NAG Fortran Library Routine Document G13DJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

G13DJF computes forecasts of a multivariate time series. It is assumed that a vector ARMA model has already been fitted to the appropriately differenced/transformed time series using G13DCF. The standard deviations of the forecast errors are also returned. A reference vector is set up so that should future series values become available the forecasts and their standard errors may be updated by calling G13DKF.

2 Specification

```
SUBROUTINE G13DJF(K, N, Z, IK, TR, ID, DELTA, IP, IQ, MEAN, PAR, LPAR,

QQ, V, LMAX, PREDZ, SEFZ, REF, LREF, WORK, LWORK,

IWORK, LIWORK, IFAIL)

INTEGER

K, N, IK, ID(K), IP, IQ, LPAR, LMAX, LREF, LWORK,

IWORK(LIWORK), LIWORK, IFAIL

real

Z(IK,N), DELTA(IK,*), PAR(LPAR), QQ(IK,K), V(IK,*),

PREDZ(IK,LMAX), SEFZ(IK,LMAX), REF(LREF), WORK(LWORK)

CHARACTER*1

TR(K), MEAN
```

3 Description

Let the vector $Z_t = (z_{1t}, z_{2t}, \dots, z_{kt})^T$, for $t = 1, 2, \dots, n$, denote a k-dimensional time series for which forecasts of $Z_{n+1}, Z_{n+2}, \dots, Z_{n+l_{\max}}$ are required. Let $W_t = (w_{1t}, w_{2t}, \dots, w_{kt})^T$ be defined as follows:

$$w_{it} = \delta_i(B)z_{it}^*, \quad i = 1, 2, \dots, k,$$

where $\delta_i(B)$ is the differencing operator applied to the *i*th series and where z_{it}^* is equal to either z_{it} , $\sqrt{z_{it}}$ or $\log_e z_{it}$ depending on whether or not a transformation was required to stabilize the variance before fitting the model.

If the order of differencing required for the *i*th series is d_i , then the differencing operator for the *i*th series is defined by $\delta_i(B) = 1 - \delta_{i1}B - \delta_{i2}B^2 - \cdots - \delta_{id_i}B^{d_i}$ where B is the backward shift operator; that is, $BZ_t = Z_{t-1}$. The differencing parameters δ_{ij} for $i = 1, 2, \dots, k; j = 1, 2, \dots, d_i$ must be supplied by the user. If the *i*th series does not require differencing, then $d_i = 0$.

 W_t is assumed to follow a multivariate ARMA model of the form:

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \dots + \phi_p(W_{t-p} - \mu) + \epsilon_t - \theta_1 \epsilon_{t-1} - \dots - \theta_q \epsilon_{t-q}, \tag{1}$$

where $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t}, \dots, \epsilon_{kt})^T$, for $t = 1, 2, \dots, n$, is a vector of k residual series assumed to be Normally distributed with zero mean and positive-definite covariance matrix Σ . The components of ϵ_t are assumed to be uncorrelated at non-simultaneous lags. The ϕ_i and θ_j are k by k matrices of parameters. The matrices ϕ_i , for $i = 1, 2, \dots, p$, are the autoregressive (AR) parameter matrices, and the matrices θ_i , for $i = 1, 2, \dots, q$, the moving average (MA) parameter matrices. The parameters in the model are thus the p (k by k) ϕ -matrices, the p (k by p) ϕ -matrices, the mean vector p and the residual error covariance matrix p. The ARMA model (1) must be both stationary and invertible; see G13DXF for a method of checking these conditions.

The ARMA model (1) may be rewritten as

$$\phi(B)(\delta(B)Z_t^* - \mu) = \theta(B)\epsilon_t,$$

where $\phi(B)$ and $\theta(B)$ are the autoregressive and moving average polynomials and $\delta(B)$ denotes the k by k diagonal matrix whose ith diagonal elements is $\delta_i(B)$ and $Z_t^* = (z_{1t}^*, z_{2t}^*, \dots, z_{kt}^*)^T$.

This may be rewritten as

$$\phi(B)\delta(B)Z_t^* = \phi(B)\mu + \theta(B)\epsilon_t$$

or

$$Z_t^* = \tau + \psi(B)\epsilon_t = \tau + \epsilon_t + \psi_1\epsilon_{t-1} + \psi_2\epsilon_{t-2} + \cdots$$

where $\psi(B) = \delta^{-1}(B)\phi^{-1}(B)\theta(B)$ and $\tau = \delta^{-1}(B)\mu$ is a vector of length k.

Forecasts are computed using a multivariate version of the procedure described in Box and Jenkins (1976). If $\hat{Z}_n^*(l)$ denotes the forecast of Z_{n+l}^* , then $\hat{Z}_n^*(l)$ is taken to be that linear function of Z_n^*, Z_{n-1}^*, \ldots which minimises the elements of $E\{e_n(l)e_n'(l)\}$ where $e_n(l) = Z_{n+l}^* - \hat{Z}_n^*(l)$ is the forecast error. $\hat{Z}_n^*(l)$ is referred to as the linear minimum mean square error forecast of Z_{n+l}^* .

The linear predictor which minimises the mean square error may be expressed as

$$\hat{Z}_n^*(l) = \tau + \psi_l \epsilon_n + \psi_{l+1} \epsilon_{n-1} + \psi_{l+2} \epsilon_{n-2} + \cdots$$

The forecast error at t for lead l is then

$$e_n(l) = Z_{n+l}^* - \hat{Z}_n^*(l) = \epsilon_{n+l} + \psi_1 \epsilon_{n+l-1} + \psi_2 \epsilon_{n+l-2} + \dots + \psi_{l-1} \epsilon_{n+1}.$$

Let $d=\max(d_i)$, for $i=1,2,\ldots,k$. Unless q=0 the routine requires estimates of ϵ_t , for $t=d+1,d+2,\ldots,n$, which are obtainable from G13DCF. The terms ϵ_t are assumed to be zero, for $t=n+1,n+2,\ldots,n+l_{\max}$. The user may use G13DKF to update these l_{\max} forecasts should further observations, Z_{n+1},Z_{n+2},\ldots , become available. Note that when l_{\max} or more further observations are available then G13DJF must be used to produce new forecasts for $Z_{n+l_{\max}+1},Z_{n+l_{\max}+2},\ldots$, should they be required.

When a transformation has been used the forecasts and their standard errors are suitably modified to give results in terms of the original series, Z_t ; see Granger and Newbold (1976).

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

Granger C W J and Newbold P (1976) Forecasting transformed series J. Roy. Statist. Soc. Ser. B 38 189-203

Wei W W S (1990) Time Series Analysis: Univariate and Multivariate Methods Addison-Wesley

5 Parameters

The output quantities, K, N, IK, IP, IQ, PAR, NPAR, QQ and V from G13DCF are suitable for input to G13DJF.

1: K – INTEGER Input

On entry: the dimension, k, of the multivariate time series.

Constraint: $K \geq 1$.

2: N – INTEGER Input

On entry: the number, n, of observations in the series, Z_t , prior to differencing.

Constraint: N > 3.

The total number of observations must exceed the total number of parameters in the model; that is

if MEAN = 'Z', then
$$N \times K > (IP + IQ) \times K \times K + K \times (K + 1)/2$$
;

if MEAN = 'M', then
$$N \times K > (IP + IQ) \times K \times K + K + K \times (K + 1)/2$$
;

(see the parameters IP, IQ and MEAN).

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3: Z(IK,N) - real array

Input

On entry: Z(i,t) must be set equal to the *i*th component of Z_t , for $i=1,2,\ldots,k;\ t=1,2,\ldots,n$. Constraints:

if TR(i) = L', then Z(i,t) > 0.0 and if TR(i) = S', then $Z(i,t) \geq 0.0$, for i = 1, 2, ..., k; t = 1, 2, ..., n.

4: IK – INTEGER

Input

On entry: the first dimension of the arrays Z, QQ, DELTA, V, PREDZ and SEFZ as declared in the (sub)program from which G13DJF is called.

Constraint: $IK \geq K$.

5: TR(K) – CHARACTER*1 array

Input

On entry: TR(i) indicates whether the ith time series is to be transformed, for i = 1, 2, ..., k.

If TR(i) = N', then no transformation is used;

If TR(i) = L', then a log transformation is used;

If TR(i) = S, then a square root transformation is used.

Constraint: TR(i) must equal either 'N', 'L' or 'S', for i = 1, 2, ..., k.

6: ID(K) – INTEGER array

Input

On entry: ID(i) must specify, d_i , the order of differencing required for the ith series.

Constraint: $0 \le ID(i) < N - max(IP, IQ)$, for i = 1, 2, ..., k.

7: DELTA(IK,*) – real array

Input

Note: the second dimension of the array DELTA must be at least $\max(1,d)$, where $d = \max(\mathrm{ID}(i))$.

On entry: if ID(i) > 0, then DELTA(i,j) must be set equal to δ_{ij} , for $j = 1, 2, ..., d_i$; i = 1, 2, ..., k. If d = 0, then DELTA is not referenced.

8: IP – INTEGER

Input

On entry: the number, p, of AR parameter matrices.

Constraint: $IP \geq 0$.

9: IQ – INTEGER

Input

On entry: the number, q, of MA parameter matrices.

Constraint: $IQ \ge 0$.

10: MEAN - CHARACTER*1

Input

On entry: MEAN must be set to 'M' if components of μ have been estimated and 'Z' if all elements of μ are to be taken as zero.

Constraint: MEAN = 'M' or 'Z'.

11: PAR(LPAR) – *real* array

Input

On entry: PAR must contain the parameter estimates read in row by row in the order $\phi_1, \phi_2, \dots, \phi_p$, $\theta_1, \theta_2, \dots, \theta_q, \mu$.

Thus, if IP > 0, then PAR($(l-1) \times k \times k + (i-1) \times k + j$) must be set equal to an estimate of the (i, j)th element of ϕ_l , for l = 1, 2, ..., p; i = 1, 2, ..., k; j = 1, 2, ..., k.

If IQ > 0, then PAR($(p \times k \times k + (l-1) \times k \times k + (i-1) \times k + j)$ must be set equal to an estimate of the (i, j)th element of θ_l , for $l = 1, 2, \ldots, q$; $i = 1, 2, \ldots, k$; $j = 1, 2, \ldots, k$.

If MEAN has been set equal to 'M', then $PAR((p+q) \times k \times k + i)$ must be set equal to an estimate of the *i*th component of μ , for i = 1, 2, ..., k.

Constraint: the first IP \times K \times K elements of PAR must satisfy the stationarity condition and the next IQ \times K \times K elements of PAR must satisfy the invertibility condition.

12: LPAR – INTEGER Input

On entry: the dimension of the array PAR as declared in the (sub)program from which G13DJF is called

Constraints:

```
if MEAN = 'Z', then LPAR \geq max(1, (IP + IQ) \times K \times K); if MEAN = 'M', then LPAR \geq (IP + IQ) \times K \times K + K.
```

13: QQ(IK,K) - real array

Input/Output

On entry: QQ(i, j) must contain an estimate of the (i, j)th element of Σ . The lower triangle only is needed.

Constraint: QQ must be positive-definite.

On exit: if IFAIL $\neq 1$, then the upper triangle is set equal to the lower triangle.

14: V(IK,*) - real array

Input

Note: the second dimension of the array V must be at least max(1, N - max(ID(i))).

On entry: V(i,t) must contain an estimate of the *i*th component of ϵ_{t+d} , for $i=1,2,\ldots,k$; $t=1,2,\ldots,n-d$, where d is the maximum value of the elements of the array ID. If q=0, then V is not used.

15: LMAX – INTEGER

Input

On entry: the number, l_{max} , of forecasts required.

Constraint: LMAX ≥ 1 .

16: PREDZ(IK,LMAX) – *real* array

Output

On exit: PREDZ(i, l) contains the forecast of $z_{i,n+l}$, for i = 1, 2, ..., k; $l = 1, 2, ..., l_{max}$.

17: SEFZ(IK,LMAX) – *real* array

Output

On exit: SEFZ(i, l) contains an estimate of the standard error of the forecast of $z_{i,n+l}$, for i = 1, 2, ..., k; $l = 1, 2, ..., l_{max}$.

18: REF(LREF) – *real* array

Output

On exit: the reference vector which may be used to update forecasts using G13DKF. The first $(LMAX-1)\times K\times K$ elements contain the ψ weight matrices, $\psi_1,\psi_2,\ldots,\psi_{l_{\max}-1}$, stored columnwise. The next $K\times LMAX$ elements contain the forecasts of the transformed series $\hat{Z}_{n+1}^*,\hat{Z}_{n+2}^*,\ldots,\hat{Z}_{n+l_{\max}}^*$ and the next $K\times LMAX$ contain the variances of the forecasts of the transformed variables. The last K elements are used to store the transformations for the series.

19: LREF – INTEGER

Input

On entry: the dimension of the array REF as declared in the (sub)program from which G13DJF is called.

Constraint: LREF \geq (LMAX - 1) \times K \times K + 2 \times K \times LMAX + K.

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20: WORK(LWORK) - real array

Workspace

21: LWORK – INTEGER

Input

On entry: the dimension of the array WORK as declared in the (sub)program from which G13DJF is called.

Constraint: if $r = \max(\text{IP}, \text{IQ})$ and $d = \max(\text{ID}(i))$, for i = 1, 2, ..., k, then LWORK $\geq \max\{\text{K}r(\text{K}r + 2), (\text{IP} + d + 2)\text{K}^2 + (\text{N} + \text{LMAX})\text{K}\}.$

22: IWORK(LIWORK) – INTEGER array

Workspace

23: LIWORK – INTEGER

Input

On entry: the dimension of the array IWORK as declared in the (sub)program from which G13DJF is called.

Constraint: LIWORK $\geq K \times max(IP, IQ)$.

24: IFAIL – INTEGER

Input/Output

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

```
On entry, K < 1,
           N < 3.
or
          IK < K.
or
           ID(i) < 0 for some i = 1, 2, ..., k,
or
           ID(i) > N - max(IP, IQ) for some i = 1, 2, ..., k,
or
           IP < 0.
or
           IQ < 0,
or
           MEAN \neq 'M' or 'Z',
or
or
           LPAR < (IP + IQ) \times K \times K + K, and MEAN = 'M',
           LPAR < (IP + IQ) \times K \times K and MEAN = 'Z',
or
           N \times K \le (IP + IQ) \times K \times K + K + K(K + 1)/2, and MEAN = 'M',
or
           N \times K \le (IP + IQ) \times K \times K + K(K + 1)/2 and MEAN = 'Z',
or
or
           LREF < (LMAX - 1) \times K \times K + 2 \times K \times LMAX + K
or
           LWORK is too small,
or
           LIWORK is too small.
```

IFAIL = 2

On entry, at least one of the first k elements of TR is not equal to 'N', 'L' or 'S'.

IFAIL = 3

On entry, one or more of the transformations requested cannot be computed; that is, the user may be trying to log or square-root a series, some of whose values are negative.

IFAIL = 4

On entry, either QQ is not positive-definite or the autoregressive parameter matrices are extremely close to or outside the stationarity region, or the moving average parameter matrices are extremely close to or outside the invertibility region. To proceed, the user must supply different parameter estimates in the arrays PAR and QQ.

IFAIL = 5

This is an unlikely exit brought about by an excessive number of iterations being needed to evaluate the eigenvalues of the matrices required to check for stationarity and invertibility; see G13DXF. All output parameters are undefined.

IFAIL = 6

This is an unlikely exit which could occur if QQ is nearly non positive-definite. In this case the standard deviations of the forecast errors may be non-positive. To proceed, the user must supply different parameter estimates in the array QQ.

IFAIL = 7

This is an unlikely exit. For one of the series, overflow will occur if the forecasts are computed. The user should check whether the transformations requested in the array TR are sensible. All output parameters are undefined.

7 Accuracy

The matrix computations are believed to be stable.

8 Further Comments

The same differencing operator does not have to be applied to all the series. For example, suppose we have k=2, and wish to apply the second order differencing operator ∇^2 to the first series and the first-order differencing operator ∇ to the second series:

$$w_{1t} = \nabla^2 z_{1t} = (1 - B)^2 z_{1t} = (1 - 2B + B^2) Z_{1t},$$
 and $w_{2t} = \nabla z_{2t} = (1 - B) z_{2t}.$

Then $d_1 = 2$, $d_2 = 1$, $d = \max(d_1, d_2) = 2$, and

$$DELTA = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 \end{bmatrix}.$$

Note: although differencing may already have been applied prior to the model fitting stage, the differencing parameters supplied in DELTA are part of the model definition and are still required by this routine to produce the forecasts.

This routine should not be used when the moving average parameters lie close to the boundary of the invertibility region. The routine does test for both invertibility and stationarity but if in doubt, the user may use G13DXF, before calling this routine, to check that the VARMA model being used is invertible.

On a successful exit, the output quantities K, LMAX, IK, REF and LREF will be suitable for input to G13DKF.

9 Example

A program to compute forecasts of the next five values in two series each of length 48. No transformation is to be used and no differencing is to be applied to either of the series. G13DCF is first called to fit an AR(1) model to the series. The mean vector μ is to be estimated and $\phi_1(2,1)$ constrained to be zero.

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9.1 Program Text

Note: the listing of the example program presented below uses **bold italicised** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
G13DJF Example Program Text
   Mark 15 Release. NAG Copyright 1991.
   .. Parameters ..
                    NIN, NOUT
   INTEGER
   PARAMETER
                     (NIN=5,NOUT=6)
   INTEGER
                     KMAX, IK, IPMAX, IQMAX, NMAX, NPARMX, LWORK,
                    LIWORK, ICM, MMAX, IDMAXL, LREF
  PARAMETER
                     (KMAX=3,IK=KMAX,IPMAX=3,IQMAX=3,NMAX=100,
                    NPARMX=(IPMAX+IQMAX) *KMAX*KMAX+KMAX,LWORK=2000,
                    LIWORK=100, ICM=NPARMX, MMAX=10, IDMAXL=2,
                    LREF=MMAX*KMAX*(KMAX+2))
   .. Local Scalars ..
  real
                     CGETOL, RLOGL
  INTEGER
                     I, I2, IDMAX, IDMIN, IFAIL, IP, IPRINT, IQ,
                     ISHOW, J, K, L, L2, LMAX, LOOP, MAXCAL, N, ND,
                    NITER, NPAR
  LOGICAL
                    EXACT, MEANL
   CHARACTER
                    MEAN
   .. Local Arrays ..
                    CM(ICM,NPARMX), DELTA(IK,IDMAXL), G(NPARMX),
  real
                    PAR(NPARMX), PREDZ(IK, MMAX), QQ(IK, KMAX),
                    REF(LREF), SEFZ(IK, MMAX), V(IK, NMAX), W(IK, NMAX),
                    WORK(LWORK), Z(IK,NMAX)
   INTEGER
                    ID(KMAX), IWORK(LIWORK)
  LOGICAL
                    PARHLD (NPARMX)
   CHARACTER
                    TR(KMAX)
   .. External Subroutines .
   EXTERNAL
                    G13DCF, G13DJF, G13DLF
   .. Intrinsic Functions ..
   INTRINSIC
                    MAX, MIN, MOD
   .. Executable Statements .
   WRITE (NOUT,*) 'G13DJF Example Program Results'
   Skip heading in data file
   READ (NIN, *)
  READ (NIN,*) K, N, IP, IQ, MEAN, LMAX
  NPAR = (IP+IQ)*K*K
   MEANL = .FALSE.
   IF (MEAN.EQ.'M' .OR. MEAN.EQ.'m') THEN
      NPAR = NPAR + K
      MEANL = .TRUE.
  END IF
  IF (K.GT.O .AND. K.LE.KMAX .AND. N.GE.1 .AND. N.LE.NMAX .AND.
       NPAR.GE.1 .AND. NPAR.LE.NPARMX .AND. LMAX.GE.1 .AND. LMAX.LE.
       MMAX) THEN
      READ (NIN, \star) (ID(I), I=1, K)
      IDMIN = 0
      IDMAX = 0
      DO 20 I = 1, K
         IDMIN = MIN(ID(I), IDMIN)
         IDMAX = MAX(ID(I), IDMAX)
20
      CONTINUE
      IF (IDMIN.GE.O .AND. IDMAX.LE.IDMAXL) THEN
         DO 40 I = 1, K
            READ (NIN, *) (Z(I,J),J=1,N)
40
         CONTINUE
         READ (NIN, \star) (TR(I), I=1, K)
         IF (IDMAX.GT.O) THEN
            DO 60 I = 1, K
               READ (NIN,*) (DELTA(I,J),J=1,ID(I))
            CONTINUE
60
         END IF
         IFAIL = 0
```

```
CALL G13DLF(K,N,Z,IK,TR,ID,DELTA,W,ND,WORK,IFAIL)
            DO 80 I = 1, NPAR
               PAR(I) = 0.0e0
               PARHLD(I) = .FALSE.
   80
            CONTINUE
            DO 120 I = 1, K
               DO 100 J = 1, I
                   QQ(I,J) = 0.0e0
 100
               CONTINUE
 120
            CONTINUE
            PARHLD(3) = .TRUE.
            EXACT = .TRUE.
            ** Set IPRINT .lt. O for no monitoring
            IPRINT = -1
            CGETOL = 0.0001e0
            MAXCAL = 40*NPAR*(NPAR+5)
            ** Set ISHOW = 0 for no printing of results from G13DCF
            ISHOW = 0
            IFAIL = 1
            CALL G13DCF(K,ND,IP,IQ,MEANL,PAR,NPAR,QQ,IK,W,PARHLD,EXACT,
                         IPRINT,CGETOL,MAXCAL,ISHOW,NITER,RLOGL,V,G,CM,
                         ICM,WORK,LWORK,IWORK,LIWORK,IFAIL)
            IF (IFAIL.EQ.O .OR. IFAIL.GE.4) THEN
               IFAIL = 0
               CALL G13DJF(K,N,Z,IK,TR,ID,DELTA,IP,IQ,MEAN,PAR,NPAR,QQ,
                            V, LMAX, PREDZ, SEFZ, REF, LREF, WORK, LWORK, IWORK,
     +
                            LIWORK, IFAIL)
               WRITE (NOUT, *)
               WRITE (NOUT,*) ' FORECAST SUMMARY TABLE'
               WRITE (NOUT, *) ' -----'
               WRITE (NOUT, *)
               WRITE (NOUT, 99998) 'Forecast origin is set at t = ', N
               WRITE (NOUT,*)
               LOOP = LMAX/5
               IF (MOD(LMAX,5).NE.0) LOOP = LOOP + 1
               DO 160 J = 1, LOOP
                  I2 = (J-1)*5
                  L2 = MIN(I2+5,LMAX)
                  WRITE (NOUT, 99997) 'Lead Time', (I, I=I2+1, L2)
                  WRITE (NOUT, *)
                  I = 1
                  WRITE (NOUT, 99996) 'Series', I, ': Forecast
     +
                    (PREDZ(1,L),L=I2+1,L2)
                  WRITE (NOUT, 99995) ': Standard Error',
                    (SEFZ(1,L),L=I2+1,L2)
                  DO 140 I = 2, K
                      WRITE (NOUT,99996) 'Series ', I, ' : Forecast
                      \label{eq:predz} $$(PREDZ(I,L),L=I2+1,L2)$$ WRITE (NOUT,99995) ': Standard Error ',
                        (SEFZ(I,L),L=I2+1,L2)
 140
                  CONTINUE
                  WRITE (NOUT, *)
 160
               CONTINUE
            END IF
         END IF
      END IF
      STOP
99998 FORMAT (1X,A,I4)
99997 FORMAT (1X,A,12X,5I10)
99996 FORMAT (1X,A,I2,A,5F10.2)
99995 FORMAT (10X,A,4(F7.2,3X),F7.2)
      END
```

G13DJF.8 [NP3546/20A]

9.2 Program Data

```
G13DJF Example Program Data
2 48 1 0 'M' 5 : K, N, IP, IQ, MEAN, M
0 0 : ID(I),I=1,K
-1.490 -1.620 5.200 6.230 6.210 5.860 4.090 3.180
2.620 1.490 1.170 0.850 -0.350 0.240 2.440 2.580
2.040 0.400 2.260 3.340 5.090 5.000 4.780 4.110
3.450 1.650 1.290 4.090 6.320 7.500 3.890 1.580
5.210 5.250 4.930 7.380 5.870 5.810 9.680 9.070
7.290 7.840 7.550 7.320 7.970 7.760 7.000 8.350
7.340 6.350 6.960 8.540 6.620 4.970 4.550 4.810
4.750 4.760 10.880 10.010 11.620 10.360 6.400 6.240
7.930 4.040 3.730 5.600 5.350 6.810 8.270 7.680
6.650 6.080 10.250 9.140 17.750 13.300 9.630 6.800
4.080 5.060 4.940 6.650 7.940 10.760 11.890 5.850
9.010 7.500 10.020 10.380 8.150 8.370 10.730 12.140 : End of time series 'N' 'N' : TR(1), TR(2)
```

9.3 Program Results

G13DJF Example Program Results

```
FORECAST SUMMARY TABLE
```

Forecast origin is set at t = 48

Lead Time		1	2	3	4	5
Series	1 : Forecast : Standard Error	7.82 1.72	7.28 2.23	6.77 2.51	6.33 2.68	5.95 2.79
Series	2 : Forecast : Standard Error	10.31 2.32	9.25 2.68	8.65 2.78	8.30 2.82	8.10 2.83

[NP3546/20A] G13DJF.9 (last)