, A, B
complex                     Z
```

3 Description

This routine evaluates an approximation to the general elliptic integral of the second kind $F(z, k', a, b)$ given by

$$F(z, k', a, b) = \int_0^z \frac{a + b\zeta^2}{(1+\zeta^2)\sqrt{(1+\zeta^2)(1+k'^2\zeta^2)}} d\zeta,$$

where a and b are real parameters, z is a complex argument whose real part is non-negative and k' is a real parameter (the *complementary modulus*). The evaluation of F is based on the Gauss transformation. Further details, in particular for the conformal mapping provided by F, can be found in Bulirsch (1960).

Special values include

$$F(z, k', 1, 1) = \int_0^z \frac{d\zeta}{\sqrt{(1+\zeta^2)(1+k'2\zeta^2)}},$$

or $F_1(z, k')$ (the *elliptic integral of the first kind*) and

$$F(z, k', 1, k'^2) = \int_0^z \frac{\sqrt{1+k'^2\zeta^2}}{(1+\zeta^2)\sqrt{1+\zeta^2}} d\zeta,$$

or $F_2(z, k')$ (the *elliptic integral of the second kind*). Note that the values of $F_1(z, k')$ and $F_2(z, k')$ are equal to $\tan^{-1}(z)$ in the trivial case $k' = 1$.

S21DAF is derived from an Algol 60 procedure given by Bulirsch (1960). Constraints are placed on the values of z and k' in order to avoid the possibility of machine overflow.

4 References

Bulirsch R (1960) Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Parameters

1: Z – ***complex*** *Input*

On entry: the argument z of the function.

Constraints:

$0.0 \leq \mathrm{Re(Z)} \leq \lambda$,
$\mathrm{ABS(Im(Z))} \leq \lambda$, where $\lambda^6 = 1/\mathrm{X02AMF}$.

2: AKP – ***real*** *Input*

On entry: the argument k' of the function.

Constraint: $\mathrm{ABS(AKP)} \leq \lambda$.

3: A – ***real*** *Input*

On entry: the argument a of the function.

4: B – ***real*** *Input*

On entry: the argument b of the function.

5: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, $\mathrm{Re(Z)} < 0.0$,
or $\mathrm{Re(Z)} > \lambda$,
or $\mathrm{ABS(Im(Z))} > \lambda$,
or $\mathrm{ABS(AKP)} > \lambda$, where $\lambda^6 = 1/\mathrm{X02AMF}$.

IFAIL $= 2$

The iterative procedure used to evaluate the integral has failed to converge. The result is returned as zero.

7 Accuracy

In principle the routine is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the Fortran intrinsic functions for elementary functions such as ATAN2 and LOG.

8 Further Comments

None.

9 Example

The example program evaluates the elliptic integral of the first kind $F_1(z,k')$ given by

$$F_1(z,k') = \int_0^z \frac{d\zeta}{\sqrt{(1+\zeta^2)(1+k'^2\zeta^2)}},$$

where $z = 1.2 + 3.7i$ and $k' = 0.5$, and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S21DAF Example Program Text.
*     Mark 20 Release. NAG Copyright 2001.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      complex          Y, Z
      real             A, AKP, B
      INTEGER          IFAIL
*     .. External Functions ..
      complex          S21DAF
      EXTERNAL         S21DAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S21DAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     Z          AKP     A',
     +  '      B                 Y                  IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) Z, AKP, A, B
      IFAIL = 0
*
      Y = S21DAF(Z,AKP,A,B,IFAIL)
*
      WRITE (NOUT,99999) Z, AKP, A, B, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,'(',F4.1,',',F4.1,' )',3F7.1,3X,'(',1P,E12.4,',',E12.4,
     +       ' )',I6)
      END
```

9.2 Program Data

```
S21DAF Example Program Data
 (1.2, 3.7)  0.5  1.0  1.0 : Values of Z, AKP, A and B
```

9.3 Program Results

```
 S21DAF Example Program Results

      Z          AKP     A      B                 Y                  IFAIL

 ( 1.2, 3.7 )    0.5    1.0    1.0   (  1.9713E+00,  5.0538E-01 )      0
```