NAG Fortran Library Routine Document

G01FCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01FCF returns the deviate associated with the given lower tail probability of the χ^2 distribution with real degrees of freedom, via the routine name.

2 Specification

```
real FUNCTION G01FCF(P, DF, IFAIL)
INTEGER              IFAIL
real                 P, DF
```

3 Description

The deviate, x_p, associated with the lower tail probability p of the χ^2 distribution with ν degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{v/2-1} dX, \quad 0 \leq x_p < \infty; \ \nu > 0.$$

The required x_p is found by using the relationship between a χ^2 distribution and a gamma distribution, i.e., a χ^2 distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

For very large values of ν, greater than 10^5, Wilson and Hilferty's Normal approximation to the χ^2 is used; see Kendall and Stuart (1969).

4 References

Best D J and Roberts D E (1975) Algorithm AS91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Volume 1)* (3rd Edition) Griffin

5 Parameters

1: P – ***real*** *Input*

On entry: the probability, p, from the required χ^2 distribution.

Constraint: $0.0 \leq \mathrm{P} < 1.0$.

2: DF – ***real*** *Input*

On entry: the degrees of freedom, ν, of the χ^2 distribution.

Constraint: $\mathrm{DF} > 0$.

3: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL $= 1$, 2, 3 or 5 on exit, then G01FCF returns 0.0.

IFAIL $= 1$

On entry, P < 0.0,
or P ≥ 1.0.

IFAIL $= 2$

On entry, DF ≤ 0.

IFAIL $= 3$

P is too close to 0 or 1 for the result to be calculated.

IFAIL $= 4$

The solution has failed to converge. The result should be a reasonable approximation.

IFAIL $= 5$

The series used to calculate the gamma probabilities has failed to converge. This is an unlikely error exit.

7 Accuracy

The results should be accurate to 5 significant digits for most parameter values. Some accuracy is lost for p close to 0.0.

8 Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to G01FFF made.

9 Example

Lower tail probabilities are read for several χ^2 distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01FCF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DF, P, X
      INTEGER          IFAIL
*     .. External Functions ..
      real             G01FCF
      EXTERNAL         G01FCF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01FCF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     P       DF      X'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) P, DF
      IFAIL = -1
*
      X = G01FCF(P,DF,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) P, DF, X
      ELSE
         WRITE (NOUT,99999) P, DF, X, ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,3F8.3,A,I1)
      END
```

9.2 Program Data

```
G01FCF Example Program Data
0.0100 20.0                 :P DF
0.4279 7.5                  :P DF
0.8694 45.0                 :P DF
```

9.3 Program Results

```
 G01FCF Example Program Results

      P       DF      X

    0.010  20.000   8.260
    0.428   7.500   6.200
    0.869  45.000  55.759
```