NAG Fortran Library Routine Document

S01BAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S01BAF returns a value of the shifted logarithmic function, $\ln(1+x)$, via the routine name.

2 Specification

```
real FUNCTION S01BAF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

This routine computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The routine is based on the Chebyshev expansion

$$\ln\frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4\sum_{k=0}^{\infty}\frac{p^{2k+1}}{2k+1}T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \dfrac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \dfrac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\dfrac{1}{\sqrt{2}} - 1, \sqrt{2} - 1\right]$.

Outside this domain, $\ln(1+x)$ is computed by the Fortran intrinsic logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Parameters

1: X – ***real*** *Input*

On entry: the argument x of the function.

Constraint: X > -1.0.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, X ≤ -1.0.

The result is returned as zero.

7 Accuracy

The returned result should be accurate almost to ***machine precision***, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in S01BAF.

8 Further Comments

Empirical tests show that the time taken for a call of S01BAF usually lies between about 1.25 and 2.5 times the time for a call to the standard Fortran function LOG.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S01BAF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. External Functions ..
      real             S01BAF
      EXTERNAL         S01BAF
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S01BAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '       X           Y'
   20 READ (NIN,*,END=40) X
      IFAIL = 0
*
      Y = S01BAF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.4)
      END
```

9.2 Program Data

```
S01BAF Example Program Data
  2.50E+0
  1.25E-1
 -9.06E-1
  1.29E-3
 -7.83E-6
  1.00E-9
```

9.3 Program Results

```
 S01BAF Example Program Results

        X           Y
   2.5000E+00  1.2528E+00
   1.2500E-01  1.1778E-01
  -9.0600E-01 -2.3645E+00
   1.2900E-03  1.2892E-03
  -7.8300E-06 -7.8300E-06
   1.0000E-09  1.0000E-09
```