NAG Fortran Library Routine Document

G01HAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01HAF returns the lower tail probability for the bivariate Normal distribution, via the routine name.

2 Specification

```
real FUNCTION G01HAF(X, Y, RHO, IFAIL)
INTEGER              IFAIL
real                 X, Y, RHO
```

3 Description

For the two random variables (X, Y) following a bivariate Normal distribution with

$$E[X] = 0, \quad E[Y] = 0, \quad E[X^2] = 1, \quad E[Y^2] = 1 \quad \text{and} \quad E[XY] = \rho,$$

the lower tail probability is defined by:

$$P(X \leq x, Y \leq y : \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^{y} \int_{-\infty}^{x} \exp\left(-\frac{(X^2 - 2\rho XY + Y^2)}{2(1-\rho^2)}\right) dX\, dY.$$

For a more detailed description of the bivariate Normal distribution and its properties see Abramowitz and Stegun (1972) and Kendall and Stuart (1969) The method used is described by Divgi (1979).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Divgi D R (1979) Calculation of univariate and bivariate normal probability functions *Ann. Statist.* **7** 903–910

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

5 Parameters

1: X – ***real*** *Input*

On entry: the first argument for which the bivariate Normal distribution function is to be evaluated, x.

2: Y – ***real*** *Input*

On entry: the second argument for which the bivariate Normal distribution function is to be evaluated, y.

3: RHO – ***real*** *Input*

On entry: the correlation coefficient, ρ.

Constraint: $-1.0 \leq \mathrm{RHO} \leq 1.0$.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, RHO < -1.0,
or RHO > 1.0.

If on exit IFAIL $= 1$ then G01HAF returns zero.

7 Accuracy

Accuracy is discussed in Divgi (1979). A higher order polynomial approximation to Mills ratio is used in G01HAF, (15 terms) than is given in Divgi (1979). This will give higher absolute accuracy of about 10 digits on machines of sufficiently high precision.

8 Further Comments

The probabilities for the univariate normal distribution can be computed using S15ABF and S15ABF.

9 Example

Values of x and y for a bivariate Normal distribution are read along with the value of ρ. The lower tail probabilities are computed.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01HAF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             PROB, RHO, X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             G01HAF
      EXTERNAL         G01HAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01HAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X       Y      RHO    PROB'
      WRITE (NOUT,*)
```

```
   20 READ (NIN,*,END=40) X, Y, RHO
      IFAIL = 0
*
      PROB = G01HAF(X,Y,RHO,IFAIL)
*
      WRITE (NOUT,99999) X, Y, RHO, PROB
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,3F8.3,F8.4)
      END
```

9.2 Program Data

```
G01HAF Example Program Data
  1.7  23.1   0.0         :X Y RHO
  0.0   0.0   0.1         :X Y RHO
  3.3  11.1   0.54        :X Y RHO
  9.1   9.1   0.17        :X Y RHO
```

9.3 Program Results

```
 G01HAF Example Program Results

      X       Y      RHO    PROB

    1.700  23.100   0.000  0.9554
    0.000   0.000   0.100  0.2659
    3.300  11.100   0.540  0.9995
    9.100   9.100   0.170  1.0000
```