

NAG Fortran Library Routine Document

S17AJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S17AJF returns a value of the derivative of the Airy function $\text{Ai}(x)$, via the routine name.

2 Specification

```
real FUNCTION S17AJF(X, IFAIL)
  INTEGER          IFAIL
  real             X
```

3 Description

This routine evaluates an approximation to the derivative of the Airy function $\text{Ai}(x)$. It is based on a number of Chebyshev expansions.

For $x < -5$,

$$\text{Ai}'(x) = \sqrt[4]{-x} \left[a(t) \cos z + \frac{b(t)}{\zeta} \sin z \right],$$

where $z = \frac{\pi}{4} + \zeta$, $\zeta = \frac{2}{3}\sqrt{-x^3}$ and $a(t)$ and $b(t)$ are expansions in variable $t = -2\left(\frac{5}{x}\right)^3 - 1$.

For $-5 \leq x \leq 0$,

$$\text{Ai}'(x) = x^2 f(t) - g(t),$$

where f and g are expansions in $t = -2\left(\frac{x}{5}\right)^3 - 1$.

For $0 < x < 4.5$,

$$\text{Ai}'(x) = e^{-11x/8} y(t),$$

where $y(t)$ is an expansion in $t = 4\left(\frac{x}{9}\right) - 1$.

For $4.5 \leq x < 9$,

$$\text{Ai}'(x) = e^{-5x/2} v(t),$$

where $v(t)$ is an expansion in $t = 4\left(\frac{x}{9}\right) - 3$.

For $x \geq 9$,

$$\text{Ai}'(x) = \sqrt[4]{-x} e^{-z} u(t),$$

where $z = \frac{2}{3}\sqrt{x^3}$ and $u(t)$ is an expansion in $t = 2\left(\frac{18}{z}\right) - 1$.

For $|x| < \text{the square of the machine precision}$, the result is set directly to $\text{Ai}'(0)$. This both saves time and avoids possible intermediate underflows.

For large negative arguments, it becomes impossible to calculate a result for the oscillating function with any accuracy and so the routine must fail. This occurs for $x < -\left(\frac{\sqrt{\pi}}{\epsilon}\right)^{4/7}$, where ϵ is the **machine precision**.

For large positive arguments, where Ai' decays in an essentially exponential manner, there is a danger of underflow so the routine must fail.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – *real* *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

X is too large and positive. On soft failure, the routine returns zero.

IFAIL = 2

X is too large and negative. On soft failure, the routine returns zero.

7 Accuracy

For negative arguments the function is oscillatory and hence absolute error is the appropriate measure. In the positive region the function is essentially exponential in character and here relative error is needed. The absolute error, E , and the relative error, ϵ , are related in principle to the relative error in the argument, δ , by

$$E \simeq |x^2 \text{Ai}(x)|\delta \quad \epsilon \simeq \left| \frac{x^2 \text{Ai}(x)}{\text{Ai}'(x)} \right| \delta.$$

In practice, approximate equality is the best that can be expected. When δ , ϵ or E is of the order of the **machine precision**, the errors in the result will be somewhat larger.

For small x , positive or negative, errors are strongly attenuated by the function and hence will be roughly bounded by the **machine precision**.

For moderate to large negative x , the error, like the function, is oscillatory; however the amplitude of the error grows like

$$\frac{|x|^{7/4}}{\sqrt{\pi}}.$$

Therefore it becomes impossible to calculate the function with any accuracy if $|x|^{7/4} > \frac{\sqrt{\pi}}{\delta}$.

For large positive x , the relative error amplification is considerable:

$$\frac{\epsilon}{\delta} \simeq \sqrt{x^3}.$$

However, very large arguments are not possible due to the danger of underflow. Thus in practice error amplification is limited.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      S17AJF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      real              X, Y
      INTEGER           IFAIL
*      .. External Functions ..
      real              S17AJF
      EXTERNAL          S17AJF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'S17AJF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '          X              Y              IFAIL'
      WRITE (NOUT,*)
20    READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S17AJF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
40    STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

S17AJF Example Program Data

```
-10.0  
-1.0  
0.0  
1.0  
5.0  
10.0  
20.0
```

9.3 Program Results

S17AJF Example Program Results

X	Y	IFAIL
-1.000E+01	9.963E-01	0
-1.000E+00	-1.016E-02	0
0.000E+00	-2.588E-01	0
1.000E+00	-1.591E-01	0
5.000E+00	-2.474E-04	0
1.000E+01	-3.521E-10	0
2.000E+01	-7.586E-27	0
