

# NAG Fortran Library Routine Document

## G05YAF

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

### 1 Purpose

To generate multi-dimensional quasi-random sequences with a uniform probability distribution.

### 2 Specification

```
SUBROUTINE G05YAF(FCALL, SEQ, ISKIP, IDIM, QUASI, IREF, IFAIL)
INTEGER          ISKIP, IDIM, IREF(2000), IFAIL
real           QUASI(IDIM)
LOGICAL          FCALL
CHARACTER*1      SEQ
```

### 3 Description

Low discrepancy (quasi-random) sequences are used in numerical integration, simulation and optimisation. Like pseudo-random numbers they are uniformly distributed but they are not statistically independent, rather they are designed to give more even distribution in multidimensional space (uniformity). Therefore they are often more efficient than pseudo-random numbers in multidimensional Monte Carlo methods.

G05YAF generates a set of points  $x^1, x^2, \dots, x^N$  with high uniformity in the  $s$ -dimensional unit cube  $I^s = [0, 1]^s$ . One measure of the uniformity is the discrepancy which is defined as follows:

Given a set of points  $x^1, x^2, \dots, x^N \in I^s$  and a subset  $G \subset I^s$ , define the counting function  $S_N(G)$  as the number of points  $x^i \in G$ . For each  $x = (x_1, x_2, \dots, x_s) \in I^s$ , let  $G_x$  be the rectangular  $s$ -dimensional region

$$G_x = [0, x_1) \times [0, x_2) \times \dots \times [0, x_s)$$

with volume  $x_1, x_2, \dots, x_s$ . Then the discrepancy of the points  $x^1, x^2, \dots, x^N$  is

$$D_N^*(x^1, x^2, \dots, x^N) = \sup_{x \in I^s} |S_N(G_x) - Nx_1, x_2, \dots, x_s|.$$

The discrepancy of the first  $N$  terms of such a sequence has the form

$$D_N^*(x^1, x^2, \dots, x^N) \leq C_s (\log N)^s + O((\log N)^{s-1}) \quad \text{for all } N \geq 2.$$

The principal aim in the construction of low-discrepancy sequences is to find sequences of points in  $I^s$  with a bound of this form where the constant  $C_s$  is as small as possible.

G05YAF generates the low-discrepancy sequences proposed by Sobol, Faure and Niederreiter. Here, both the Sobol and Niederreiter sequences are implemented in binary arithmetic and make use of the (extended F77) bitwise exclusive-or operation, where possible (see Users' Note).

### 4 References

Fox B L (1986) Implementation and Relative Efficiency of Quasirandom Sequence Generators *ACM Trans. Math. Software* **12** (4) 362–376

Brately P and Fox B L (1988) Algorithm 659: Implementing Sobol's Quasirandom Sequence Generator *ACM Trans. Math. Software* **14** (1) 88–100

## 5 Parameters

- 1: FCALL – LOGICAL *Input*  
*On entry:* if FCALL = .TRUE. then first call for initialisation, and there is no output via array QUASI.  
 If FCALL = .FALSE. then the sequence has been initialised by a prior call to G05YAF with FCALL = .TRUE.. Random numbers are output via array QUASI.
  
- 2: SEQ – CHARACTER\*1 *Input*  
*On entry:* the type of sequence to generate.  
     If SEQ = 'S', a Sobol sequence.  
     If SEQ = 'N', a Niederreiter sequence.  
     If SEQ = 'F', a Faure sequence.  
*Constraint:* SEQ = 'S', 'N' or 'F'.
  
- 3: ISKIP – INTEGER *Input*  
*On entry:* the number of terms in the sequence to skip on initialisation. SKIP is not referenced when SEQ = 'F'.  
 When ISKIP = 0, all the terms of the sequence are generated. If ISKIP =  $k$ , the first  $k$  terms of the sequence are ignored and the first term of the sequence now corresponds to the  $k$ th term of the sequence when ISKIP = 0.  
*Constraint:* ISKIP  $\geq 0$ , if SEQ = 'N' or SEQ = 'S' and FCALL = .TRUE..
  
- 4: IDIM – INTEGER *Input*  
*On entry:* the number of dimensions required.  
*Constraint:*  $1 \leq \text{IDIM} \leq 40$ .
  
- 5: QUASI(IDIM) – *real* array *Output*  
*On exit:* the random numbers. If FCALL = .FALSE., QUASI( $k$ ) contains the random number for the  $k$ th dimension.
  
- 6: IREF(2000) – INTEGER array *Workspace*  
*On entry/on exit:* workspace used to contain information between calls to the routine. The contents of this array should not be changed.
  
- 7: IFAIL – INTEGER *Input/Output*  
*On entry:* IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.  
*On exit:* IFAIL = 0 unless the routine detects an error (see Section 6).  
 For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL  $\neq 0$  on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

## 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, ISKIP < 0, if SEQ = 'N' or SEQ = 'S' and FCALL = .TRUE.,  
or IDIM < 1,  
or IDIM > 40,  
or SEQ ≠ 'F' or SEQ ≠ 'N' or SEQ ≠ 'S'.

IFAIL = 2

On entry, the array IREF has not been correctly initialised.

IFAIL = 3

The value of ISKIP is too large.

IFAIL = 4

There have been too many calls in the sequence.

IFAIL = 5

An internal error has occurred within the routine. Please contact NAG.

## 7 Accuracy

Not applicable.

## 8 Further Comments

The maximum length of the generated sequences is  $2^{29} - 1$ , this should be adequate for practical purposes. In the case of the Niederreiter generator the routine jumps to the appropriate starting point, while for the Sobol generator it simply steps through the sequence. In consequence the Sobol generator with large values of ISKIP will take a significant amount of time.

## 9 Example

This example program approximates the integral

$$\int_0^1 \dots \int_0^1 \prod_{i=1}^s |4x_i - 2| dx_1, dx_2, \dots, dx_s = 1,$$

where  $s$  is the number of dimensions.

### 9.1 Program Text

**Note:** the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      G05YAF Example Program Text
*      Mark 20 Release. NAG Copyright 2001.
*      .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*      .. Local Scalars ..
      real              SUM, VSBL
      INTEGER          I, IDIM, IFAIL, NTIMES, SKIP
```

```

      CHARACTER          SEQ
*    .. Local Arrays ..
      real              QUASI(15)
      INTEGER           IREF(2000)
*    .. External Functions ..
      real              FUN
      EXTERNAL          FUN
*    .. External Subroutines ..
      EXTERNAL          G05YAF
*    .. Intrinsic Functions ..
      INTRINSIC         real
*    .. Executable Statements ..
      WRITE (NOUT,*) 'G05YAF Example Program Results'
      IDIM = 15
      NTIMES = 10000
      SEQ = 'S'
      IF (SEQ.EQ.'N' .OR. SEQ.EQ.'n') THEN
        SKIP = 1000
      ELSE
        SKIP = 0
      END IF
      IFAIL = 0
*
      CALL G05YAF(.TRUE.,SEQ,SKIP,IDIM,QUASI,IREF,IFAIL)
*
      SUM = 0.0e0
      DO 20 I = 1, NTIMES
*
        CALL G05YAF(.FALSE.,SEQ,SKIP,IDIM,QUASI,IREF,IFAIL)
*
        SUM = SUM + FUN(IDIM,QUASI)
20  CONTINUE
      VSBL = SUM/real(NTIMES)
      WRITE (NOUT,*)
      WRITE (NOUT,99999) 'Value of integral = ', VSBL
      STOP
*
99999 FORMAT (A,F8.3)
      END
*
      real FUNCTION FUN(IDIM,X)
*    .. Scalar Arguments ..
      INTEGER           IDIM
*    .. Array Arguments ..
      real              X(IDIM)
*    .. Local Scalars ..
      real              TMP
      INTEGER           J
*    .. Intrinsic Functions ..
      INTRINSIC         ABS
*    .. Executable Statements ..
      TMP = 1.0e0
      DO 20 J = 1, IDIM
        TMP = TMP*ABS(4.0e0*X(J)-2.0e0)
20  CONTINUE
      FUN = TMP
      RETURN
      END

```

## 9.2 Program Data

None.

## 9.3 Program Results

G05YAF Example Program Results

Value of integral =     1.020