NAG Fortran Library Routine Document

S11ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1   Purpose

S11ACF returns the value of the inverse hyperbolic cosine, $\text{arccosh}\,x$, via the routine name. The result is in the principal positive branch.

2   Specification

```
real FUNCTION S11ACF(X, IFAIL)
INTEGER            IFAIL
real               X
```

3   Description

The routine calculates an approximate value for the inverse hyperbolic cosine, $\text{arccosh}\,x$. It is based on the relation

$$\text{arccosh}\,x = \ln(x + \sqrt{x^2 - 1}).$$

This form is used directly for $1 < x < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $x \geq 10^k$, $\sqrt{x^2 - 1}$ is equal to $\sqrt{x}$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\text{arccosh}\,x = \ln 2 + \ln x.$$

4   References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5   Parameters

1:    X – **real**   *Input*

On entry: the argument x of the function.

Constraint: $X \geq 1.0$.

2:    IFAIL – INTEGER   *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

The routine has been called with an argument less than 1.0, for which arccosh x is not defined. The result returned is zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\sqrt{x^2 - 1} \, \text{arccosh} \, x} \times \delta \right|.$$

That is the relative error in the argument is amplified by a factor at least $\dfrac{x}{\sqrt{x^2 - 1} \, \text{arccosh} \, x}$ in the result.

The equality should apply if δ is greater than the **machine precision** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation it is possible that an extra figure may be lost in internal calculation and round-off. The behaviour of the amplification factor is shown in the following graph:
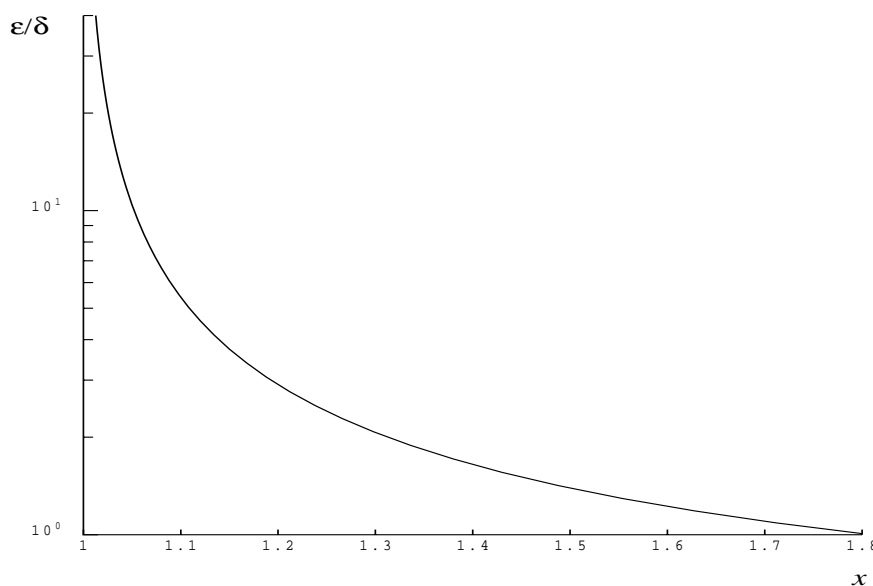

Figure 1

It should be noted that for $x > 2$ the factor is always less than 1.0. For large x we have the absolute error E in the result, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by **machine precision**. More significantly for x close to 1, $x - 1 \sim \delta$, the above analysis becomes inapplicable due to the fact that both function and argument are bounded, $x \geq 1$, arccosh $x \geq 0$. In this region we have

$$E \sim \sqrt{\delta}.$$

That is, there will be approximately half as many decimal places correct in the result as there were correct figures in the argument.

8   Further Comments

None.

9   Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1   Program Text

Note: the listing of the example program presented below uses **bold italicised** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      S11ACF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
       INTEGER          NIN, NOUT
       PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
       real             X, Y
       INTEGER          IFAIL
*      .. External Functions ..
       real             S11ACF
       EXTERNAL         S11ACF
*      .. Executable Statements ..
       WRITE (NOUT,*) 'S11ACF Example Program Results'
*      Skip heading in data file
       READ (NIN,*)
       WRITE (NOUT,*)
       WRITE (NOUT,*) '     X              Y         IFAIL'
       WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
       IFAIL = 1
*
       Y = S11ACF(X,IFAIL)
*
       WRITE (NOUT,99999) X, Y, IFAIL
       GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
       END
```

9.2   Program Data

```
S11ACF Example Program Data
          1.00
          2.0
          5.0
         10.0
         -0.5
```

9.3   Program Results

```
 S11ACF Example Program Results

     X              Y         IFAIL

   1.000E+00   0.000E+00        0
   2.000E+00   1.317E+00        0
   5.000E+00   2.292E+00        0
   1.000E+01   2.993E+00        0
  -5.000E-01   0.000E+00        1
```