NAG Fortran Library Routine Document

G05HNF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G05HNF generates a given number of terms of an exponential GARCH(p,q) process (see Engle and Ng (1993)).

2 Specification

```
 SUBROUTINE G05HNF(DIST, NUM, IP, IQ, THETA, DF, HT, ET, FCALL, RVEC,
1                  IGEN, ISEED, RWSAV, IFAIL)
 INTEGER           NUM, IP, IQ, IGEN, ISEED(4), IFAIL
 real              THETA(2*IQ+IP+1), DF, HT(NUM), ET(NUM), RVEC(40),
1                  RWSAV(9)
 LOGICAL           FCALL
 CHARACTER*1       DIST
```

3 Description

An exponential GARCH(p,q) process is represented by:

$$ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i z_{t-i} + \sum_{i=1}^{q} \phi_i(|z_{t-i}| - E[|z_{t-i}|]) + \sum_{j=1}^{p} \beta_j ln(h_{t-j}), \quad t = 1, \ldots, T,$$

where $z_t = \frac{\epsilon_t}{\sqrt{h_t}}$, $E[|z_{t-i}|]$ denotes the expected value of $|z_{t-i}|$, and $\epsilon_t|\psi_{t-1} = N(0, h_t)$ or $\epsilon_t|\psi_{t-1} = S_t(df, h_t)$. Here S_t is a standardised Student's t-distribution with df degrees of freedom and variance h_t, T is the number of observations in the sequence, ϵ_t is the *observed* value of the GARCH(p,q) process at time t, h_t is the conditional variance at time t, and ψ_t the set of all information up to time t.

One of the initialisation routines G05KBF (for a repeatable sequence if computed sequentially) or G05KCF (for a non-repeatable sequence) must be called prior to the first call to G05HNF.

4 References

Engle R (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation *Econometrica* **50** 987–1008

Bollerslev T (1986) Generalised autoregressive conditional heteroskedasticity *Journal of Econometrics* **31** 307–327

Engle R and Ng V (1993) Measuring and Testing the Impact of News on Volatility *Journal of Finance* **48** 1749–1777

Hamilton J (1994) *Time Series Analysis* Princeton University Press

Glosten L, Jagannathan R and Runkle D (1993) Relationship between the expected value and the volatility of nominal excess return on stocks *Journal of Finance* **48** 1779–1801

5 Parameters

1: DIST – CHARACTER*1 *Input*

On entry: the type of distribution to use for ϵ_t.

DIST = 'N'

Then a Normal distribution is used.

DIST = 'T'

Then a Student's t-distribution is used.

Constraint: DIST = 'N' or 'T'.

2: NUM – INTEGER *Input*

On entry: the number of terms in the sequence, T.

Constraint: NUM > 0.

3: IP – INTEGER *Input*

On entry: the number of coefficients, β_i, for $i = 1, \ldots, p$.

Constraints:

$2 \times \mathrm{IQ} + \mathrm{IP} + 1 \leq 20$,
IP ≥ 0.

4: IQ – INTEGER *Input*

On entry: the number of coefficients, α_i, for $i = 1, \ldots, q$.

Constraints:

$2 \times \mathrm{IQ} + \mathrm{IP} + 1 \leq 20$,
IQ ≥ 1.

5: THETA(2*IQ+IP+1) – ***real*** array *Input*

On entry: the initial parameter estimates for the vector θ. The first element must contain the coefficient α_o and the next IQ elements must contain the autoregressive coefficients α_i, for $i = 1, \ldots, q$. The next IQ elements must contain the coefficients ϕ_i, for $i = 1, \ldots, q$. The next IP elements must contain the moving average coefficients β_j, for $j = 1, \ldots, p$.

6: DF – ***real*** *Input*

On entry: the number of degrees of freedom for the Student's t-distribution. It is not referenced if DIST = 'N'.

Constraint: DF > 2.

7: HT(NUM) – ***real*** array *Output*

On exit: the conditional variances h_t, for $t = 1, \ldots, T$ for the GARCH(p, q) sequence.

8: ET(NUM) – ***real*** array *Output*

On exit: the observations ϵ_t, for $t = 1, \ldots, T$ for the GARCH(p, q) sequence.

9: FCALL – LOGICAL *Input*

On entry: if FCALL = .TRUE., a new sequence is to be generated, otherwise a given sequence is to be continued using the information in RVEC.

10: RVEC(40) – ***real*** array *Input/Output*

On entry: the array contains information required to continue a sequence if FCALL = .FALSE..

On exit: contains information that can be used in a subsequent call of G05HNF, with FCALL = .FALSE..

11: IGEN – INTEGER *Input*

On entry: must contain the identification number for the generator to be used to return a pseudo-random number and should remain unchanged following initialisation by a prior call to one of the routines G05KBF or G05KCF.

12: ISEED(4) – INTEGER array *Input/Output*

On entry: contains values which define the current state of the selected generator.

On exit: contains updated values defining the new state of the selected generator.

13: RWSAV(9) – ***real*** array *Workspace*

14: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, IP < 0,
or IQ < 1,
or DF ≤ 2,
or NUM ≤ 0,
or DIST $\neq$ 'N' and DIST $\neq$ 'T',
or $2 \times \mathrm{IQ} + \mathrm{IP} + 1 > 20$.

IFAIL = 2

Invalid sequence generated, use different parameters.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

See Section 9 of the document for G13FGF.