NAG Fortran Library Routine Document

G01EDF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EDF returns the probability for the lower or upper tail of the F or variance-ratio distribution with real degrees of freedom, via the routine name.

2 Specification

```
real FUNCTION G01EDF(TAIL, F, DF1, DF2, IFAIL)
INTEGER              IFAIL
real                 F, DF1, DF2
CHARACTER*1          TAIL
```

3 Description

The lower tail probability for the F, or variance-ratio distribution, with ν_1 and ν_2 degrees of freedom, $P(\mathrm{F} \leq f : \nu_1, \nu_2)$, is defined by:

$$P(F \leq f : \nu_1, \nu_2) = \frac{\nu_1^{\nu_1/2}\nu_2^{\nu_2/2}\Gamma((\nu_1+\nu_2)/2)}{\Gamma(\nu_1/2)\Gamma(\nu_2/2)} \int_0^f F^{(\nu_1-2)/2}(\nu_1 F + \nu_2)^{-(\nu_1+\nu_2)/2} dF,$$

for $\nu_1,\ \nu_2 > 0,\ f \geq 0$.

The probability is computed by means of a transformation to a beta distribution, $P_\beta(B \leq \beta : a, b)$:

$$P(F \leq f : \nu_1, \nu_2) = P_\beta\left(B \leq \frac{\nu_1 f}{\nu_1 f + \nu_2} : \nu_1/2, \nu_2/2\right)$$

and using a call to G01EEF.

For very large values of both ν_1 and ν_2, greater than 10^5, a normal approximation is used. If only one of ν_1 or ν_2 is greater than 10^5 then a χ^2 approximation is used, see Abramowitz and Stegun (1972).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Parameters

1: TAIL – CHARACTER*1 *Input*

On entry: indicates whether an upper or lower tail probability is required.

If TAIL = 'L', then the lower tail probability is returned, i.e., $P(\mathrm{F} \leq f : \nu_1, \nu_2)$.

If TAIL = 'U', then the upper tail probability is returned, i.e., $P(\mathrm{F} \geq f : \nu_1, \nu_2)$.

Constraint: TAIL = 'L' or 'U'.

2: F – ***real*** *Input*

On entry: the value of the F variate, f.

Constraint: F ≥ 0.

3: DF1 – ***real*** *Input*

On entry: the degrees of freedom of the numerator variance, ν_1.

Constraint: DF1 > 0.0.

4: DF2 – ***real*** *Input*

On entry: the degrees of freedom of the denominator variance, ν_2.

Constraint: DF2 > 0.0.

5: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL $\neq 0$ on exit, the recommended value is -1. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL $= 1$, 2 or 3 on exit, then G01EDF returns 0.0.

IFAIL $= 1$

On entry, TAIL $\neq$ 'L' or 'U'.

IFAIL $= 2$

On entry, F < 0.0.

IFAIL $= 3$

On entry, DF1 ≤ 0.0,
or DF2 ≤ 0.0.

IFAIL $= 4$

F is too far out into the tails for the probability to be evaluated exactly. The result tends to approach 1.0 if x is large, or 0.0 if x is small. The result returned is a good approximation to the required solution.

7 Accuracy

The result should be accurate to 5 significant digits.

8 Further Comments

For higher accuracy G01EEF can be used along with the transformations given in Section 3.

9 Example

Values from, and degrees of freedom for F-distributions are read, the lower-tail probabilities computed, and all these values printed, until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01EDF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DF1, DF2, F, PROB
      INTEGER          IFAIL
*     .. External Functions ..
      real             G01EDF
      EXTERNAL         G01EDF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01EDF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   F      DF1     DF2    PROB'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) F, DF1, DF2
      IFAIL = -1
*
      PROB = G01EDF('Lower',F,DF1,DF2,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) F, DF1, DF2, PROB
      ELSE
         WRITE (NOUT,99999) F, DF1, DF2, PROB, ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,F6.3,2F8.3,F8.4,A,I1)
      END
```

9.2 Program Data

```
G01EDF Example Program Data
  5.5   1.5   25.5
 39.9   1.0    1.0
 2.5   20.25   1.0
```

9.3 Program Results

```
 G01EDF Example Program Results

    F      DF1     DF2    PROB

  5.500   1.500  25.500  0.9837
 39.900   1.000   1.000  0.9000
  2.500  20.250   1.000  0.5342
```