NAG Fortran Library Routine Document

F07QNF (ZSPSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07QNF (ZSPSV) computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n symmetric matrix stored in packed format and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07QNF (UPLO, N, NRHS, AP, IPIV, B, LDB, INFO)
INTEGER            N, NRHS, IPIV(*), LDB, INFO
complex*16         AP(*), B(LDB,*)
CHARACTER*1        UPLO
```

The routine may be called by its LAPACK name ***zspsv***.

3 Description

The diagonal pivoting method is used to factor A as $A = UDU^T$, if UPLO = 'U' or $A = LDL^T$, if UPLO = 'L', where U (or L) is a product of permutation and unit upper (lower) triangular matrices, D is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: if UPLO = 'U', the upper triangle of A is stored.

If UPLO = 'L', the lower triangle of A is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: N $\geq$ 0.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

4: AP$(*)$ – ***complex*16*** array *Input/Output*

Note: the dimension of the array AP must be at least $\max(\mathrm{N} \times (\mathrm{N}+1)/2)$.

On entry: the upper or lower triangle of the symmetric matrix A, packed columnwise in a linear array. The jth column of A is stored in the array AP as follows:

if UPLO = 'U', $\mathrm{AP}(i+(j-1)\times j/2) = a_{ij}$ for $1 \leq i \leq j$;
if UPLO = 'L', $\mathrm{AP}(i+(j-1)\times(2n-j)/2) = a_{ij}$ for $j \leq i \leq n$.

On exit: the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = UDU^T$ or $A = LDL^T$ as computed by F07QRF (ZSPTRF), stored as a packed triangular matrix in the same storage format as A.

5: IPIV$(*)$ – INTEGER array *Output*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On exit: details of the interchanges and the block structure of D, as determined by F07QRF (ZSPTRF). If $\mathrm{IPIV}(k) > 0$, then rows and columns k and $\mathrm{IPIV}(k)$ were interchanged, and $D(k,k)$ is a 1 by 1 diagonal block. If UPLO = 'U' and $\mathrm{IPIV}(k) = \mathrm{IPIV}(k-1) < 0$, then rows and columns $k-1$ and $-\mathrm{IPIV}(k)$ were interchanged and $D(k-1:k, k-1:k)$ is a 2 by 2 diagonal block. If UPLO = 'L' and $\mathrm{IPIV}(k) = \mathrm{IPIV}(k+1) < 0$, then rows and columns $k+1$ and $-\mathrm{IPIV}(k)$ were interchanged and $D(k:k+1, k:k+1)$ is a 2 by 2 diagonal block.

6: B(LDB,$*$) – ***complex*16*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$. To solve the equations $Ax = b$, where b is a single right-hand side, B may be supplied as a one-dimensional array with length $\mathrm{LDB} = \max(1, \mathrm{N})$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO $= 0$, the n by r solution matrix X.

7: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07QNF (ZSPSV) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

8: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) and Chapter 11 of Higham (2002) for further details.

F07QPF (ZSPSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04DJF solves $Ax = b$ and returns a forward error bound and condition estimate. F04DJF calls F07QNF (ZSPSV) to solve the equations.

8 Further Comments

The total number of floating point operations is approximately $\frac{4}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this routine is F07PAF (DSPSV).

9 Example

To solve the equations

$$Ax = b,$$

where A is the complex symmetric matrix

$$A = \begin{pmatrix} -0.56 + 0.12i & -1.54 - 2.86i & 5.32 - 1.59i & 3.80 + 0.92i \\ -1.54 - 2.86i & -2.83 - 0.03i & -3.52 + 0.58i & -7.86 - 2.96i \\ 5.32 - 1.59i & -3.52 + 0.58i & 8.86 + 1.81i & 5.14 - 0.64i \\ 3.80 + 0.92i & -7.86 - 2.96i & 5.14 - 0.64i & -0.39 - 0.71i \end{pmatrix}$$

and

$$b = \begin{pmatrix} -6.43 + 19.24i \\ -0.49 - 1.47i \\ -48.18 + 66.00i \\ -55.64 + 41.22i \end{pmatrix}.$$

Details of the factorization of A are also output.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07QNF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NMAX
      PARAMETER        (NMAX=8)
      CHARACTER        UPLO
      PARAMETER        (UPLO='U')
*     .. Local Scalars ..
```

```
      INTEGER          I, IFAIL, INFO, J, N
*     .. Local Arrays ..
      COMPLEX *16      AP((NMAX*(NMAX+1))/2), B(NMAX)
      INTEGER          IPIV(NMAX)
      CHARACTER        CLABS(1), RLABS(1)
*     .. External Subroutines ..
      EXTERNAL         X04DDF, ZSPSV
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07QNF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read the upper or lower triangular part of the matrix A from
*        data file
*
         IF (UPLO.EQ.'U') THEN
            READ (NIN,*) ((AP(I+(J*(J-1))/2),J=I,N),I=1,N)
         ELSE IF (UPLO.EQ.'L') THEN
            READ (NIN,*) ((AP(I+((2*N-J)*(J-1))/2),J=1,I),I=1,N)
         END IF
*
*        Read b from data file
*
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
*
         CALL ZSPSV(UPLO,N,1,AP,IPIV,B,N,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
            WRITE (NOUT,99999) (B(I),I=1,N)
*
*           Print details of factorization
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04DDF(UPLO,'Non-unit diagonal',N,AP,'Bracketed',
     +                  'F7.4','Details of factorization','Integer',
     +                  RLABS,'Integer',CLABS,80,0,IFAIL)
*
*           Print pivot indices
*
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Pivot indices'
            WRITE (NOUT,99998) (IPIV(I),I=1,N)
*
         ELSE
            WRITE (NOUT,99997) 'The diagonal block ', INFO,
     +        ' of D is zero'
         END IF
      ELSE
         WRITE (NOUT,*) 'NMAX too small'
      END IF
      STOP
*
99999 FORMAT ((3X,4(' (',F7.4,',',F7.4,')',:)))
99998 FORMAT (1X,7I11)
99997 FORMAT (1X,A,I3,A)
      END
```

9.2 Program Data

```
F07QNF Example Program Data
   4                                                          :Value of N
 ( -0.56,  0.12) ( -1.54, -2.86) (  5.32, -1.59) (  3.80,  0.92)
                 ( -2.83 ,-0.03) ( -3.52,  0.58) ( -7.86, -2.96)
                                 (  8.86,  1.81) (  5.14, -0.64)
                                                 ( -0.39 ,-0.71) :End matrix A
 ( -6.43, 19.24) ( -0.49, -1.47) (-48.18, 66.00) (-55.64, 41.22) :End vector b
```

9.3 Program Results

```
 F07QNF Example Program Results

 Solution
    (-4.0000, 3.0000) ( 3.0000,-2.0000) (-2.0000, 5.0000) ( 1.0000,-1.0000)

 Details of factorization
                     1                  2                  3                  4
 1  (-2.0954,-2.2011) (-0.1071,-0.3157) (-0.4823, 0.0150) ( 0.4426, 0.1936)
 2                    ( 4.4079, 5.3991) (-0.6078, 0.2811) ( 0.5279,-0.3715)
 3                                      (-2.8300,-0.0300) (-7.8600,-2.9600)
 4                                                        (-0.3900,-0.7100)

 Pivot indices
           1          2         -2         -2
```