NAG Fortran Library Routine Document

F07MAF (DSYSV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

F07MAF (DSYSV) computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric matrix and X and B are n by r matrices.

2 Specification

```
SUBROUTINE F07MAF (UPLO, N, NRHS, A, LDA, IPIV, B, LDB, WORK, LWORK,
1                  INFO)
INTEGER            N, NRHS, LDA, IPIV(*), LDB, LWORK, INFO
double precision   A(LDA,*), B(LDB,*), WORK(*)
CHARACTER*1        UPLO
```

The routine may be called by its LAPACK name ***dsysv***.

3 Description

The diagonal pivoting method is used to factor A as $A = UDU^T$, if UPLO = 'U' or $A = LDL^T$, if UPLO = 'L', where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1 by 1 and 2 by 2 diagonal blocks. The factored form of A is then used to solve the system of equations $AX = B$.

Note that, in general, different permutations (pivot sequences) and diagonal block structures are obtained for UPLO = 'U' or 'L'

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: UPLO – CHARACTER*1 *Input*

On entry: if UPLO = 'U', the upper triangle of A is stored.

If UPLO = 'L', the lower triangle of A is stored.

Constraint: UPLO = 'U' or 'L'.

2: N – INTEGER *Input*

On entry: n, the number of linear equations, i.e., the order of the matrix A.

Constraint: N $\geq$ 0.

3: NRHS – INTEGER *Input*

On entry: r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathrm{NRHS} \geq 0$.

4: A(LDA,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array A must be at least $\max(1, \mathrm{N})$.

On entry: the symmetric matrix A.

If UPLO = 'U', the leading n by n upper triangular part of A contains the upper triangular part of the matrix A, and the strictly lower triangular part of A is not referenced.

If UPLO = 'L', the leading n by n lower triangular part of A contains the lower triangular part of the matrix A, and the strictly upper triangular part of A is not referenced.

On exit: if INFO = 0, the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $\mathrm{A} = UDU^T$ or $\mathrm{A} = LDL^T$ as computed by F07MDF (DSYTRF).

5: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07MAF (DSYSV) is called.

Constraint: $\mathrm{LDA} \geq \max(1, \mathrm{N})$.

6: IPIV($*$) – INTEGER array *Output*

Note: the dimension of the array IPIV must be at least $\max(1, \mathrm{N})$.

On exit: details of the interchanges and the block structure of D, as determined by F07MDF (DSYTRF). If $\mathrm{IPIV}(k) > 0$, then rows and columns k and $\mathrm{IPIV}(k)$ were interchanged, and $D(k,k)$ is a 1 by 1 diagonal block. If UPLO = 'U' and $\mathrm{IPIV}(k) = \mathrm{IPIV}(k-1) < 0$, then rows and columns $k-1$ and $-\mathrm{IPIV}(k)$ were interchanged and $D(k-1:k, k-1:k)$ is a 2 by 2 diagonal block. If UPLO = 'L' and $\mathrm{IPIV}(k) = \mathrm{IPIV}(k+1) < 0$, then rows and columns $k+1$ and $-\mathrm{IPIV}(k)$ were interchanged and $D(k:k+1, k:k+1)$ is a 2 by 2 diagonal block.

7: B(LDB,$*$) – ***double precision*** array *Input/Output*

Note: the second dimension of the array B must be at least $\max(1, \mathrm{NRHS})$.

On entry: the n by r right-hand side matrix B.

On exit: if INFO = 0, the n by r solution matrix X.

8: LDB – INTEGER *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07MAF (DSYSV) is called.

Constraint: $\mathrm{LDB} \geq \max(1, \mathrm{N})$.

9: WORK($*$) – ***double precision*** array *Workspace*

Note: the dimension of the array WORK must be at least $\max(1, \mathrm{LWORK})$.

On exit: if INFO = 0, $\mathrm{WORK}(1)$ returns the optimal LWORK.

10: LWORK – INTEGER *Input*

On entry: the dimension of the array WORK as declared in the (sub)program from which F07MAF (DSYSV) is called.

$\mathrm{LWORK} \geq 1$, and for best performance $\mathrm{LWORK} \geq \max(1, \mathrm{N} \times nb)$, where nb is the optimal blocksize for F07MDF (DSYTRF).

If LWORK $= -1$, a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued.

11: INFO – INTEGER *Output*

On exit: INFO $= 0$ unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If INFO $= -i$, the ith argument had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO $= i$, d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution could not be computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* (1999) for further details.

F07MBF (DSYSVX) is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, F04BHF solves $Ax = b$ and returns a forward error bound and condition estimate. F04BHF calls F07MAF (DSYSV) to solve the equations.

8 Further Comments

The total number of floating point operations is approximately $\frac{1}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogues of F07MAF (DSYSV) are F07MNF (ZHESV) for Hermitian matrices, and F07NNF (ZSYSV) for symmetric matrices.

9 Example

To solve the equations

$$Ax = b,$$

where A is the symmetric matrix

$$A = \begin{pmatrix} -1.81 & 2.06 & 0.63 & -1.15 \\ 2.06 & 1.15 & 1.87 & 4.20 \\ 0.63 & 1.87 & -0.21 & 3.87 \\ -1.15 & 4.20 & 3.87 & 2.07 \end{pmatrix}$$

and

$$b = \begin{pmatrix} 0.96 \\ 6.07 \\ 8.38 \\ 9.50 \end{pmatrix}.$$

Details of the factorization of A are also output.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     F07MAF Example Program Text
*     Mark 21 Release. NAG Copyright 2004.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
      INTEGER          NB, NMAX
      PARAMETER        (NB=64,NMAX=8)
      INTEGER          LDA, LWORK
      PARAMETER        (LDA=NMAX,LWORK=NB*NMAX)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, INFO, J, N
*     .. Local Arrays ..
      DOUBLE PRECISION A(LDA,NMAX), B(NMAX), WORK(LWORK)
      INTEGER          IPIV(NMAX)
*     .. External Subroutines ..
      EXTERNAL         DSYSV, X04CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'F07MAF Example Program Results'
      WRITE (NOUT,*)
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) N
      IF (N.LE.NMAX) THEN
*
*        Read the upper triangular part of the matrix A from data file
*
         READ (NIN,*) ((A(I,J),J=I,N),I=1,N)
*
*        Read b from data file
*
         READ (NIN,*) (B(I),I=1,N)
*
*        Solve the equations Ax = b for x
*
         CALL DSYSV('Upper',N,1,A,LDA,IPIV,B,N,WORK,LWORK,INFO)
*
         IF (INFO.EQ.0) THEN
*
*           Print solution
*
            WRITE (NOUT,*) 'Solution'
            WRITE (NOUT,99999) (B(I),I=1,N)
*
*           Print details of factorization
*
            WRITE (NOUT,*)
            IFAIL = 0
            CALL X04CAF('Upper','Non-unit diagonal',N,N,A,LDA,
```

```
     +                 'Details of the factorization',IFAIL)
*
*           Print pivot indices
*
            WRITE (NOUT,*)
            WRITE (NOUT,*) 'Pivot indices'
            WRITE (NOUT,99998) (IPIV(I),I=1,N)
*
         ELSE
            WRITE (NOUT,99997) 'The diagonal block ', INFO,
     +        ' of D is zero'
         END IF
       ELSE
          WRITE (NOUT,*) 'NMAX too small'
       END IF
       STOP
*
99999 FORMAT ((3X,7F11.4))
99998 FORMAT (1X,7I11)
99997 FORMAT (1X,A,I3,A)
      END
```

9.2 Program Data

```
F07MAF Example Program Data
  4                          :Value of N
 -1.81   2.06   0.63  -1.15
         1.15   1.87   4.20
               -0.21   3.87
                       2.07 :End of matrix A
  0.96   6.07   8.38   9.50 :End of vector b
```

9.3 Program Results

```
 F07MAF Example Program Results

 Solution
       -5.0000    -2.0000     1.0000     4.0000

 Details of the factorization
             1          2          3          4
 1      0.4074     0.3031    -0.5960     0.6537
 2                -2.5907     0.8115     0.2230
 3                            1.1500     4.2000
 4                                       2.0700

 Pivot indices
           1          2         -2         -2
```