NAG Fortran Library Routine Document

F08WSF (CGGHRD/ZGGHRD)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F08WSF (CGGHRD/ZGGHRD) reduces a pair of complex matrices (A, B), where B is upper triangular, to the generalized upper Hessenberg form using unitary transformations.

2 Specification

```
      SUBROUTINE F08WSF(COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,
     1                  LDZ, INFO)
      ENTRY      cgghrd (COMPQ, COMPZ, N, ILO, IHI, A, LDA, B, LDB, Q, LDQ, Z,
     1                  LDZ, INFO)
      INTEGER           N, ILO, IHI, LDA, LDB, LDQ, LDZ, INFO
      complex           A(LDA,*), B(LDB,*), Q(LDQ,*), Z(LDZ,*)
      CHARACTER*1       COMPQ, COMPZ
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

F08WSF (CGGHRD/ZGGHRD) is usually the third step in the solution of the complex generalized eigenvalue problem

$$Ax = \lambda Bx.$$

The (optional) first step balances the two matrices using F08WVF (CGGBAL/ZGGBAL). In the second step, matrix B is reduced to upper triangular form using the QR factorization routine F08ASF (CGEQRF/ZGEQRF) and this unitary transformation Q is applied to matrix A by calling F08AUF (CUNMQR/ZUNMQR).

F08WSF (CGGHRD/ZGGHRD) reduces a pair of complex matrices (A, B), where B is triangular, to the generalized upper Hessenberg form using unitary transformations. This two-sided transformation is of the form

$$Q^H AZ = H$$
$$Q^H BZ = T$$

where H is an upper Hessenberg matrix, T is an upper triangular matrix and Q and Z are unitary matrices determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1, so that

$$Q_1 AZ_1^H = (Q_1 Q)H(Z_1 Z)^H,$$
$$Q_1 BZ_1^H = (Q_1 Q)T(Z_1 Z)^H.$$

4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Moler C B and Stewart G W (1973) An algorithm for generalized matrix eigenproblems *SIAM J. Numer. Anal.* **10** 241–256

5 Parameters

1: COMPQ – CHARACTER*1 *Input*

On entry: specifies the form of the computed unitary matrix Q, as follows:

if COMPQ = 'N', do not compute Q;

if COMPQ = 'I', the unitary matrix Q is returned;

if COMPQ = 'V', Q must contain a unitary matrix Q_1, and the product Q_1Q is returned.

Constraint: COMPQ = 'N', 'I' or 'V'.

2: COMPZ – CHARACTER*1 *Input*

On entry: specifies the form of the computed unitary matrix Z, as follows:

if COMPZ = 'N', do not compute Z;

if COMPZ = 'I', the unitary matrix Z is returned;

if COMPZ = 'V', Z must contain a unitary matrix Z_1, and the product Z_1Z is returned.

Constraint: COMPZ = 'N', 'I' or 'V'.

3: N – INTEGER *Input*

On entry: n, the order of the matrices A and B.

Constraint: $\mathrm{N} \geq 0$.

4: ILO – INTEGER *Input*
5: IHI – INTEGER *Input*

On entry: i_{lo} and i_{hi} as determined by a previous call to F08WVF (CGGBAL/ZGGBAL). Otherwise, they should be set to 1 and n, respectively.

Constraints:

$1 \leq \mathrm{ILO} \leq \mathrm{IHI} \leq \mathrm{N}$ if $\mathrm{N} > 0$;
$\mathrm{ILO} = 1$ and $\mathrm{IHI} = 0$ if $\mathrm{N} = 0$.

6: A(LDA,*) – ***complex*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the matrix A of the matrix pair (A, B). Usually, this is the matrix A returned by F08AUF (CUNMQR/ZUNMQR).

On exit: A is overwritten by the upper Hessenberg matrix H.

7: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F08WSF (CGGHRD/ZGGHRD) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

8: B(LDB,*) – ***complex*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{N})$.

On entry: the upper triangular matrix B of the matrix pair (A, B). Usually, this is the matrix B returned by the QR factorization routine F08ASF (CGEQRF/ZGEQRF).

On exit: B is overwritten by the upper triangular matrix T.

9: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F08WSF (CGGHRD/ZGGHRD) is called.

Constraint: LDB $\geq \max(1, N)$.

10: Q(LDQ,*) – ***complex*** array *Input/Output*

Note: the second dimension of the array Q must be at least $\max(1, N)$.

On entry: if COMPQ = 'N', Q is not referenced; if COMPQ = 'V', Q must contain a unitary matrix Q_1.

On exit: if COMPQ = 'I', Q contains the unitary matrix Q; if COMPQ = 'V', Q is overwritten by $Q_1 Q$.

11: LDQ – INTEGER *Input*

On entry: the first dimension of the array Q as declared in the (sub)program from which F08WSF (CGGHRD/ZGGHRD) is called.

Constraints:

LDQ ≥ 1 if COMPQ = 'N',
LDQ $\geq \max(1, N)$ if COMPQ = 'I' or 'V'.

12: Z(LDZ,*) – ***complex*** array *Input/Output*

Note: the second dimension of the array Z must be at least $\max(1, N)$.

On entry: if COMPZ = 'N', Z is not referenced; if COMPZ = 'V', Z must contain a unitary matrix Z_1.

On exit: if COMPZ = 'I', Z contains the unitary matrix Z; if COMPZ = 'V', Z is overwritten by $Z_1 Z$.

13: LDZ – INTEGER *Input*

On entry: the first dimension of the array Z as declared in the (sub)program from which F08WSF (CGGHRD/ZGGHRD) is called.

Constraints:

LDZ ≥ 1 if COMPZ = 'N',
LDZ $\geq \max(1, N)$ if COMPZ = 'V' or 'I'.

14: INFO – INTEGER *Output*

On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7 Accuracy

The reduction to the generalized Hessenberg form is implemented using unitary transformations which are backward stable.

8 Further Comments

This routine is usually followed by F08XSF (CHGEQZ/ZHGEQZ) which implements the QZ algorithm for computing generalized eigenvalues of a reduced pair of matrices.

The real analogue of this routine is F08WEF (SGGHRD/DGGHRD).

9 Example

See Section 9 of the documents for F08XSF (CHGEQZ/ZHGEQZ) and F08YXF (CTGEVC/ZTGEVC).