NAG Fortran Library Routine Document

G13CCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13CCF calculates the smoothed sample cross spectrum of a bivariate time series using one of four lag windows: rectangular, Bartlett, Tukey or Parzen.

2 Specification

```
SUBROUTINE G13CCF(NXY, MTXY, PXY, IW, MW, IS, IC, NC, CXY, CYX, KC, L,
1                 NXYG, XG, YG, NG, IFAIL)
INTEGER           NXY, MTXY, IW, MW, IS, IC, NC, KC, L, NXYG, NG, IFAIL
real              PXY, CXY(NC), CYX(NC), XG(NXYG), YG(NXYG)
```

3 Description

The smoothed sample cross spectrum is a complex valued function of frequency ω, $f_{xy}(\omega) = cf(\omega) + iqf(\omega)$, defined by its real part or co-spectrum

$$cf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S)\cos(\omega k)$$

and imaginary part or quadrature spectrum

$$qf(\omega) = \frac{1}{2\pi} \sum_{k=-M+1}^{M-1} w_k C_{xy}(k+S)\sin(\omega k)$$

where $w_k = w_{-k}$, $k = 0, 1, \ldots, M-1$, is the smoothing lag window as defined in the description of G13CAF. The alignment shift S is recommended to be chosen as the lag k at which the cross covariances $c_{xy}(k)$ peak, so as to minimize bias.

The results are calculated for frequency values

$$\omega_j = \frac{2\pi j}{L}, \quad j = 0, 1, \ldots, [L/2],$$

where [ ] denotes the integer part.

The cross covariances $c_{xy}(k)$ may be supplied by the user, or constructed from supplied series $x_1, x_2, \ldots, x_n$; $y_1, y_2, \ldots, y_n$ as

$$c_{xy}(k) = \frac{\sum_{t=1}^{n-k} x_t y_{t+k}}{n}, \quad k \geq 0$$

$$c_{xy}(k) = \frac{\sum_{t=1-k}^{n} x_t y_{t+k}}{n} = c_{yx}(-k), \quad k < 0$$

this convolution being carried out using the finite Fourier transform.

The supplied series may be mean and trend corrected and tapered before calculation of the cross covariances, in exactly the manner described in G13CAF for univariate spectrum estimation. The results are corrected for any bias due to tapering.

The bandwidth associated with the estimates is not returned. It will normally already have been calculated in previous calls of G13CAF for estimating the univariate spectra of y_t and x_t.

4 References

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden-Day

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

1: NXY – INTEGER *Input*

On entry: the length, n, of the time series x and y.

Constraint: NXY ≥ 1.

2: MTXY – INTEGER *Input*

On entry: if cross covariances are to be calculated by the routine (IC $= 0$), MTXY must specify whether the data is to be initially mean or trend corrected.

MTXY $= 0$

For no correction,

MTXY $= 1$

For mean correction,

MTXY $= 2$

For trend correction.

If cross covariances are supplied (IC $\neq 0$), MTXY is not used.

Constraint: $0 \leq$ MTXY ≤ 2 if IC $= 0$.

3: PXY – ***real*** *Input*

On entry: if cross covariances are to be calculated by the routine (IC $= 0$), PXY must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. A value of 0.0 implies no tapering. If cross covariances are supplied (IC $\neq 0$), PXY is not used.

Constraint: $0.0 \leq$ PXY ≤ 1.0, if IC $= 0$.

4: IW – INTEGER *Input*

On entry: the choice of lag window. IW $= 1$ for rectangular, 2 for Bartlett, 3 for Tukey or 4 for Parzen.

Constraint: $1 \leq$ IW ≤ 4.

5: MW – INTEGER *Input*

On entry: the 'cut-off' point, M, of the lag window, relative to any alignment shift that has been applied. Windowed cross covariances at lags $(-\text{MW} + \text{IS})$ or less, and at lags $(\text{MW} + \text{IS})$ or greater are zero.

Constraints:

MW ≥ 1,
MW $+ |$IS$| \leq$ NXY.

6: IS – INTEGER *Input*

On entry: the alignment shift, S, between the x and y series. If x leads y, the shift is positive.

Constraint: $-$MW $<$ IS $<$ MW.

7: IC – INTEGER *Input*

On entry: indicates whether cross covariances are to be calculated in the routine or supplied in the call to the routine.

IC $= 0$

Cross covariances are to be calculated.

IC $\neq 0$

Cross covariances are to be supplied.

8: NC – INTEGER *Input*

On entry: the number of cross covariances to be calculated in the routine or supplied in the call to the routine.

Constraint: MW + |IS| $\leq$ NC $\leq$ NXY.

9: CXY(NC) – ***real*** array *Input/Output*

On entry: if IC $\neq 0$, then CXY must contain the NC cross covariances between values in the y series and earlier values in time in the x series, for lags from 0 to $(\text{NC} - 1)$. If IC $= 0$CXY need not be set.

On exit: if IC $= 0$, CYX will contain the NC calculated cross covariances.

If IC $\neq 0$, the contents of CXY will be unchanged.

10: CYX(NC) – ***real*** array *Input/Output*

On entry: if IC $\neq 0$, then CYX must contain the NC cross covariances between values in the y series and later values in time in the x series, for lags from 0 to $(\text{NC} - 1)$. If IC $= 0$, CYX need not be set.

On exit: if IC $= 0$, CYX will contain the NC calculated cross covariances.

If IC $\neq 0$, the contents of CYX will be unchanged.

11: KC – INTEGER *Input*

On entry: if IC $= 0$, KC must specify the order of the fast Fourier transform (FFT) used to calculate the cross covariances. KC should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq n +$ NC.

If IC $\neq 0$, that is if covariances are supplied, then KC is not used.

Constraints:

KC $\geq$ NXY + NC.
The largest prime factor of KC must not exceed 19, and the total number of prime factors of KC, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF and C06EBF which perform the FFT.

12: L – INTEGER *Input*

On entry: the frequency division, L, of the spectral estimates as $\dfrac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the cross covariances. L should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$.

Constraints:

L $\geq 2 \times$ MW $- 1$.
The largest prime factor of L must not exceed 19, and the total number of prime factors of L, counting repetitions, must not exceed 20. These two restrictions are imposed by C06EAF which performs the FFT.

13: NXYG – INTEGER *Input*

On entry: the length of the smaller of the arrays XG and YG, as declared in the (sub)program from which G13CCF is called.

Constraints:

if IC $= 0$, cross covariances are to be calculated, NXYG $\geq \max(\mathrm{KC}, \mathrm{L})$,
if IC $\neq 0$, cross covariances are to be supplied, NXYG $\geq$ L.

14: XG(NXYG) – ***real*** array *Input/Output*

On entry: if the cross covariances are to be calculated, then XG must contain the NXY data points of the x series. If covariances are supplied, XG need not be set.

On exit: contains the real parts of the NG complex spectral estimates in elements XG(1) to XG(NG), and XG$(\mathrm{NG}+1)$ to XG(NXYG) contain 0.0. The y series leads the x series.

15: YG(NXYG) – ***real*** array *Input/Output*

On entry: if cross covariances are to be calculated, then YG must contain the NXY data points of the y series. If covariances are supplied, YG need not be set.

On exit: contains the imaginary parts of the NG complex spectral estimates in elements YG(1) to YG(NG), and YG$(\mathrm{NG}+1)$ to YG(NXYG) contain 0.0. The y series leads the x series.

16: NG – INTEGER *Output*

On exit: the number, $[L/2]+1$, of complex spectral estimates, whose separate parts are held in XG and YG.

17: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, NXY < 1,
or MTXY < 0 and IC $= 0$,
or MTXY > 2 and IC $= 0$,
or PXY < 0.0 and IC $= 0$,
or PXY > 1.0 and IC $= 0$,
or IW ≤ 0,
or IW > 4,
or MW < 1,
or MW $+ |\mathrm{IS}| >$ NXY,
or $|\mathrm{IS}| \geq$ MW,
or NC $<$ MW $+ |\mathrm{IS}|$,
or NC $>$ NXY,
or NXYG $< \max(\mathrm{KC}, \mathrm{L})$ and IC $= 0$,

or NXYG $<$ L and IC $\neq 0$.

IFAIL $= 2$

On entry, KC $<$ NXY + NC,
or KC has a prime factor exceeding 19,
or KC has more than 20 prime factors, counting repetitions.

This error only occurs when IC $= 0$.

IFAIL $= 3$

On entry, L $< 2 \times$ MW $- 1$,
or L has a prime factor exceeding 19,
or L has more than 20 prime factors, counting repetitions.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

G13CCF carries out two FFTs of length KC by calls to C06EAF and C06EBF to calculate the sample cross covariances and one FFT of length L to calculate the sample spectrum. The timing of G13CCF is therefore dependent on the choice of these values. The time taken for an FFT of length n is approximately proportional to $n \log n$ (but see Section 8 of the document for C06EAF for further details).

9 Example

The example program reads 2 time series of length 296. It then selects mean correction, a 10% tapering proportion, the Parzen smoothing window and a cut-off point of 35 for the lag window. The alignment shift is set to 3 and 50 cross covariances are chosen to be calculated. The program then calls G13CCF to calculate the cross spectrum and then prints the cross covariances and cross spectrum.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13CCF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXYG, NCMAX
      PARAMETER        (NXYG=350,NCMAX=50)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             PXY
      INTEGER          I, IC, IFAIL, II, IS, IW, KC, L, MTXY, MW, NC,
     +                 NG, NXY
*     .. Local Arrays ..
      real             CXY(NCMAX), CYX(NCMAX), XG(NXYG), YG(NXYG)
*     .. External Subroutines ..
      EXTERNAL         G13CCF
*     .. Intrinsic Functions ..
      INTRINSIC        MIN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13CCF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NXY, NC, IC
      IF (NXY.GT.0 .AND. NXY.LE.NXYG .AND. NC.GT.0 .AND. NC.LE.NCMAX)
     +    THEN
```

```
         IF (IC.EQ.0) THEN
            READ (NIN,*) (XG(I),I=1,NXY)
            READ (NIN,*) (YG(I),I=1,NXY)
         ELSE
            READ (NIN,*) (CXY(I),I=1,NC)
            READ (NIN,*) (CYX(I),I=1,NC)
         END IF
*        Set parameters for call to G13CCF
*        Mean correction and 10 percent taper
         MTXY = 1
         PXY = 0.1e0
*        Parzen window and zero covariance at lag 35
         IW = 4
         MW = 35
*        Alignment shift of 3, 50 covariances to be calculated
         IS = 3
         KC = 350
         L = 80
         IFAIL = 0
*
         CALL G13CCF(NXY,MTXY,PXY,IW,MW,IS,IC,NC,CXY,CYX,KC,L,NXYG,XG,
     +               YG,NG,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,*) '                  Returned cross covariances'
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     + 'Lag     XY       YX   Lag     XY       YX   Lag     XY       YX'
         DO 20 I = 1, NC, 3
            WRITE (NOUT,99999) (II-1,CXY(II),CYX(II),II=I,MIN(I+2,NC))
   20    CONTINUE
         WRITE (NOUT,*)
         WRITE (NOUT,*) '                     Returned sample spectrum'
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     +'       Real  Imaginary       Real  Imaginary       Real  Imaginar
     +y'
         WRITE (NOUT,*)
     +'Lag    part     part  Lag    part     part  Lag    part     part'
         DO 40 I = 1, NG, 3
            WRITE (NOUT,99999) (II-1,XG(II),YG(II),II=I,MIN(I+2,NG))
   40    CONTINUE
      END IF
      STOP
*
99999 FORMAT (1X,I3,2F9.4,I4,2F9.4,I4,2F9.4)
      END
```

9.2 Program Data

```
G13CCF Example Program Data
   296 50 0
 -0.109  0.000  0.178  0.339  0.373  0.441  0.461  0.348
  0.127 -0.180 -0.588 -1.055 -1.421 -1.520 -1.302 -0.814
 -0.475 -0.193  0.088  0.435  0.771  0.866  0.875  0.891
  0.987  1.263  1.775  1.976  1.934  1.866  1.832  1.767
  1.608  1.265  0.790  0.360  0.115  0.088  0.331  0.645
  0.960  1.409  2.670  2.834  2.812  2.483  1.929  1.485
  1.214  1.239  1.608  1.905  2.023  1.815  0.535  0.122
  0.009  0.164  0.671  1.019  1.146  1.155  1.112  1.121
  1.223  1.257  1.157  0.913  0.620  0.255 -0.280 -1.080
 -1.551 -1.799 -1.825 -1.456 -0.944 -0.570 -0.431 -0.577
 -0.960 -1.616 -1.875 -1.891 -1.746 -1.474 -1.201 -0.927
 -0.524  0.040  0.788  0.943  0.930  1.006  1.137  1.198
  1.054  0.595 -0.080 -0.314 -0.288 -0.153 -0.109 -0.187
 -0.255 -0.299 -0.007  0.254  0.330  0.102 -0.423 -1.139
 -2.275 -2.594 -2.716 -2.510 -1.790 -1.346 -1.081 -0.910
 -0.876 -0.885 -0.800 -0.544 -0.416 -0.271  0.000  0.403
  0.841  1.285  1.607  1.746  1.683  1.485  0.993  0.648
  0.577  0.577  0.632  0.747  0.999  0.993  0.968  0.790
```

```
 0.399 -0.161 -0.553 -0.603 -0.424 -0.194 -0.049  0.060
 0.161  0.301  0.517  0.566  0.560  0.573  0.592  0.671
 0.933  1.337  1.460  1.353  0.772  0.218 -0.237 -0.714
-1.099 -1.269 -1.175 -0.676  0.033  0.556  0.643  0.484
 0.109 -0.310 -0.697 -1.047 -1.218 -1.183 -0.873 -0.336
 0.063  0.084  0.000  0.001  0.209  0.556  0.782  0.858
 0.918  0.862  0.416 -0.336 -0.959 -1.813 -2.378 -2.499
-2.473 -2.330 -2.053 -1.739 -1.261 -0.569 -0.137 -0.024
-0.050 -0.135 -0.276 -0.534 -0.871 -1.243 -1.439 -1.422
-1.175 -0.813 -0.634 -0.582 -0.625 -0.713 -0.848 -1.039
-1.346 -1.628 -1.619 -1.149 -0.488 -0.160 -0.007 -0.092
-0.620 -1.086 -1.525 -1.858 -2.029 -2.024 -1.961 -1.952
-1.794 -1.302 -1.030 -0.918 -0.798 -0.867 -1.047 -1.123
-0.876 -0.395  0.185  0.662  0.709  0.605  0.501  0.603
 0.943  1.223  1.249  0.824  0.102  0.025  0.382  0.922
 1.032  0.866  0.527  0.093 -0.458 -0.748 -0.947 -1.029
-0.928 -0.645 -0.424 -0.276 -0.158 -0.033  0.102  0.251
 0.280  0.000 -0.493 -0.759 -0.824 -0.740 -0.528 -0.204
 0.034  0.204  0.253  0.195  0.131  0.017 -0.182 -0.262
53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9
51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0
```

9.3 Program Results

```
G13CCF Example Program Results

                  Returned cross covariances

Lag     XY       YX   Lag     XY       YX   Lag     XY       YX
  0  -1.6700  -1.6700    1  -2.0581  -1.3606    2  -2.4859  -1.1383
  3  -2.8793  -0.9926    4  -3.1473  -0.9009    5  -3.2239  -0.8382
  6  -3.0929  -0.7804    7  -2.7974  -0.7074    8  -2.4145  -0.6147
  9  -2.0237  -0.5080   10  -1.6802  -0.4032   11  -1.4065  -0.3159
 12  -1.2049  -0.2554   13  -1.0655  -0.2250   14  -0.9726  -0.2238
 15  -0.9117  -0.2454   16  -0.8658  -0.2784   17  -0.8180  -0.3081
 18  -0.7563  -0.3257   19  -0.6750  -0.3315   20  -0.5754  -0.3321
 21  -0.4701  -0.3308   22  -0.3738  -0.3312   23  -0.3023  -0.3332
 24  -0.2665  -0.3384   25  -0.2645  -0.3506   26  -0.2847  -0.3727
 27  -0.3103  -0.3992   28  -0.3263  -0.4152   29  -0.3271  -0.4044
 30  -0.3119  -0.3621   31  -0.2837  -0.2919   32  -0.2568  -0.2054
 33  -0.2427  -0.1185   34  -0.2490  -0.0414   35  -0.2774   0.0227
 36  -0.3218   0.0697   37  -0.3705   0.1039   38  -0.4083   0.1356
 39  -0.4197   0.1805   40  -0.3920   0.2460   41  -0.3241   0.3319
 42  -0.2273   0.4325   43  -0.1216   0.5331   44  -0.0245   0.6199
 45   0.0528   0.6875   46   0.1074   0.7329   47   0.1448   0.7550
 48   0.1713   0.7544   49   0.1943   0.7349

                    Returned sample spectrum

       Real  Imaginary       Real  Imaginary       Real  Imaginary
Lag    part      part  Lag    part      part  Lag    part      part
  0  -6.5500   0.0000    1  -5.4267  -1.9842    2  -3.1323  -2.7307
  3  -1.2649  -2.3998    4  -0.2102  -1.7520    5   0.3411  -1.1903
```

```
  6   0.6063  -0.7420   7   0.6178  -0.3586   8   0.4391  -0.1008
  9   0.2422   0.0061  10   0.1233   0.0409  11   0.0574   0.0529
 12   0.0174   0.0452  13  -0.0008   0.0289  14  -0.0058   0.0161
 15  -0.0051   0.0084  16  -0.0027   0.0040  17  -0.0010   0.0015
 18  -0.0006   0.0006  19  -0.0005   0.0003  20  -0.0003   0.0003
 21  -0.0003   0.0004  22  -0.0003   0.0003  23  -0.0003   0.0002
 24  -0.0004   0.0001  25  -0.0004  -0.0000  26  -0.0003  -0.0001
 27  -0.0002  -0.0001  28  -0.0001   0.0001  29  -0.0002   0.0003
 30  -0.0003   0.0002  31  -0.0002   0.0001  32  -0.0001   0.0000
 33  -0.0000  -0.0000  34   0.0001  -0.0001  35   0.0001  -0.0002
 36   0.0001  -0.0001  37   0.0001  -0.0001  38   0.0001  -0.0001
 39   0.0001  -0.0001  40   0.0001   0.0000
```