NAG Fortran Library Routine Document

G01GBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01GBF returns the lower tail probability for the non-central Student's t-distribution, via the routine name.

2 Specification

```
real FUNCTION G01GBF(T, DF, DELTA, TOL, MAXIT, IFAIL)
INTEGER             MAXIT, IFAIL
real                T, DF, DELTA, TOL
```

3 Description

The lower tail probability of the non-central Student's t-distribution with ν degrees of freedom and non-centrality parameter δ, $P(\mathrm{T} \leq t : \nu; \delta)$, is defined by

$$P(T \leq t : \nu; \delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma(\frac{1}{2}\nu) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}.$$

The probability is computed in one of two ways.

(i) When $t = 0.0$, the relationship to the normal is used:

$$P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2} du.$$

(ii) Otherwise the series expansion described in equation 9 of Amos (1964) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 References

Amos D E (1964) Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

5 Parameters

1: T – ***real*** *Input*

On entry: the deviate from the Student's t-distribution with ν degrees of freedom, t.

2: DF – ***real*** *Input*

On entry: the degrees of freedom of the Student's t-distribution, ν.

Constraint: DF ≥ 1.0.

3: DELTA – ***real*** *Input*

On entry: the non-centrality parameter of the Students t-distribution, δ.

4: TOL – ***real*** *Input*

On entry: the absolute accuracy required by the user in the results. If G01GBF is entered with TOL greater than or equal to 1.0 or less than $10\times$ ***machine precision*** (see X02AJF), then the value of $10\times$ ***machine precision*** is used instead.

5: MAXIT – INTEGER *Input*

On entry: the maximum number of terms that are used in each of the summations.

Suggested value: 100. See Section 8 for further comments.

Constraint: MAXIT ≥ 1.

6: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If on exit IFAIL $= 0$, then G01GBF returns 0.0.

IFAIL $= 1$

On entry, DF < 1.0.

IFAIL $= 2$

On entry, MAXIT < 1.

IFAIL $= 3$

One of the series has failed to converge. Reconsider the requested tolerance and/or maximum number of iterations.

IFAIL $= 4$

The probability is too small to calculate accurately.

7 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than TOL. There may also be some loss of accuracy due to calculation of gamma functions.

8 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/(t^2+\nu)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two-tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2,\ \lambda = \delta^2,\ \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to G01GDF.

Note that this routine only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

9 Example

Values from, and degrees of freedom for, and non-centrality parameter of the non-central Student's t-distributions are read, the lower tail probabilities calculated and all these values printed until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01GBF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DELTA, DF, PROB, T, TOL
      INTEGER          IFAIL, MAXIT
*     .. External Functions ..
      real             G01GBF
      EXTERNAL         G01GBF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01GBF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     T       DF    DELTA    PROB'
      WRITE (NOUT,*)
      TOL = 0.5e-5
      MAXIT = 50
   20 READ (NIN,*,END=40) T, DF, DELTA
      IFAIL = 0
*
      PROB = G01GBF(T,DF,DELTA,TOL,MAXIT,IFAIL)
*
      WRITE (NOUT,99999) T, DF, DELTA, PROB
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,3F8.3,F8.4)
      END
```

9.2 Program Data

```
G01GBF Example Program Data
 -1.528  20.0  2.0           :T DF DELTA
 -0.188   7.5  1.0           :T DF DELTA
  1.138  45.0  0.0           :T DF DELTA
```

9.3 Program Results

```
 G01GBF Example Program Results

      T        DF    DELTA    PROB

   -1.528  20.000   2.000  0.0003
   -0.188   7.500   1.000  0.1189
    1.138  45.000   0.000  0.8694
```