NAG Fortran Library Routine Document

G01EMF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EMF returns the probability associated with the lower tail of the distribution of the Studentized range statistic, via the routine name.

2 Specification

```
real FUNCTION G01EMF(Q, V, IR, IFAIL)
INTEGER              IR, IFAIL
real                 Q, V
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as:

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e}$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{largest}$ and $\bar{T}_{smallest}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{largest} - \bar{T}_{smallest}}{\sqrt{MS_{error}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - qx)]^{r-1} \, dy \right\} dx$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2) 2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) \, dt.$$

The above two-dimensional integral is evaluated using D01DAF with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - q)]^{r-1} \, dy.$$

This integral is evaluated using D01AMF.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32 (2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw-Hill

5 Parameters

1: Q – ***real*** *Input*

On entry: the Studentized range statistic, q.

Constraint: Q > 0.0.

2: V – ***real*** *Input*

On entry: the number of degrees of freedom for the experimental error, v.

Constraint: V ≥ 1.0.

3: IR – INTEGER *Input*

On entry: the number of groups, r.

Constraint: IR ≥ 2.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

If on exit IFAIL $= 1$, then G01EMF returns to 0.0.

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, Q ≤ 0.0,
or V < 1.0,
or IR < 2.

IFAIL $= 2$

There is some doubt as to whether full accuracy has been achieved.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless IFAIL = 2. When IFAIL = 2 it is usual that the returned value will be a good estimate of the true value.

8 Further Comments

None.

9 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q, ν and r.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01EMF Example Program Text
*     Mark 15 Release. NAG Copyright 1991.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             Q, V, VALP
      INTEGER          I, IFAIL, IR
*     .. External Functions ..
      real             G01EMF
      EXTERNAL         G01EMF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01EMF Example Program Results '
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '  Q       V    IR    Quantile '
      WRITE (NOUT,*)
      DO 20 I = 1, 3
         READ (NIN,*) Q, V, IR
         IFAIL = -1
*
         VALP = G01EMF(Q,V,IR,IFAIL)
*
         IF (IFAIL.EQ.0) THEN
            WRITE (NOUT,99999) Q, V, IR, VALP
         END IF
   20 CONTINUE
      STOP
*
99999 FORMAT (1X,F7.4,2X,F4.1,1X,I3,1X,F10.4)
      END
```

9.2 Program Data

```
G01EMF Example Program Data
4.6543  10.0   5
2.8099  60.0  12
4.2636   5.0   4
```

9.3 Program Results

```
 G01EMF Example Program Results

    Q        V     IR    Quantile

  4.6543  10.0   5      0.9500
  2.8099  60.0  12      0.3000
  4.2636   5.0   4      0.9000
```