NAG Fortran Library Routine Document

G13BCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13BCF calculates cross correlations between two time series.

2 Specification

```
SUBROUTINE G13BCF(X, Y, NXY, NL, S, RO, R, STAT, IFAIL)
INTEGER           NXY, NL, IFAIL
real              X(NXY), Y(NXY), S, RO, R(NL), STAT
```

3 Description

Given two series $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ the routine calculates the cross correlations between x_t and lagged values of y_t:

$$r_{xy}(l) = \frac{\sum_{t=1}^{n-l}(x_t - \bar{x})(y_{t+l} - \bar{y})}{n s_x s_y}, \quad l = 0, 1, \ldots, L,$$

where

$$\bar{x} = \frac{\sum_{t=1}^{n} x_t}{n}$$

$$s_x{}^2 = \frac{\sum_{t=1}^{n}(x_t - \bar{x})^2}{n}$$

and similarly for y.

The ratio of standard deviations s_y/s_x is also returned, and a portmanteau statistic is calculated:

$$\text{STAT} = n \sum_{i=1}^{L} r_{xy}(l)^2.$$

Provided n is large, L much less than n, and both x_t, y_t are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross correlations between the series are zero, STAT has a χ^2 distribution with L degrees of freedom. Values of STAT in the upper tail of this distribution provide evidence against the null hypothesis.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

5 Parameters

1: X(NXY) – ***real*** array *Input*

On entry: the n values of the x series.

2: Y(NXY) – ***real*** array *Input*

On entry: the n values of the y series.

3: NXY – INTEGER *Input*

On entry: the length of the time series, n.

Constraint: NXY ≥ 2.

4: NL – INTEGER *Input*

On entry: the maximum lag for calculating cross correlations, L.

Constraint: $1 \leq \text{NL} < \text{NXY}$.

5: S – ***real*** *Output*

On exit: the ratio of the standard deviation of the y series to the standard deviation of the x series, s_y/s_x.

6: R0 – ***real*** *Output*

On exit: the cross correlation between the x and y series at lag zero.

7: R(NL) – ***real*** array *Output*

On exit: the cross correlations between the x and y series at lags 1 to L, $r_{xy}(l)$ for $l = 1, 2, \ldots, L$.

8: STAT – ***real*** *Output*

On exit: the statistic for testing for absence of cross correlation.

9: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, NXY ≤ 1,
or NL < 1,
or NL $\geq$ NXY.

IFAIL = 2

One or both of the x and y series have zero variance and hence cross correlations cannot be calculated.

7 Accuracy

All computations are believed to be stable.

8 Further Comments

The time taken by the routine is approximately proportional to nL.

9 Example

The example program reads two time series of length 20. It calculates and prints the cross correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13BCF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NXYMAX, NLMAX
      PARAMETER        (NXYMAX=20,NLMAX=15)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             R0XY, R0YX, STATXY, STATYX, SXY, SYX
      INTEGER          I, IFAIL, NL, NXY
*     .. Local Arrays ..
      real             RXY(NLMAX), RYX(NLMAX), X(NXYMAX), Y(NXYMAX)
*     .. External Subroutines ..
      EXTERNAL         G13BCF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13BCF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
*     Read series length and number of lags
      READ (NIN,*) NXY, NL
      IF (NXY.GT.2 .AND. NXY.LE.NXYMAX .AND. NL.GT.0 .AND. NL.LE.NLMAX)
     +    THEN
*        Read series
         READ (NIN,*) (X(I),I=1,NXY)
         READ (NIN,*) (Y(I),I=1,NXY)
*        Call routine to calculate cross correlations between X and Y
         IFAIL = 0
*
         CALL G13BCF(X,Y,NXY,NL,SXY,R0XY,RXY,STATXY,IFAIL)
*
         IFAIL = 0
*
*        Call routine to calculate cross correlations between Y and X
         CALL G13BCF(Y,X,NXY,NL,SYX,R0YX,RYX,STATYX,IFAIL)
*
         WRITE (NOUT,*)
         WRITE (NOUT,*)
     +     '                              Between        Between'
         WRITE (NOUT,*)
     +     '                              X and Y        Y and X'
         WRITE (NOUT,*)
         WRITE (NOUT,99999) 'Standard deviation ratio', SXY, SYX
         WRITE (NOUT,*)
```

```
         WRITE (NOUT,*) 'Cross correlation at lag'
         WRITE (NOUT,99999) '                        0', R0XY, R0YX
         WRITE (NOUT,99998) (I,RXY(I),RYX(I),I=1,NL)
         WRITE (NOUT,*)
         WRITE (NOUT,99997) 'Test statistic          ', STATXY, STATYX
      END IF
      STOP
*
99999 FORMAT (1X,A,F10.4,F15.4)
99998 FORMAT (21X,I4,F10.4,F15.4)
99997 FORMAT (1X,A,F10.4,F15.4)
      END
```

9.2 Program Data

```
G13BCF Example Program Data
    20    15
  0.02  0.05  0.08  0.03 -0.05  0.11 -0.01 -0.08 -0.08 -0.11
 -0.18 -0.19 -0.09  0.03  0.10  0.15 -0.14  0.07  0.09  0.16
  3.18  3.21  3.26  3.25  3.08  3.01  3.06  3.17  3.12  3.04
  3.26  3.45  3.33  3.70  3.31  3.81  3.33  2.96  3.28  3.10
```

9.3 Program Results

```
 G13BCF Example Program Results

                            Between        Between
                            X and Y        Y and X

 Standard deviation ratio    2.0053         0.4987

 Cross correlation at lag
                         0    0.0568         0.0568
                         1    0.0438        -0.0151
                         2   -0.3762         0.3955
                         3   -0.4864         0.3417
                         4   -0.6294         0.5486
                         5   -0.3871         0.2291
                         6   -0.1690         0.3190
                         7   -0.0678         0.1980
                         8    0.0962         0.0438
                         9    0.0788        -0.1428
                        10    0.2910        -0.1376
                        11    0.0950        -0.0387
                        12    0.0547        -0.0380
                        13    0.1855        -0.1551
                        14    0.0243        -0.1536
                        15    0.0034        -0.0696

 Test statistic             22.1269        17.2917
```