# NAG Fortran Library Routine Document

# F12ADF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

# 1 Purpose

F12ADF is an option setting routine in a suite of routines consisting of F12ADF, F12AAF, F12ABF, F12ACF and F12AEF, and may be used to supply individual optional parameters to F12ABF and F12ACF. The initialization routine F12AAF **must** have been called prior to calling F12ADF.

## 2 Specification

SUBROUTINE F12ADF (STR, ICOMM, COMM, IFAIL)INTEGERICOMM(\*), IFAILdouble precisionCOMM(\*)CHARACTER\*(\*)STR

# **3** Description

F12ADF may be used to supply values for optional parameters to F12ABF and F12ACF. It is only necessary to call F12ADF for those parameters whose values are to be different from their default values. One call to F12ADF sets one parameter value.

Each optional parameter is defined by a single character string consisting of one or more items. The items associated with a given option must be separated by spaces, or equals signs [=]. Alphabetic characters may be upper or lower case. The string

'Pointers = Yes'

is an example of a string used to set an optional parameter. For each option the string contains one or more of the following items:

- (a) a mandatory keyword;
- (b) a phrase that qualifies the keyword;
- (c) a number that specifies an INTEGER or *double precision* value. Such numbers may be up to 16 contiguous characters in Fortran's I, F, E or D format.

F12ADF does not have an equivalent routine from the ARPACK package which passes options by directly setting values to scalar parameters or to specific elements of array arguments. F12ADF is intended to make the passing of options more transparent and follows the same principle as the single option setting routines in Chapter E04.

The setup routine F12AAF must be called prior to the first call to F12ADF and all calls to F12ADF must preced the first call to F12ABF, the reverse communication iterative solver.

A complete list of optional parameters, their abbreviations, synonyms and default values is given in Section 10.

#### 4 References

Lehoucq R B (2001) Implicitly Restarted Arnoldi Methods and Subspace Iteration SIAM Journal on Matrix Analysis and Applications 23 551–562

Lehoucq R B and Scott J A (1996) An evaluation of software for computing eigenvalues of sparse nonsymmetric matrices *Preprint MCS-P547-1195* Argonne National Laboratory

Lehoucq R B and Sorensen D C (1996) Deflation Techniques for an Implicitly Restarted Arnoldi Iteration SIAM Journal on Matrix Analysis and Applications **17** 789–821

Lehoucq R B, Sorensen D C and Yang C (1998) ARPACK Users' Guide: Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods SIAM, Philidelphia

#### **5** Parameters

1: STR – CHARACTER\*(\*)

On entry: a single valid option string (as described in Section 3 above and in Section 10).

- 2: ICOMM(\*) INTEGER array *Communication Array* ICOMM, on initial entry, must remain unchanged following a call to the setup routine F12AAF.
- 3: COMM(\*) *double precision* array

COMM, on initial entry, must remain unchanged following a call to the setup routine F12AAF.

4: IFAIL – INTEGER

Input/Output

Communication Array

Input

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.

# 6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

The string passed in STR contains an ambiguous keyword.

 $\mathrm{IFAIL}=2$ 

The string passed in STR contains a keyword that could not be recognized.

 $\mathrm{IFAIL}=3$ 

The string passed in STR contains a second keyword that could not be recognized.

 $\mathrm{IFAIL} = 4$ 

The initialization routine F12AAF has not been called or a communication array has become corrupted.

# 7 Accuracy

Not applicable.

# 8 Further Comments

None.

F12ADF.2

# 9 Example

The example solves  $Ax = \lambda Bx$  in shifted-inverse mode, where A and B are derived from the finite element discretization of the one-dimensional convection-diffusion operator  $\frac{d^2u}{dx^2} + \rho \frac{du}{dx}$  on the interval [0, 1], with zero Dirichlet boundary conditions.

The shift  $\sigma$  is a real number, and the operator used in the shifted-inverse iterative proces is  $OP = (A - \sigma B)^{-1}B$ .

#### 9.1 Program Text

```
F12ADF Example Program Text
     Mark 21 Release. NAG Copyright 2004.
      .. Parameters ..
                       IMON, LICOMM, NIN, NOUT
      TNTEGER
                       (IMON=0,LICOMM=140,NIN=5,NOUT=6)
     PARAMETER
     INTEGER
                      MAXN, MAXNCV, LDV
     PARAMETER
                       (MAXN=256,MAXNCV=30,LDV=MAXN)
                      LCOMM
      INTEGER
     PARAMETER
                       (LCOMM=3*MAXN+3*MAXNCV*MAXNCV+6*MAXNCV+60)
     DOUBLE PRECISION ONE, SIX, TWO
                     (ONE=1.0D+0,SIX=6.0D+0,TWO=2.0D+0)
     PARAMETER
      .. Local Scalars ..
*
     DOUBLE PRECISION H, RHO, S, S1, S2, S3, SIGMAI, SIGMAR
                       IFAIL, IFAIL1, INFO, IREVCM, J, N, NCONV, NCV,
     INTEGER
     +
                       NEV, NITER, NSHIFT, NX
       . Local Arrays .
*
     DOUBLE PRECISION COMM(LCOMM), D(MAXNCV,3), DD(MAXN), DL(MAXN),
                       DU(MAXN), DU2(MAXN), MX(MAXN), RESID(MAXN),
     +
                       V(LDV,MAXNCV), X(MAXN)
     INTEGER
                       ICOMM(LICOMM), IPIV(MAXN)
      .. External Functions ..
     DOUBLE PRECISION DNRM2
     EXTERNAL
                      DNRM2
      .. External Subroutines ..
*
                       DCOPY, DGTTRF, DGTTRS, F12AAF, F12ABF, F12ACF,
     EXTERNAL
                       F12ADF, F12AEF, MV
     +
      .. Intrinsic Functions ..
*
     INTRINSIC
                       DBLE
      .. Executable Statements ..
     WRITE (NOUT, *) 'F12ADF Example Program Results'
      WRITE (NOUT, *)
      Skip heading in data file
      READ (NIN, *)
     READ (NIN,*) NX, NEV, NCV, RHO, SIGMAR, SIGMAI
     N = NX * NX
      IF (N.LT.1 .OR. N.GT.MAXN) THEN
         WRITE (NOUT, 99999) 'N is out of range: N = ', N
      ELSE IF (NCV.GT.MAXNCV) THEN
        WRITE (NOUT,99999) 'NCV is out of range: NCV = ', NCV
      ELSE
         IFAIL = 0
         CALL F12AAF(N,NEV,NCV,ICOMM,LICOMM,COMM,ICOMM,IFAIL)
         Set the mode.
*
         CALL F12ADF('SHIFTED REAL', ICOMM, COMM, IFAIL)
         Set problem type
*
         CALL F12ADF('GENERALIZED', ICOMM, COMM, IFAIL)
         Construct C = A - SIGMA*I, and factor C using DGTTRF/F07CDF.
         H = ONE/DBLE(N+1)
         S = RHO/TWO
         S1 = -ONE/H - S - SIGMAR*H/SIX
S2 = TWO/H - 4.0D+0*SIGMAR*H/SIX
         S3 = -ONE/H + S - SIGMAR*H/SIX
         DO 20 J = 1, N - 1
            DL(J) = S1
            DD(J) = S2
            DU(J) = S3
  20
         CONTINUE
```

#### F12ADF

```
DD(N) = S2
         CALL DGTTRF(N,DL,DD,DU,DU2,IPIV,INFO)
         IREVCM = 0
         IFAIL = -1
   40
         CONTINUE
         CALL F12ABF(IREVCM, RESID, V, LDV, X, MX, NSHIFT, COMM, ICOMM, IFAIL)
         IF (IREVCM.NE.5) THEN
           IF (IREVCM.EQ.-1) THEN
              Perform x <--- OP*x = inv[A-SIGMA*M]*M*x using DGGTRS/
*
              FO7CEF
               CALL MV(N,X)
               CALL DGTTRS('N',N,1,DL,DD,DU,DU2,IPIV,X,N,INFO)
           ELSE IF (IREVCM.EQ.1) THEN
           Perform x <--- OP*x = inv[A-SIGMA*M]*M*x.
*
               CALL DGTTRS('N',N,1,DL,DD,DU,DU2,IPIV,MX,N,INFO)
               CALL DCOPY(N,MX,1,X,1)
           ELSE IF (IREVCM.EQ.2) THEN
               Perform y <--- M*x
              CALL MV(N, X)
            ELSE IF (IREVCM.EQ.4 .AND. IMON.NE.0) THEN
              Output monitoring information
               CALL F12AEF(NITER,NCONV,D,D(1,2),D(1,3),ICOMM,COMM)
              WRITE (6,99998) NITER, NCONV, DNRM2(NEV,D(1,3),1)
            END IF
           GO TO 40
         END IF
         IF (IFAIL.EQ.0) THEN
           Post-Process using F12ACF to compute eigenvalues/vectors.
*
            IFAIL1 = 0
           CALL F12ACF(NCONV,D,D(1,2),V,LDV,SIGMAR,SIGMAI,RESID,V,LDV,
                       COMM, ICOMM, IFAIL1)
     +
           Print computed eigenvalues.
            WRITE (NOUT, 99996) NCONV
           DO 60 J = 1, NCONV
              WRITE (NOUT, 99995) J, D(J,1), D(J,2)
   60
           CONTINUE
         ELSE
           WRITE (NOUT, 99997) IFAIL
        END IF
     END IF
     STOP
99999 FORMAT (1X,A,I5)
99998 FORMAT (1X,'Iteration',1X,I3,', No. converged =',1X,I3,', norm o',
            'f estimates =',E16.8)
    +
END
*
      SUBROUTINE MV(N,V)
      Compute the in-place matrix vector multiplication X < ---M * X,
     where M is mass matrix formed by using piecewise linear elements
*
*
     on [0,1].
      .. Parameters ..
*
     DOUBLE PRECISION ONE, FOUR, SIX
     PARAMETER
                   (ONE=1.0D0,FOUR=4.0D+0,SIX=6.0D+0)
      .. Scalar Arguments ..
*
      INTEGER
                   Ν
      .. Array Arguments ..
     DOUBLE PRECISION V(N)
      .. Local Scalars ..
*
      DOUBLE PRECISION H, VM1, VV
      INTEGER
                   J
      .. External Subroutines ..
*
     EXTERNAL
                   DSCAL
      .. Intrinsic Functions ..
      INTRINSIC
                   DBLE
```

```
* .. Executable Statements ..
VM1 = V(1)
V(1) = (FOUR*V(1)+V(2))/SIX
DO 20 J = 2, N - 1
VV = V(J)
V(J) = (VM1+FOUR*VV+V(J+1))/SIX
VM1 = VV
20 CONTINUE
V(N) = (VM1+FOUR*V(N))/SIX
*
H = ONE/DBLE(N+1)
CALL DSCAL(N,H,V,1)
RETURN
END
```

#### 9.2 Program Data

```
F12ADF Example Program Data
10 4 10 10.0 1.0 0.0 : Values for NX NEV NCV RHO SIGMAR and SIGMAI
```

#### 9.3 Program Results

F12ADF Example Program Results

The 4 generalized Ritz values closest to unity are: 1 ( 34.8634 , 0.0000 ) 2 ( 64.4479 , 0.0000 )

2	(	64.4479	,	0.0000	)
3	(	113.7872	,	0.0000	)
4	(	182.9293	,	0.0000	)

# **10 Optional Parameters**

Several optional parameters for the computational routines F12ABF and F12ACF define choices in the problem specification or the algorithm logic. In order to reduce the number of formal parameters of F12ABF and F12ACF these optional parameters have associated *default values* that are appropriate for most problems. Therefore, the user need only specify those optional parameters whose values are to be different from their default values.

The remainder of this section can be skipped by users who wish to use the default values for *all* optional parameters. A complete list of optional parameters and their default values is given in Section 10.1.

Optional parameters may be specified by calling F12ADF prior to a call to F12ABF, but after a call to F12AAF. One call is necessary for each optional parameter.

All optional parameters not specified by the user are set to their default values. Optional parameters specified by the user are unaltered by F12ABF and F12ACF (unless they define invalid values) and so remain in effect for subsequent calls unless altered by the user.

#### 10.1 Optional parameter checklist and default values

The following list gives the valid options. For each option, we give the keyword, any essential optional qualifiers and the default value. A definition for each option can be found in Section 10.2. The minimum abbreviation of each keyword is underlined. The qualifier may be omitted. The letters i and r denote INTEGER and *double precision* values required with certain options. The number  $\epsilon$  is a generic notation for *machine precision* (see X02AJF).

<b>Optional Parameters</b>	Default Values
<u>Adv</u> isory	Default = the value returned by X04ABF
Defaults	
Exact Shifts	Default = Exact Shifts
<b>Gen</b> eralized	
Initial Residual	
Iteration Limit	Default = 300

Largest Imaginary	
Largest Magnitude	Default = Largest Magnitude
Largest Real	8 8
List	Default = Nolist
Monitoring	Default = -1
Nolist	
Pointers	Default = No
Print Level	Default = 0
Random Residual	Default = Random Residual
Regular	Default = Regular
Regular Inverse	C C
Shifted Inverse Imaginary	
Shifted Inverse Real	
Smallest Imaginary	
Smallest Magnitude	
Smallest Real	
Standard	Default = <b>Standard</b>
Supplied Shifts	
Tolerance	$Default = \epsilon$
Vectors	Default = Ritz

# 10.2 Description of the Optional Parameters

#### Advisory

The output channel for advisory messages.

## Defaults

This special keyword may be used to reset all optional parameters to their default values.

#### Exact Shifts Supplied Shifts

During the Arnoldi iterative process, shifts are applied internally as part of the implicit restarting scheme. The shift strategy used by default and selected by the option **Exact Shifts** is strongly recommended over the alternative option **Supplied Shifts** (see Lehoucq *et al.* (1998) for details of shift strategies).

i

If **Exact Shifts** are used then these are computed internally by the algorithm in the implicit restarting scheme.

If **Supplied Shifts** are used then, during the Arnoldi iterative process, you must supply shifts through array arguments of F12ABF when F12ABF returns with IREVCM = 3; the real and imaginary parts of the shifts are returned in X and MX respectively (or in COMM when the option **Pointers** = Yes is set). This option should only be used by experienced users since this requires some algorithmic knowledge and because more operations are usually required than for the implicit shift scheme. Details on the use of explicit shifts and further references on shift strategies are available in Lehoucq *et al.* (1998).

# Iteration Limit

i

Default = 300

The limit on the number of Arnoldi iterations that can be performed before F12ABF exits. If not all requested eigenvalues have converged to within **Tolerance** and the number of Arnoldi iterations has reached this limit then F12ABF exits with a error; F12ACF can still be called subsequently to return the number of converged eigenvalues, the converged eigenvalues and, if requested, the corresponding eigenvectors.

Default = Exact Shifts

Default = the value returned by X04ABF

Largest Magnitude

Largest Real Largest Imaginary Smallest Magnitude

The Arnoldi iterative method converges on a number of eigenvalues with given properties. The default is for F12ABF to compute the eigenvalues of largest magnitude using option Largest Magnitude. Alternatively, eigenvalues may be chosen which have Largest Real part, Largest Imaginary part, Smallest Magnitude, Smallest Real part or Smallest Imaginary part.

Note that these options select the eigenvalue properties for eigenvalues of OP (and B for Generalized problems), the linear operator determined by the computational mode and problem type.

Normally each optional parameter specification is not printed to the advisory channel as it is supplied. List may be used to enable printing and Nolist may be used to suppress the printing.

#### Monitoring

List

Nolist

If i > 0, monitoring information is output to channel number i during the solution of each problem; this may be the same as the **Advisory** channel number. The type of information produced is dependent on the value of Print Level, see the description of Print Level in this section for details of the information produced. Please see X04ACF to associate a file with a given channel number.

#### Pointers

During the iterative process and reverse communication calls to F12ABF, required data can be communicated to and from F12ABF in one of two ways. When Pointers = No is selected (the default) then the array arguments X and MX are used to supply you with required data and used to return computed values back to F12ABF. For example, when IREVCM = 1 F12ABF returns the vector x in X and the matrix-vector product Bx in MX and expects the result of the linear operation OP(x) to be returned in X.

If **Pointers** = Yes is selected then the data is passed through sections of the array argument COMM. The section corresponding to X when **Pointers** = No begins at a location given by the first element of ICOMM; similarly the section corresponding to MX begins at a location given by the second element of ICOMM. This option allows F12ABF to perform fewer copy operations on each intermediate exit and entry, but can also lead to less elegant code in the calling program.

#### **Print Level**

This controls the amount of printing produced by F12ADF as follows.

- = 0No output except error messages. If you want to suppress all output, set **Print Level** = 0.
- $\geq 0$ The set of selected options.
- = 2Problem and timing statistics on final exit from F12ABF.
- $\geq 5$ A single line of summary output at each Arnoldi iteration.
- > 10If **Monitoring** > 0, then at each iteration, the length and additional steps of the current Arnoldi factorization and the number of converged Ritz values; during re-orthogonalisation, the norm of initial/restarted starting vector.
- $\geq 20$ Problem and timing statistics on final exit from F12ABF. If **Monitoring** > 0, then at each iteration, the number of shifts being applied, the eigenvalues and estimates of the Hessenberg matrix H, the size of the Arnoldi basis, the wanted Ritz values and associated Ritz estimates and the shifts applied; vector norms prior to and following re-orthogonalisation.

Default = Largest Magnitude

Default = Nolist

Default = -1

Default = No

Default = 0

- $\geq$  30 If **Monitoring** > 0, then on final iteration, the norm of the residual; when computing the real Schur form, the eigenvalues and Ritz estimates both before and after sorting; for each iteration, the norm of residual for compressed factorization and the compressed upper Hessenberg matrix *H*; during re-orthogonalisation, the initial/restarted starting vector; during the Arnoldi iteration loop, a restart is flagged and the number of the residual requiring iterative refinement; while applying shifts, some indices.
- $\geq$  40 If **Monitoring** > 0, then during the Arnoldi iteration loop, the Arnoldi vector number and norm of the current residual; while applying shifts, key measures of progress and the order of H; while computing eigenvalues of H, the last rows of the Schur and eigenvector matrices; when computing implicit shifts, the eigenvalues and Ritz estimates of H.
- $\geq$  50 During Arnoldi iteration loop: norms of key components and the active column of *H*, norms of residuals during iterative refinement, the final upper Hessenberg matrix *H*; while applying shifts: number of shifts, shift values, block indices, updated matrix *H*; while computing eigenvalues of *H*: the matrix *H*, the computed eigenvalues and Ritz estimates.

# Random Residual

To begin the Arnoldi iterative process, F12ABF requires an initial residual vector. By default F12ABF provides its own random initial residual vector; this option can also be set using **Random Residual**. Alternatively, you can supply an initial residual vector (perhaps from a previous computation) to F12ABF through the array argument RESID; this option can be set using **Random Residual**.

Default = **Regular** 

Default = **Random Residual** 

# Regular<br/>RegularInverseShiftedInverseRealShiftedInverseImaginary

These options define the computational mode which in turn defines the form of operation OP(x) to be performed when F12ABF returns with IREVCM = -1 or IREVCM = 1 and the matrix-vector product Bx when F12ABF returns with IREVCM = 2.

Given a **Standard** eigenvalue problem in the form  $Ax = \lambda x$  then the following modes are available with the appropriate operator OP(x).

Regular	OP = A
Shifted Real	$OP = (A - \sigma I)^{-1}$ where $\sigma$ is real

Given a Generalized eigenvalue problem in the form  $Ax = \lambda Bx$  then the following modes are available with the appropriate operator OP(x).

Regular Inverse	$OP = B^{-1}A$
Shifted Real with real shift	$OP = (A - \sigma B)^{-1}B$ , where $\sigma$ is real
Shifted Real with complex shift	$OP = Real((A - \sigma B)^{-1}B)$ , where $\sigma$ is complex
Shifted Imaginary	$OP = Imag((A - \sigma B)^{-1}B)$ , where $\sigma$ is complex

#### Standard Generalized

The problem to be solved is either a standard eigenvalue problem,  $Ax = \lambda x$ , or a generalized eigenvalue problem,  $Ax = \lambda Bx$ . The option **Standard** should be used when a standard eigenvalue problem is being solved and the option **Generalized** should be used when a generalized eigenvalue problem is being solved.

#### **Tolerance**

An approximate eigenvalue has deemed to have converged when the corresponding Ritz estimate is within **Tolerance** relative to the magnitude of the eigenvalue.

Default =  $\epsilon$ 

Default = **Standard** 

#### Vectors

The routine F12ACF can optionally compute the Schur vectors and/or the eigenvectors corresponding to the converged eigenvalues. To turn off computation of any vectors the option **Vectors** = None should be set. To compute only the Schur vectors (at very little extra cost), the option **Vectors** = Schur should be set and these will be returned in the array argument V of F12ACF. To compute the eigenvectors (Ritz vectors) corresponding to the eigenvalue estimates, the option **Vectors** = Ritz should be set and these will be returned in the array argument Z of F12ACF; if the array argument V is passed to F12ACF in place of Z then the Schur vectors in V are overwritten by the eigenvectors computed by F12ACF.