NAG Fortran Library Routine Document

G05LEF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G05LEF generates a vector of pseudo-random numbers taken from a beta distribution with a and b.

2 Specification

```
SUBROUTINE G05LEF(A, B, N, X, IGEN, ISEED, IFAIL)
INTEGER           N, IGEN, ISEED(4), IFAIL
real              A, B, X(*)
```

3 Description

The beta distribution has PDF (probability density function)

$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\, x^{a-1}(1-x)^{b-1} \quad \text{if } 0 \leq x \leq 1;\ a, b > 0.0,$$

$$f(x) = 0 \quad \text{otherwise.}$$

One of four algorithms is used to generate the variates depending on the values of a and b. Let α be the maximum and β be the minimum of a and b. Then the algorithms are as follows:

If $\alpha < 0.5$, Jhnk's algorithm is used, see for example Dagpunar (1988). This generates the beta variate as $u_1^{1/a} / \left(u_1^{1/a} + u_2^{1/b} \right)$, where u_1 and u_2 are uniformly distributed random variates;

If $\beta > 1$, the algorithm BB given by Cheng (1978) is used. This involves the generation of an observation from a beta distribution of the second kind by the envelope rejection method using a log-logistic target distribution and then transforming it to a beta variate;

If $\alpha > 1$ and $\beta < 1$, the switching algorithm given by Atkinson (1979) is used. The two target distributions used are $f_1(x) = \beta x^{\beta}$ and $f_2(x) = \alpha(1-x)^{\beta-1}$, along with the approximation to the switching parameter of $t = (1-\beta)/(\alpha+1-\beta)$;

In all other cases, Cheng's BC algorithm (see Cheng (1978)) is used with modifications suggested by Dagpunar (1988). This algorithm is similar to BB, used when $\beta > 1$, but is tuned for small values of a and b.

One of the initialisation routines G05KBF (for a repeatable sequence if computed sequentially) or G05KCF (for a non-repeatable sequence) must be called prior to the first call to G05LEF.

4 References

Atkinson A C (1979) A family of switching algorithms for the computer generation of beta random variates *Biometrika* **66** 141–5

Cheng R C H (1978) Generating beta variates with nonintegral shape parameters *Comm. ACM* **21** 317–322

Dagpunar J (1988) *Principles of Random Variate Generation* Oxford University Press

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Parameters

1: A – ***real*** *Input*

On entry: the parameter, a, of the beta distribution.

Constraint: A > 0.0.

2: B – ***real*** *Input*

On entry: the parameter, b, of the beta distribution.

Constraint: B > 0.0.

3: N – INTEGER *Input*

On entry: the number, n, of pseudo-random numbers to be generated.

Constraint: N ≥ 0.

4: X(*) – ***real*** array *Output*

Note: the dimension of the array X must be at least $\max(1, \mathrm{N})$.

On exit: the n pseudo-random numbers from the specified beta distribution.

5: IGEN – INTEGER *Input*

On entry: must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialisation by a prior call to one of the routines G05KBF or G05KCF.

6: ISEED(4) – INTEGER array *Input/Output*

On entry: contains values which define the current state of the selected generator.

On exit: contains updated values defining the new state of the selected generator.

7: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, A ≤ 0.0.

IFAIL $= 2$

On entry, B ≤ 0.0.

IFAIL $= 3$

On entry, N < 0.

7 Accuracy

Not applicable.

8 Further Comments

To generate an observation, y, from the beta distribution of the second kind from an observation, x, generated by G05LEF the transformation, $y = x/(1-x)$, may be used.

9 Example

The example program prints a set of five pseudo-random numbers from a beta distribution with parameters $a = 2.0$ and $b = 2.0$, generated by a single call to G05LEF, after initialisation by G05KBF.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G05LEF Example Program Text
*     Mark 20 Release. NAG Copyright 2001.
*     .. Parameters ..
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
      INTEGER          N
      PARAMETER        (N=5)
*     .. Local Scalars ..
      INTEGER          IFAIL, IGEN, J
*     .. Local Arrays ..
      real             X(N)
      INTEGER          ISEED(4)
*     .. External Subroutines ..
      EXTERNAL         G05KBF, G05LEF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G05LEF Example Program Results'
      WRITE (NOUT,*)
*     Initialise the seed to a repeatable sequence
      ISEED(1) = 1762543
      ISEED(2) = 9324783
      ISEED(3) = 42344
      ISEED(4) = 742355
*     IGEN identifies the stream.
      IGEN = 1
      CALL G05KBF(IGEN,ISEED)
*
      IFAIL = 0
      WRITE (NOUT,*) 'Beta Dist --- A=2.0, B=2.0'
*
      CALL G05LEF(2.0e0,2.0e0,N,X,IGEN,ISEED,IFAIL)
*
      WRITE (NOUT,99999) (X(J),J=1,N)
      STOP
*
99999 FORMAT (1X,F10.4)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 G05LEF Example Program Results

 Beta Dist --- A=2.0, B=2.0
     0.4334
     0.8888
     0.5604
     0.3799
     0.5064
```