NAG Fortran Library Routine Document

F07AHF (SGERFS/DGERFS)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of **bold italicised** terms and other implementation-dependent details.

1 Purpose

F07AHF (SGERFS/DGERFS) returns error bounds for the solution of a real system of linear equations with multiple right-hand sides, $AX = B$ or $A^T X = B$. It improves the solution by iterative refinement, in order to reduce the backward error as much as possible.

2 Specification

```
SUBROUTINE F07AHF(TRANS, N, NRHS, A, LDA, AF, LDAF, IPIV, B, LDB, X,
1                 LDX, FERR, BERR, WORK, IWORK, INFO)
ENTRY     sgerfs (TRANS, N, NRHS, A, LDA, AF, LDAF, IPIV, B, LDB, X,
1                 LDX, FERR, BERR, WORK, IWORK, INFO)
INTEGER           N, NRHS, LDA, LDAF, IPIV(*), LDB, LDX, IWORK(*), INFO
real              A(LDA,*), AF(LDAF,*), B(LDB,*), X(LDX,*), FERR(*),
1                 BERR(*), WORK(*)
CHARACTER*1       TRANS
```

The ENTRY statement enables the routine to be called by its LAPACK name.

3 Description

This routine returns the backward errors and estimated bounds on the forward errors for the solution of a real system of linear equations with multiple right-hand sides $AX = B$ or $A^T X = B$. The routine handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of the routine in terms of a single right-hand side b and solution x.

Given a computed solution x, the routine computes the *component-wise backward error* β. This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$
$$|\delta a_{ij}| \le \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \le \beta |b_i|.$$

Then the routine estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the F07 Chapter Introduction.

4 References

Golub G H and van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5   Parameters

1:    TRANS – CHARACTER*1   *Input*

On entry: indicates the form of the linear equations for which X is the computed solution as follows:

if TRANS = 'N', then the linear equations are of the form $AX = B$;

if TRANS = 'T' or 'C', then the linear equations are of the form $A^T X = B$.

Constraint: TRANS = 'N', 'T' or 'C'.

2:    N – INTEGER   *Input*

On entry: n, the order of the matrix A.

Constraint: $N \geq 0$.

3:    NRHS – INTEGER   *Input*

On entry: r, the number of right-hand sides.

Constraint: $NRHS \geq 0$.

4:    A(LDA,*) – **real** array   *Input*

Note: the second dimension of the array A must be at least $\max(1, N)$.

On entry: the n by n original matrix A as supplied to F07ADF (SGETRF/DGETRF).

5:    LDA – INTEGER   *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which F07AHF (SGERFS/DGERFS) is called.

Constraint: $LDA \geq \max(1, N)$.

6:    AF(LDAF,*) – **real** array   *Input*

Note: the second dimension of the array AF must be at least $\max(1, N)$.

On entry: the LU factorization of A, as returned by F07ADF (SGETRF/DGETRF).

7:    LDAF – INTEGER   *Input*

On entry: the first dimension of the array AF as declared in the (sub)program from which F07AHF (SGERFS/DGERFS) is called.

Constraint: $LDAF \geq \max(1, N)$.

8:    IPIV(*) – INTEGER array   *Input*

Note: the dimension of the array IPIV must be at least $\max(1, N)$.

On entry: the pivot indices, as returned by F07ADF (SGETRF/DGETRF).

9:    B(LDB,*) – **real** array   *Input*

Note: the second dimension of the array B must be at least $\max(1, NRHS)$.

On entry: the n by r right-hand side matrix B.

10:    LDB – INTEGER   *Input*

On entry: the first dimension of the array B as declared in the (sub)program from which F07AHF (SGERFS/DGERFS) is called.

Constraint: $LDB \geq \max(1, N)$.

11:   X(LDX,*) – ***real*** array   *Input/Output*

   Note: the second dimension of the array X must be at least $\max(1, \text{NRHS})$.

   On entry: the n by r solution matrix X, as returned by F07AEF (SGETRS/DGETRS).

   On exit: the improved solution matrix X.

12:   LDX – INTEGER   *Input*

   On entry: the first dimension of the array X as declared in the (sub)program from which F07AHF (SGERFS/DGERFS) is called.

   Constraint: $\text{LDX} \geq \max(1, \text{N})$.

13:   FERR(*) – ***real*** array   *Output*

   Note: the dimension of the array FERR must be at least $\max(1, \text{NRHS})$.

   On exit: $\text{FERR}(j)$ contains an estimated error bound for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

14:   BERR(*) – ***real*** array   *Output*

   Note: the dimension of the array BERR must be at least $\max(1, \text{NRHS})$.

   On exit: $\text{BERR}(j)$ contains the component-wise backward error bound β for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

15:   WORK(*) – ***real*** array   *Workspace*

   Note: the dimension of the array WORK must be at least $\max(1, 3 * \text{N})$.

16:   IWORK(*) – INTEGER array   *Workspace*

   Note: the dimension of the array IWORK must be at least $\max(1, \text{N})$.

17:   INFO – INTEGER   *Output*

   On exit: INFO = 0 unless the routine detects an error (see Section 6).

6   Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

   If $\text{INFO} = -i$, the ith parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

7   Accuracy

The bounds returned in FERR are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8   Further Comments

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations. Each step of iterative refinement involves an additional $6n^2$ operations. At most 5 steps of iterative refinement are performed, but usually only 1 or 2 steps are required.

Estimating the forward error involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ operations.

The complex analogue of this routine is F07AVF (CGERFS/ZGERFS).

9 Example

To solve the system of equations $AX = B$ using iterative refinement and to compute the forward and backward error bounds, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 9.52 & 18.47 \\ 24.35 & 2.25 \\ 0.77 & -13.28 \\ -6.22 & -6.21 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by F07ADF (SGETRF/DGETRF).

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*       F07AHF Example Program Text
*       Mark 15 Release. NAG Copyright 1991.
*       .. Parameters ..
        INTEGER           NIN, NOUT
        PARAMETER         (NIN=5,NOUT=6)
        INTEGER           NMAX, NRHMAX, LDA, LDAF, LDB, LDX
        PARAMETER         (NMAX=8,NRHMAX=NMAX,LDA=NMAX,LDAF=NMAX,LDB=NMAX,
       +                  LDX=NMAX)
        CHARACTER         TRANS
        PARAMETER         (TRANS='N')
*       .. Local Scalars ..
        INTEGER           I, IFAIL, INFO, J, N, NRHS
*       .. Local Arrays ..
        real              A(LDA,NMAX), AF(LDAF,NMAX), B(LDB,NRHMAX),
       +                  BERR(NRHMAX), FERR(NRHMAX), WORK(3*NMAX),
       +                  X(LDX,NMAX)
        INTEGER           IPIV(NMAX), IWORK(NMAX)
*       .. External Subroutines ..
        EXTERNAL          F06QFF, sgerfs, sgetrf, sgetrs, X04CAF
*       .. Executable Statements ..
        WRITE (NOUT,*) 'F07AHF Example Program Results'
*       Skip heading in data file
        READ (NIN,*)
        READ (NIN,*) N, NRHS
        IF (N.LE.NMAX .AND. NRHS.LE.NRHMAX) THEN
*
*          Read A and B from data file, and copy A to AF and B to X
*
           READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
           READ (NIN,*) ((B(I,J),J=1,NRHS),I=1,N)
*
           CALL F06QFF('General',N,N,A,LDA,AF,LDAF)
*
           CALL F06QFF('General',N,NRHS,B,LDB,X,LDX)
*
*          Factorize A in the array AF
*
           CALL sgetrf(N,N,AF,LDAF,IPIV,INFO)
*
           WRITE (NOUT,*)
           IF (INFO.EQ.0) THEN
*
*             Compute solution in the array X
*
```

```
                CALL sgetrs(TRANS,N,NRHS,AF,LDAF,IPIV,X,LDX,INFO)
*
*               Improve solution, and compute backward errors and
*               estimated bounds on the forward errors
*
                CALL sgerfs(TRANS,N,NRHS,A,LDA,AF,LDAF,IPIV,B,LDB,X,LDX,
     +                      FERR,BERR,WORK,IWORK,INFO)
*
*               Print solution
*
                IFAIL = 0
*
                CALL X04CAF('General',' ',N,NRHS,X,LDX,'Solution(s)',IFAIL)
*
                WRITE (NOUT,*)
                WRITE (NOUT,*) 'Backward errors (machine-dependent)'
                WRITE (NOUT,99999) (BERR(J),J=1,NRHS)
                WRITE (NOUT,*)
     +            'Estimated forward error bounds (machine-dependent)'
                WRITE (NOUT,99999) (FERR(J),J=1,NRHS)
            ELSE
                WRITE (NOUT,*) 'The factor U is singular'
            END IF
         END IF
         STOP
*
99999 FORMAT ((3X,1P,7e11.1))
         END
```

9.2 Program Data

```
F07AHF Example Program Data
  4  2                       :Values of N and NRHS
  1.80   2.88   2.05  -0.89
  5.25  -2.95  -0.95  -3.80
  1.58  -2.69  -2.90  -1.04
 -1.11  -0.66  -0.59   0.80   :End of matrix A
  9.52  18.47
 24.35   2.25
  0.77 -13.28
 -6.22  -6.21                 :End of matrix B
```

9.3 Program Results

```
 F07AHF Example Program Results

 Solution(s)
            1         2
 1      1.0000    3.0000
 2     -1.0000    2.0000
 3      3.0000    4.0000
 4     -5.0000    1.0000

 Backward errors (machine-dependent)
       5.6E-17    6.2E-17
 Estimated forward error bounds (machine-dependent)
       2.4E-14    3.3E-14
```