NAG Fortran Library Routine Document

G13CDF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13CDF calculates the smoothed sample cross spectrum of a bivariate time series using spectral smoothing by the trapezium frequency (Daniell) window.

2 Specification

```
SUBROUTINE G13CDF(NXY, MTXY, PXY, MW, IS, PW, L, KC, XG, YG, NG, IFAIL)
INTEGER            NXY, MTXY, MW, IS, L, KC, NG, IFAIL
real               PXY, PW, XG(KC), YG(KC)
```

3 Description

The supplied time series may be mean and trend corrected and tapered as in the description of G13CBF before calculation of the unsmoothed sample cross-spectrum

$$f_{xy}^{*}(\omega) = \frac{1}{2\pi n}\left\{\sum_{t=1}^{n} y_t \exp(i\omega t)\right\} \times \left\{\sum_{t=1}^{n} x_t \exp(-i\omega t)\right\}$$

for frequency values $\omega_j = \frac{2\pi j}{K}$, $0 \leq \omega_j \leq \pi$.

A correction is made for bias due to any tapering.

As in the description of G13CBF for univariate frequency window smoothing, the smoothed spectrum is returned as a subset of these frequencies,

$$\nu_l = \frac{2\pi l}{L}, \quad l = 0, 1, \ldots, [L/2]$$

where [ ] denotes the integer part.

Its real part or co-spectrum $cf(\nu_l)$, and imaginary part or quadrature spectrum $qf(\nu_l)$ are defined by

$$f_{xy}(\nu_l) = cf(\nu_l) + iqf(\nu_l) = \sum_{|\omega_k| < \frac{\pi}{M}} \tilde{w}_k f_{xy}^{*}(\nu_l + \omega_k)$$

where the weights $\tilde{w}_k$ are similar to the weights w_k defined for G13CBF, but allow for an implicit alignment shift S between the series:

$$\tilde{w}_k = w_k \exp(-2\pi iSk/L).$$

It is recommended that S is chosen as the lag k at which the cross covariances $c_{xy}(k)$ peak, so as to minimize bias.

If no smoothing is required, the integer M, which determines the frequency window width $\frac{2\pi}{M}$, should be set to n.

The bandwidth of the estimates will normally have been calculated in a previous call of G13CBF for estimating the univariate spectra of y_t and x_t.

4 References

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

1: NXY – INTEGER *Input*

On entry: the length, n, of the time series x and y.

Constraint: NXY ≥ 1.

2: MTXY – INTEGER *Input*

On entry: whether the data is to be initially mean or trend corrected.

MTXY $= 0$

For no correction.

MTXY $= 1$

For mean correction.

MTXY $= 2$

For trend correction.

Constraint: $0 \leq$ MTXY ≤ 2.

3: PXY – ***real*** *Input*

On entry: the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.

A value of 0.0 implies no tapering.

Constraint: $0.0 \leq$ PXY ≤ 1.0.

4: MW – INTEGER *Input*

On entry: the frequency width, M, of the smoothing window as $\frac{2\pi}{M}$.

A value of n implies that no smoothing is to be carried out.

Constraint: $1 \leq$ MW $\leq$ NXY.

5: IS – INTEGER *Input*

On entry: the alignment shift, S, between the x and y series. If x leads y, the shift is positive.

Constraint: $-$L $<$ IS $<$ L.

6: PW – ***real*** *Input*

On entry: the shape parameter, p, of the trapezium frequency window.

A value of 0.0 gives a triangular window, and a value of 1.0 a rectangular window.

If MW $=$ NXY (i.e., no smoothing is carried out) then PW is not used.

Constraint: $0.0 \leq$ PW ≤ 1.0 if MW $\neq$ NXY.

7: L – INTEGER *Input*

On entry: the frequency division, L, of smoothed cross spectral estimates as $\frac{2\pi}{L}$.

Constraints:

L ≥ 1,
L must be a factor of KC (see below).

8: KC – INTEGER *Input*

On entry: the order of the fast Fourier transform (FFT) used to calculate the spectral estimates. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2n$, provided $m \leq 20$.

Constraints:

KC $\geq 2 \times$ NXY,
KC must be a multiple of L. The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF and C06EBF which perform the FFT.

9: XG(KC) – ***real*** array *Input/Output*

On entry: the NXY data points of the x series.

On exit: the real parts of the NG cross spectral estimates in elements XG(1) to XG(NG), and XG(NG + 1) to XG(KC) contain 0.0. The y series leads the x series.

10: YG(KC) – ***real*** array *Input/Output*

On entry: the NXY data points of the y series.

On exit: the imaginary parts of the NG cross spectral estimates in elements YG(1) to YG(NG), and YG(NG + 1) to YG(KC) contain 0.0. The y series leads the x series.

11: NG – INTEGER *Output*

On exit: the number of spectral estimates, $[L/2] + 1$, whose separate parts are held in XG and YG.

12: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, NXY < 1,
or MTXY < 0,
or MTXY > 2,
or PXY < 0.0,
or PXY > 1.0,

or MW < 1,
or MW > NXY,
or PW < 0.0 and MW $\neq$ NXY,
or PW > 1.0 and MW $\neq$ NXY,
or L < 1,
or $|IS| \geq L$.

IFAIL = 2

On entry, KC < 2 $\times$ NXY,
or KC is not a multiple of L,
or KC has a prime factor exceeding 19,
or KC has more than 20 prime factors, counting repetitions.

IFAIL = 3

This indicates that a serious error has occurred. Check all array subscripts in calls to G13CDF. Seek expert help.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CDF carries out an FFT of length KC to calculate the sample cross spectrum. The time taken by the routine for this is approximately proportional to KC $\times$ log(KC) (but see routine document Section for further details).

9 Example

The example program reads two time series of length 296. It selects mean correction and a 10% tapering proportion. It selects a $2\pi/16$ frequency width of smoothing window, a window shape parameter of 0.5 and an alignment shift of 3. It then calls G13CDF to calculate the smoothed sample cross spectrum and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13CDF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXYMAX, L, KC
      PARAMETER        (NXYMAX=300,L=80,KC=8*L)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             PW, PXY
      INTEGER          I, IFAIL, IS, J, MTXY, MW, NG, NXY
*     .. Local Arrays ..
      real             XG(KC), YG(KC)
*     .. External Subroutines ..
      EXTERNAL         G13CDF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13CDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NXY
      IF (NXY.GT.0 .AND. NXY.LE.NXYMAX) THEN
         READ (NIN,*) (XG(I),I=1,NXY)
```

```
          READ (NIN,*) (YG(I),I=1,NXY)
*         Set parameters for call to G13CDF
*         Mean correction and 10 percent taper
          MTXY = 1
          PXY = 0.1e0
*         Window shape parameter and zero covariance at lag 16
          PW = 0.5e0
          MW = 16
*         Alignment shift of 3
          IS = 3
          IFAIL = 0
*
          CALL G13CDF(NXY,MTXY,PXY,MW,IS,PW,L,KC,XG,YG,NG,IFAIL)
*
          WRITE (NOUT,*)
          WRITE (NOUT,*) '                          Returned sample spectrum'
          WRITE (NOUT,*)
          WRITE (NOUT,*)
     +'      Real  Imaginary       Real  Imaginary       Real  Imaginary
     +'
          WRITE (NOUT,*)
     + '      part     part         part     part         part     part'
          WRITE (NOUT,99999) (J,XG(J),YG(J),J=1,NG)
       END IF
       STOP
*
99999 FORMAT (1X,I3,F8.4,F9.4,I5,F8.4,F9.4,I5,F8.4,F9.4)
      END
```

9.2 Program Data

```
G13CDF Example Program Data
   296
 -0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
  0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
 -0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
  0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
  1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
  0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
  1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
  0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
  1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
 -1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
 -0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
 -0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
  1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
 -0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
 -2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
 -0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
  0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
  0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
  0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
  0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
  0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
 -1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
  0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
  0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
  0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
 -2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
 -0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
 -1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
 -1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
 -0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
 -1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
 -0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
  0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
  1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
 -0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
  0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
  0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262
```

```
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0
```

9.3 Program Results

```
G13CDF Example Program Results

                            Returned sample spectrum

         Real  Imaginary         Real  Imaginary         Real  Imaginary
         part     part           part     part           part     part
     1 -6.1563   0.0000     2 -5.5905  -2.0119     3 -3.2711  -2.7963
     4 -1.1803  -2.3264     5 -0.2061  -1.8132     6  0.3434  -1.1357
     7  0.6200  -0.7351     8  0.5967  -0.3449     9  0.4523  -0.0984
    10  0.2391   0.0177    11  0.1129   0.0402    12  0.0564   0.0523
    13  0.0134   0.0443    14 -0.0032   0.0299    15 -0.0057   0.0148
    16 -0.0057   0.0069    17 -0.0033   0.0038    18 -0.0011   0.0012
    19 -0.0004   0.0001    20 -0.0004   0.0002    21 -0.0003   0.0001
    22 -0.0001   0.0002    23 -0.0002   0.0003    24 -0.0002   0.0002
    25 -0.0002   0.0000    26 -0.0004   0.0000    27 -0.0002  -0.0002
    28 -0.0001  -0.0000    29 -0.0001   0.0002    30 -0.0001   0.0002
    31 -0.0002   0.0003    32 -0.0002   0.0001    33 -0.0001   0.0000
    34 -0.0000  -0.0000    35  0.0000  -0.0001    36  0.0001  -0.0001
    37  0.0001  -0.0001    38  0.0001  -0.0001    39  0.0000  -0.0001
    40  0.0000  -0.0001    41  0.0001   0.0000
```