NAG Fortran Library Routine Document

G01EBF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G01EBF returns the lower tail, upper tail or two-tail probability for the Student's t-distribution with real degrees of freedom, via the routine name.

2 Specification

```
real FUNCTION G01EBF(TAIL, T, DF, IFAIL)
INTEGER              IFAIL
real                 T, DF
CHARACTER*1          TAIL
```

3 Description

The lower tail probability for the Student's t-distribution with ν degrees of freedom, $P(\mathrm{T} \leq t : \nu)$ is defined by:

$$P(T \leq t : \nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\,\Gamma(\nu/2)} \int_{-\infty}^{t} \left[1 + \frac{T^2}{\nu}\right]^{-(\nu+1)/2} dT, \quad \nu \geq 1.$$

Computationally, there are two situations:

(a) when $\nu < 20$, a transformation of the beta distribution, $P_\beta(B \leq \beta : a, b)$ is used;

$$P(T \leq t : \nu) = \tfrac{1}{2} P_\beta\left(B \leq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \quad \text{when} \quad t < 0.0$$

or

$$P(T \leq t : \nu) = \tfrac{1}{2} + \tfrac{1}{2} P_\beta\left(B \geq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \quad \text{when } t > 0.0$$

(b) when $\nu \geq 20$, an asymptotic normalising expansion of the Cornish–Fisher type is used to evaluate the probability, see Hill (1970).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's t-distribution *Comm. ACM* **13** 617–619

5 Parameters

1: TAIL – CHARACTER*1 *Input*

On entry: indicates which tail the returned probability should represent.

If TAIL = 'U', the upper tail probability is returned, i.e., $P(\mathrm{T} \geq t : \nu)$.

If TAIL = 'S', the two-tail (significance level) probability is returned, i.e., $P(\mathrm{T} \geq |t| : \nu) + P(\mathrm{T} \leq -|t| : \nu)$.

If TAIL = 'C', the two-tail (confidence interval) probability is returned, i.e., $P(\mathrm{T} \leq |t| : \nu) - P(\mathrm{T} \leq -|t| : \nu)$.

If TAIL = 'L', the lower tail probability is returned, i.e., $P(\mathrm{T} \leq t : \nu)$.

Constraint: TAIL = 'U', 'S', 'C' or 'L'.

2: T – ***real*** *Input*

On entry: the value of the Student's t variate, t.

3: DF – ***real*** *Input*

On entry: the degrees of freedom, ν, of the Student's t-distribution.

Constraint: DF ≥ 1.

4: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

If IFAIL $\neq 0$, then G01EBF returns 0.0.

IFAIL = 1

On entry, TAIL $\neq$ 'U', 'S', 'C' or 'L'.

IFAIL = 2

On entry, DF < 1.

7 Accuracy

The computed probability should be accurate to 5 significant places for reasonable probabilities but there will be some loss of accuracy for very low probabilities (less than 10^{-10}), see Hastings and Peacock (1975).

8 Further Comments

The probabilities could also be obtained by using the appropriate transformation to a Beta distribution (see Abramowitz and Stegun (1972)) and using G01EEF. This routine allows the user to set the required accuracy.

9 Example

Values from, and degrees of freedom for Student's t-distributions are read along with the required tail. The probabilities are calculated and printed until the end of data is reached.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G01EBF Example Program Text
*     Mark 14 Release.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             DF, PROB, T
      INTEGER          IFAIL
      CHARACTER        TAIL
*     .. External Functions ..
      real             G01EBF
      EXTERNAL         G01EBF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G01EBF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   T      DF     PROB  TAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) T, DF, TAIL
      IFAIL = 0
*
      PROB = G01EBF(TAIL,T,DF,IFAIL)
*
      WRITE (NOUT,99999) T, DF, PROB, TAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,F6.3,F8.3,F8.4,2X,A1)
      END
```

9.2 Program Data

```
G01EBF Example Program Data
0.85  20.0  'L'    :T DF TAIL
0.85  20.0  'S'    :T DF TAIL
0.85  20.0  'C'    :T DF TAIL
0.85  20.0  'U'    :T DF TAIL
```

9.3 Program Results

```
 G01EBF Example Program Results

    T      DF     PROB  TAIL

  0.850  20.000  0.7973  L
  0.850  20.000  0.4054  S
  0.850  20.000  0.5946  C
  0.850  20.000  0.2027  U
```