# NAG Fortran Library Routine Document

# F08UFF (SPBSTF/DPBSTF)

**Note:** before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

## 1    Purpose

F08UFF (SPBSTF/DPBSTF) computes a split Cholesky factorization of a real symmetric positive-definite band matrix.

## 2    Specification

```
SUBROUTINE F08UFF(UPLO, N, KB, BB, LDBB, INFO)
ENTRY       spbstf (UPLO, N, KB, BB, LDBB, INFO)
INTEGER         N, KB, LDBB, INFO
real            BB(LDBB,*)
CHARACTER*1     UPLO
```

The ENTRY statement enables the routine to be called by its LAPACK name.

## 3    Description

This routine computes a split Cholesky factorization of a real symmetric positive-definite band matrix $B$. It is designed to be used in conjunction with F08UEF (SSBGST/DSBGST).

The factorization has the form $B = S^T S$, where $S$ is a band matrix of the same bandwidth as $B$ and the following structure: $S$ is upper triangular in the first $(n+k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & s_{64} & s_{65} & s_{66} & & & \\ & & & & s_{75} & s_{76} & s_{77} & & \\ & & & & & s_{86} & s_{87} & s_{88} & \\ & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

## 4    References

None.

## 5    Parameters

1:    UPLO – CHARACTER*1                                                                      *Input*

*On entry*: indicates whether the upper or lower triangular part of $B$ is stored as follows:

if UPLO = 'U', the upper triangular part of $B$ is stored;

if UPLO = 'L', the lower triangular part of $B$ is stored.

*Constraint*: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

*On entry*: $n$, the order of the matrix $B$.

*Constraint*: $N \geq 0$.

3: KB – INTEGER *Input*

*On entry*: $k$, the number of super-diagonals of the matrix $B$ if UPLO = 'U', or the number of sub-diagonals if UPLO = 'L'.

*Constraint*: $KB \geq 0$.

4: BB(LDBB,*) – *real* array *Input/Output*

**Note:** the second dimension of the array BB must be at least $\max(1, N)$.

*On entry*: the $n$ by $n$ symmetric positive-definite band matrix $B$, stored in rows 1 to $k + 1$. More precisely, if UPLO = 'U', the elements of the upper triangle of $B$ within the band must be stored with element $b_{ij}$ in $BB(k + 1 + i - j, j)$ for $\max(1, j - k) \leq i \leq j$; if UPLO = 'L', the elements of the lower triangle of $B$ within the band must be stored with element $b_{ij}$ in $BB(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k)$.

*On exit*: $B$ is overwritten by the elements of its split Cholesky factor $S$.

5: LDBB – INTEGER *Input*

*On entry*: the first dimension of the array BB as declared in the (sub)program from which F08UFF (SPBSTF/DPBSTF) is called.

*Constraint*: $LDBB \geq KB + 1$.

6: INFO – INTEGER *Output*

*On exit*: INFO = 0 unless the routine detects an error (see Section 6).

# 6 Error Indicators and Warnings

INFO < 0

If INFO $= -i$, the $i$th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, the factorization could not be completed, because the updated element $b_{ii}$ would be the square root of a negative number. Hence $B$ is not positive-definite. This may indicate an error in forming the matrix $B$.

# 7 Accuracy

The computed factor $S$ is the exact factor of a perturbed matrix $B + E$, where

$$|E| \leq c(k + 1)\varepsilon |S^T| \, |S|,$$

$c(k + 1)$ is a modest linear function of $k + 1$, and $\varepsilon$ is the ***machine precision***. It follows that $|e_{ij}| \leq c(k + 1)\varepsilon \sqrt{(b_{ii} b_{jj})}$.

## 8     Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to this routine may be followed by a call to F08UEF (SSBGST/DSBGST) to solve the generalized eigenproblem $Az = \lambda Bz$, where $A$ and $B$ are banded and $B$ is positive-definite.

The complex analogue of this routine is F08UTF (CPBSTF/ZPBSTF).

## 9     Example

See Section 9 of the document for F08UEF (SSBGST/DSBGST).