NAG Fortran Library Routine Document

G01ECF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01ECF returns the lower or upper tail probability for the χ^2 distribution with real degrees of freedom, via the routine name.

2 Specification

```
real FUNCTION G01ECF(TAIL, X, DF, IFAIL)
INTEGER              IFAIL
real                 X, DF
CHARACTER*1          TAIL
```

3 Description

The lower tail probability for the χ^2 distribution with ν degrees of freedom, $P(\mathrm{X} \leq x : \nu)$ is defined by:

$$P(X \leq x : \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{0.0}^{x} X^{\nu/2-1} e^{-X/2} dX, \quad x \geq 0, \ \nu > 0.$$

To calculate $P(\mathrm{X} \leq x : \nu)$ a transformation of a gamma distribution is employed, i.e., a χ^2 distribution with ν degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter $\nu/2$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

5 Parameters

1: TAIL – CHARACTER*1 *Input*

On entry: indicates whether the upper or lower tail probability is required.

If TAIL = 'L', the lower tail probability is returned, i.e., $P(\mathrm{X} \leq x : \nu)$.

If TAIL = 'U', the upper tail probability is returned, i.e., $P(\mathrm{X} \geq x : \nu)$.

Constraint: TAIL = 'L' or 'U'.

2: X – ***real*** *Input*

On entry: the value of the χ^2 variate, x, with ν degrees of freedom.

Constraint: X $\geq$ 0.0.

3: DF – ***real*** *Input*

On entry: the degrees of freedom, ν, of the χ^2 distribution.

Constraint: DF $>$ 0.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if IFAIL ≠ 0 on exit, the recommended value is −1. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL = 1, 2 or 3 on exit, then G01ECF returns 0.0.

IFAIL = 1

On entry, TAIL ≠ 'L' or 'U'.

IFAIL = 2

On entry, X < 0.0.

IFAIL = 3

On entry, DF ≤ 0.0.

IFAIL = 4

The solution has failed to converge while calculating the Gamma variate. The result returned should represent an approximation to the solution.

7 Accuracy

A relative accuracy of 5 significant figures is obtained in most cases.

8 Further Comments

For higher accuracy the transformation described in Section 3 may be used with a direct call to S14BAF.

9 Example

Values from various χ^2 distributions are read, the lower-tail probabilities calculated, and all these values printed out, until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01ECF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
```

```
      real              DF, PROB, X
      INTEGER           IFAIL
*     .. External Functions ..
      real              G01ECF
      EXTERNAL          G01ECF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01ECF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   X       DF    PROB'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X, DF
      IFAIL = -1
*
      PROB = G01ECF('Lower',X,DF,IFAIL)
*
      IF (IFAIL.EQ.0) THEN
         WRITE (NOUT,99999) X, DF, PROB
      ELSE
         WRITE (NOUT,99999) X, DF, PROB, ' NOTE: IFAIL = ', IFAIL
      END IF
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,F6.3,F8.3,F8.4,A,I1)
      END
```

9.2 Program Data

```
G01ECF Example Program Data
  8.26   20.0
  6.2     7.5
 55.76   45.0
```

9.3 Program Results

```
 G01ECF Example Program Results

    X       DF    PROB

  8.260  20.000  0.0100
  6.200   7.500  0.4279
 55.760  45.000  0.8694
```