NAG Fortran Library Routine Document

S15ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S15ABF returns the value of the cumulative Normal distribution function, $P(x)$, via the routine name.

2 Specification

```
real FUNCTION S15ABF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du.$$

The routine is based on the fact that

$$P(x) = \tfrac{1}{2}\mathrm{erfc}\left(\frac{-x}{\sqrt{2}}\right)$$

and it calls S15ADF to obtain a value of erfc for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***real*** *Input*

On entry: the argument x of the function.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

There are no failure exits from this routine. The parameter IFAIL is included for consistency with other routines in this chapter.

7 Accuracy

Because of its close relationship with erfc, the accuracy of this routine is very similar to that in S15ADF. If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}\delta \right|$$

so that the relative error in the argument, x, is amplified by a factor, $\frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by ***machine precision***.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E, is given by

$$|E| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\delta \right|$$

so absolute accuracy can be guaranteed for all x.

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S15ABF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S15ABF
      EXTERNAL         S15ABF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S15ABF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S15ABF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
```

```
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S15ABF Example Program Data
          -20.0
           -1.0
            0.0
            1.0
            2.0
           20.0
```

9.3 Program Results

```
 S15ABF Example Program Results

      X           Y        IFAIL

   -2.000E+01   2.754E-89      0
   -1.000E+00   1.587E-01      0
    0.000E+00   5.000E-01      0
    1.000E+00   8.413E-01      0
    2.000E+00   9.772E-01      0
    2.000E+01   1.000E+00      0
```