

NAG Fortran Library Routine Document

E04VJF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

E04VJF may be used before E04VHF to determine the sparsity pattern for the Jacobian.

2 Specification

```
SUBROUTINE E04VJF (NF, N, USRFUN, IAFUN, JAVAR, A, LENA, NEA, IGFUN,
1          JGVAR, LENG, NEG, X, XLOW, XUPP, CW, LENCW, IW,
2          LENIW, RW, LENRW, CUSER, IUSER, RUSER, IFAIL)

INTEGER           NF, N, IAFUN(LENA), JAVAR(LENA), LENA, NEA,
1                  IGFUN(LENG), JGVAR(LENG), LENG, NEG, LENCW,
2                  IW(LENIW), LENIW, LENRW, IUSER(*), IFAIL
double precision A(LENA), X(N), XLOW(N), XUPP(N), RW(LENRW), RUSER(*)
CHARACTER*8      CW(LENCW), CUSER(*)
EXTERNAL          USRFUN
```

3 Description

When using E04VHF, if you set the optional parameter **Derivative Option** to 0 and your function routine USRFUN provides none of the derivatives, you may need to call E04VJF to determine the input arrays IAFUN, JAVAR, A, IGFUN and JGVAR. These arrays define the pattern of non-zeros in the Jacobian matrix. A typical sequence of calls could be

```
CALL E04VGF (CW, LENCW, ... )
CALL E04VJF (NF, N, ... )
CALL E04VLF ('Derivative Option' = 0, CW, ... )
CALL E04VHF (START, NF, ... )
```

E04VJF determines the sparsity pattern for the Jacobian and identifies the constant elements automatically. To do so, E04VJF approximates the problem functions, $F(x)$, at three random perturbations of the given initial point x . If an element of the approximate Jacobian is the same at all three points, then it is taken to be constant. If it is zero, it is taken to be identically zero. Since the random points are not chosen close together, the heuristic will correctly classify the Jacobian elements in the vast majority of cases. In general, E04VJF finds that the Jacobian can be permuted to the form:

$$\begin{pmatrix} G(x) & A_3 \\ A_2 & A_4 \end{pmatrix},$$

where A_2 , A_3 and A_4 are constant. Note that $G(x)$ might contain elements that are also constant, but E04VJF must classify them as nonlinear. This is because E04VHF ‘removes’ linear variables from the calculation of F by setting them to zero before calling USRFUN. A knowledgeable user would be able to move such elements from $F(x)$ in USRFUN and enter them as part of IAFUN, JAVAR and A for E04VHF.

4 References

Hock W and Schittkowski K (1981) *Test Examples for Nonlinear Programming Codes. Lecture Notes in Economics and Mathematical Systems* **187** Springer–Verlag

5 Parameters

Note: all optional parameters are described in detail in Section 11.2 of the document for E04VHF.

1: NF – INTEGER *Input*

On entry: nf , the number of problem functions in $F(x)$, including the objective function (if any) and the linear and nonlinear constraints. Simple upper and lower bounds on x can be defined using the parameters XLOW and XUPP defined below and should not be included in F .

Constraint: $NF > 0$.

2: N – INTEGER *Input*

On entry: n , the number of variables.

Constraint: $N > 0$.

3: USRFUN – SUBROUTINE, supplied by the user. *External Procedure*

USRFUN must define the problem functions $F(x)$. This subroutine is passed to E04VJF as the external parameter USRFUN.

Its specification is:

```
SUBROUTINE USRFUN (STATUS, N, X, NEEDF, NF, F, NEEDG, LENG, G,
1                   CUSER, IUSER, RUSER)
INTEGER           STATUS, N, NEEDF, NF, NEEDG, LENG, IUSER(*)
double precision X(N), F(NF), G(LENG), RUSER(*)
CHARACTER*8       CUSER(*)
```

1: STATUS – INTEGER *Input/Output*

On entry: indicates the first call to USRFUN.

If $STATUS = 0$, there is nothing special about the current call to USRFUN; if $STATUS = 1$, E04VJF is calling your subroutine for the *first* time. Some data may need to be input or computed and saved.

On exit: may be used to indicate that you are unable to evaluate F at the current x . (For example, the problem functions may not be defined there).

E04VJF evaluates $F(x)$ at random perturbation of the initial point x , say x_p . If the functions cannot be evaluated at x_p , you can set $STATUS = -1$, E04VJF will use another random perturbation.

If for some reason you wish to terminate the current problem, set $STATUS \leq -2$.

2: N – INTEGER *Input*

On entry: n , the number of variables, as defined in the call to E04VJF.

3: X(N) – **double precision** array *Input*

On entry: the variables x at which the problem functions are to be calculated. *The array x must not be altered.*

4: NEEDF – INTEGER *Input*

On entry: indicates if F must be assigned during the call to USRFUN (see F below).

5: NF – INTEGER *Input*

On entry: nf , the number of problem functions.

6:	F(NF) – double precision array	<i>Input/Output</i>
	<i>On entry:</i> this will be set by E04VJF.	
	<i>On exit:</i> the computed $F(x)$ according to the setting of NEEDF.	
	If NEEDF=0, F is not required and is ignored.	
	If NEEDF > 0, the components of $F(x)$ must be calculated and assigned to F. E04VJF will always call USRFUN with NEEDF > 0.	
	To simplify the code, you may ignore the value of NEEDF and compute $F(x)$ on every entry to USRFUN.	
7:	NEEDG – INTEGER	<i>Input</i>
	<i>On entry:</i> E04VJF will call USRFUN with NEEDG = 0 to indicate that G is not required.	
8:	LENG – INTEGER	<i>Input</i>
	<i>On entry:</i> the dimension of the array G as declared in the (sub)program from which E04VJF is called.	
9:	G(LENG) – double precision array	
	E04VJF will always call USRFUN with NEEDG = 0. G will not be used but must be declared correctly.	
10:	CUSER(*) – CHARACTER*8 array	<i>User Workspace</i>
11:	IUSER(*) – INTEGER array	<i>User Workspace</i>
12:	RUSER(*) – double precision array	<i>User Workspace</i>
	<i>USRFUN is called from E04VJF with the parameters CUSER, IUSER and RUSER as supplied to E04VJF. These parameters are not touched by E04VJF and can be used as an alternative to COMMON.</i>	

USRFUN must be declared as EXTERNAL in the (sub)program from which E04VJF is called. Parameters denoted as *Input* must **not** be changed by this procedure.

4:	IAFUN(LENA) – INTEGER array	<i>Output</i>
5:	JAVAR(LENA) – INTEGER array	<i>Output</i>
6:	A(LENA) – double precision array	<i>Output</i>
	<i>On exit:</i> define the coordinates (i, j) and values A_{ij} of the non-zero elements of the linear part A of the function $F(x) = f(x) + Ax$.	
	In particular, the NEA triples $(IAFUN(k), JAVAR(k), A(k))$ define the row and column indices $i = IAFUN(k)$ and $j = JAVAR(k)$ of the element $A_{ij} = A(k)$.	
7:	LENA – INTEGER	<i>Input</i>
	<i>On entry:</i> the dimension of the arrays IAFUN, JAVAR and A that hold (i, j, A_{ij}) as declared in the (sub)program from which E04VJF is called. LENA should be an <i>overestimate</i> of the number of elements in the linear part of the Jacobian.	
	<i>Constraint:</i> LENA ≥ 1 .	
8:	NEA – INTEGER	<i>Output</i>
	<i>On exit:</i> is the number of non-zero entries in A such that $F(x) = f(x) + Ax$.	
9:	IGFUN(LENG) – INTEGER array	<i>Output</i>
10:	JGVAR(LENG) – INTEGER array	<i>Output</i>
	<i>On exit:</i> define the coordinates (i, j) of the non-zero elements of G, the nonlinear part of the derivatives $J(x) = G(x) + A$ of the function $F(x) = f(x) + Ax$.	

11: LENG – INTEGER *Input*

On entry: the dimension of the arrays IGFUN and JGVAR that define the varying Jacobian elements (i, j, G_{ij}) as declared in the (sub)program from which E04VJF is called. LENG should be an *overestimate* of the number of elements in the nonlinear part of the Jacobian.

Constraint: $\text{LENG} \geq 1$.

12: NEG – INTEGER *Output*

On exit: the number of non-zero entries in G .

13: X(N) – **double precision** array *Input*

On entry: an initial estimate of the variables x . The contents of x will be used by E04VJF in the call of USRFUN, and so each element of X should be within the bounds given by XLOW and XUPP.

14: XLOW(N) – **double precision** array *Input*

15: XUPP(N) – **double precision** array *Input*

On entry: contain the lower and upper bounds l_x and u_x on the variables x .

To specify a non-existent lower bound $[l_x]_j = -\infty$, set $\text{XLOW}(j) \leq -\text{bigbnd}$, where **bigbnd** is the **Infinite Bound Size**. To specify a non-existent upper bound $\text{XUPP}(j) \geq \text{bigbnd}$.

To fix the j th variable (say $x_j = \beta$, where $|\beta| < \text{bigbnd}$), set $\text{XLOW}(j) = \text{XUPP}(j) = \beta$.

16: CW(LENCW) – CHARACTER*8 array *Communication Array*

17: LENCW – INTEGER *Input*

On entry: the dimension of the array CW as declared in the (sub)program from which E04VJF is called.

Constraint: $\text{LENCW} \geq 600$.

18: IW(LENIW) – INTEGER array *Communication Array*

19: LENIW – INTEGER *Input*

On entry: the dimension of the array IW as declared in the (sub)program from which E04VJF is called.

Constraint: $\text{LENIW} \geq 600$.

20: RW(LENRW) – **double precision** array *Communication Array*

21: LENRW – INTEGER *Input*

On entry: the dimension of the array RW as declared in the (sub)program from which E04VJF is called.

Constraint: $\text{LENRW} \geq 600$.

22: CUSER(*) – CHARACTER*8 array *User Workspace*

23: IUSER(*) – INTEGER array *User Workspace*

24: RUSER(*) – **double precision** array *User Workspace*

Note: the dimension of the array CUSER, IUSER and RUSER must be at least 1.

CUSER, IUSER and RUSER are not used by E04VJF, but are passed directly to USRFUN and may be used to communicate with E04VJF.

25: IFAIL – INTEGER *Input/Output*

On initial entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On final exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, because for this routine the values of the output parameters may be useful even if $\text{IFAIL} \neq 0$ on exit, the recommended value is -1 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $\text{IFAIL} = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

$\text{IFAIL} = 1$

The initialization routine E04VGF has not been called or at least one of LENCW, LENIW or LENRW is less than 600.

$\text{IFAIL} = 2$

An input parameter is invalid. The output message provides more details of the invalid parameter.

$\text{IFAIL} = 3$

Undefined user-supplied function.

The user has indicated that the problem functions are undefined by setting $\text{STATUS} = -1$ on exit from USRFUN. This exit occurs if E04VJF is unable to find a point at which the functions are defined.

$\text{IFAIL} = 4$

User requested termination.

The user has indicated the wish to terminate the call to E04VJF by setting STATUS to a value < -1 on exit from USRFUN.

$\text{IFAIL} = 5$

Either LENA or LENG is too small, resulting in insufficient array to store the Jacobian information. Increase LENA and/or LENG

$\text{IFAIL} = 6$

Unable to estimate the Jacobian structure.

$\text{IFAIL} = 7$

Internal memory allocation failed when attempting to obtain the required workspace. Please contact NAG.

$\text{IFAIL} = 8$

Internal memory allocation insufficient. Please contact NAG.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

This example shows how to call E04VJF to determine the sparsity pattern of the Jacobian before calling E04VJF to solve a sparse nonlinear programming problem without providing the Jacobian information in USRFUN.

This is a reformulation of Problem 74 from Hock and Schittkowski (1981) and involves the minimization of the nonlinear function

$$f(x) = 10^{-6}x_3^3 + \frac{2}{3} \times 10^{-6}x_4^3 + 3x_3 + 2x_4$$

subject to the bounds

$$\begin{aligned} -0.55 &\leq x_1 \leq 0.55, \\ -0.55 &\leq x_2 \leq 0.55, \\ 0 &\leq x_3 \leq 1200, \\ 0 &\leq x_4 \leq 1200, \end{aligned}$$

to the nonlinear constraints

$$\begin{aligned} 1000 \sin(-x_1 - 0.25) + 1000 \sin(-x_2 - 0.25) - x_3 &= -894.8, \\ 1000 \sin(x_1 - 0.25) + 1000 \sin(x_1 - x_2 - 0.25) - x_4 &= -894.8, \\ 1000 \sin(x_2 - 0.25) + 1000 \sin(x_2 - x_1 - 0.25) &= -1294.8, \end{aligned}$$

and to the linear constraints

$$\begin{aligned} -x_1 + x_2 &\geq -0.55, \\ x_1 - x_2 &\geq -0.55. \end{aligned}$$

The initial point, which is infeasible, is

$$x_0 = (0, 0, 0, 0)^T,$$

and $f(x_0) = 0$.

The optimal solution (to five figures) is

$$x^* = (0.11887, -0.39623, 679.94, 1026.0)^T,$$

and $f(x^*) = 5126.4$. All the nonlinear constraints are active at the solution.

The formulation of the problem combines the constraints and the objective into a single vector (F).

$$F = \begin{pmatrix} 1000 \sin(-x_1 - 0.25) & +1000 \sin(-x_2 - 0.25) - x_3 \\ 1000 \sin(x_1 - 0.25) & +1000 \sin(x_1 - x_2 - 0.25) - x_4 \\ 1000 \sin(x_2 - 0.25) & +1000 \sin(x_2 - x_1 - 0.25) \\ -x_1 + x_2 & \\ x_1 - x_2 & \\ 10^{-6}x_3^3 + \frac{2}{3} \times 10^{-6}x_4^3 + 3x_3 + 2x_4 & \end{pmatrix}$$

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      E04VJF Example Program Text
*      Mark 21 Release. NAG Copyright 2004.
*      IMPLICIT          NONE
*      .. Parameters ..
*      INTEGER            NIN, NOUT
*      PARAMETER          (NIN=5,NOUT=6)
*      INTEGER            NMAX, NFMAX, LENAMX, LENGMX
*      PARAMETER          (NMAX=100,NFMAX=100,LENAMX=300,LENGMX=300)
*      INTEGER            LENCW, LENIW, LENRW
*      PARAMETER          (LENCW=600,LENIW=600,LENRW=600)
*      .. Local Scalars ..
*      DOUBLE PRECISION   OBJADD, SINF
```

```

      INTEGER          I, IFAIL, LENA, LENG, N, NEA, NEG, NF, NFNAME,
+                  NINF, NS, NXNAME, OBJROW, START
      CHARACTER*8     PROB
*     .. Local Arrays ..
      DOUBLE PRECISION A(LENAMX), F(NFMAX), FLOW(NFMAX), FMUL(NFMAX),
+                  FUPP(NFMAX), RUSER(1), RW(LENRW), X(NMAX),
+                  XLOW(NMAX), XMUL(NMAX), XUPP(NMAX)
      INTEGER          FSTATE(NFMAX), IAFUN(LENAMX), IGFUN(LENGMX),
+                  IUSER(1), IW(LENIW), JAVAR(LENAMX),
+                  JGVAR(LENGMX), XSTATE(NMAX)
      CHARACTER*8     CUSER(1), CW(LENCW), FNAMES(NFMAX), XNAMES(NMAX)
*     .. External Subroutines ..
      EXTERNAL         E04VGF, E04VHF, E04VJF, E04VLF, E04VMF, USRFUN
*     .. Executable Statements ..
      WRITE (NOUT,*) 'E04VJF Example Program Results'
*     Skip heading in data file
      READ (NIN,*) 
      READ (NIN,*) N, NF
*
      IF (N.LE.NMAX .AND. NF.LE.NFMAX) THEN
*
*     Call E04VGF to initialise E04VJF.
      IFAIL = -1
      CALL E04VGF(CW,LENCW,IW,LENIW,RW,LENRW,IFAIL)
*
      LENA = LENAMX
      LENG = LENGMX
*
*     Read the bounds on the variables.
      DO 20 I = 1, N
         READ (NIN,*) XLOW(I), XUPP(I)
20    CONTINUE
*
      DO 40 I = 1, N
         X(I) = 0.0D0
40    CONTINUE
*
*     Determine the Jacobian structure.
      IFAIL = 0
      CALL E04VJF(NF,N,USRFUN,IAFUN,JAVAR,A,LENA,NEA,IGFUN,JGVAR,
+                  LENG,NEG,X,XLOW,XUPP,CW,LENCW,IW,LENIW,RW,LENRW,
+                  CUSER,IUSER,RUSER,IFAIL)
*
*     Print the Jacobian structure.
      WRITE (NOUT,*)
      WRITE (NOUT,99999) NEA
      WRITE (NOUT,99998)
      WRITE (NOUT,99997)
      DO 60 I = 1, NEA
         WRITE (NOUT,99996) I, IAFUN(I), JAVAR(I), A(I)
60    CONTINUE
      WRITE (NOUT,*)
      WRITE (NOUT,99995) NEG
      WRITE (NOUT,99994)
      WRITE (NOUT,99993)
      DO 80 I = 1, NEG
         WRITE (NOUT,99992) I, IGFUN(I), JGVAR(I)
80    CONTINUE
*
*     Now that we have the determined the structure of the
*     Jacobian, set up the information necessary to solve
*     the optimization problem.
      START = 0
      NFNAME = 1
      NXNAME = 1
      PROB = 'E04VJFE'
      OBJADD = 0.0
      DO 100 I = 1, N
         X(I) = 0.0D0
         XSTATE(I) = 0
         XMUL(I) = 0.0D0

```

```

100      CONTINUE
        DO 120 I = 1, NF
            F(I) = 0.0D0
            FSTATE(I) = 0
            FMUL(I) = 0.0D0
120      CONTINUE
*
*      The row containing the objective function.
        READ (NIN,*) OBJROW
*
*      Read the bounds on the functions.
        DO 140 I = 1, NF
            READ (NIN,*) FLOW(I), FUPP(I)
140      CONTINUE
*
*      By default E04VHF does not print monitoring
*      information. Set the print file unit or the summary
*      file unit to get information.
        CALL E04VMF('Print file',NOUT,CW,IW,RW,IFAIL)
*
*      Tell E04VHF that we supply no derivatives in USRFUN.
        CALL E04VLF('Derivative option 0',CW,IW,RW,IFAIL)
*
*      Solve the problem.
        IFAIL = -1
        CALL E04VHF(START,NF,N,NXNAME,NFNAME,OBJADD,OBJROW,PROB,USRFUN,
+                  IAFUN,JAVAR,A,LENA,NEA,IGFUN,JGVAR,LENG,NEG,XLOW,
+                  XUPP,XNAMES,FLOW,FUPP,FNAMES,X,XSTATE,XMUL,F,
+                  FSTATE,FMUL,NS,NINF,SINF,CW,LENCW,IW,LENIW,RW,
+                  LENRW,CUSER,IUSER,RUSER,IFAIL)
*
        WRITE (NOUT,*)
        WRITE (NOUT,99991) IFAIL
        IF (IFAIL.EQ.0 .OR. IFAIL.EQ.4) THEN
            WRITE (NOUT,99990) F(OBJROW)
            WRITE (NOUT,99989) (X(I),I=1,N)
        END IF
    END IF
*
        STOP
*
99999 FORMAT (1X,'NEA (the number of non-zero entries in A) = ',I3)
99998 FORMAT (1X,' I IAFUN(I) JAVAR(I) A(I)')
99997 FORMAT (1X,'---- ----- ----- -----')
99996 FORMAT (1X,I3,2I10,1P,E18.4)
99995 FORMAT (1X,'NEG (the number of non-zero entries in G) = ',I3)
99994 FORMAT (1X,' I IGFUN(I) JGVAR(I)')
99993 FORMAT (1X,'---- ----- -----')
99992 FORMAT (1X,I3,2I10)
99991 FORMAT (1X,'On exit from E04VHF, IFAIL = ',I5)
99990 FORMAT (1X,'Final objective value = ',F11.1)
99989 FORMAT (1X,'Optimal X = ',7F9.2)
    END

    SUBROUTINE USRFUN(STATUS,N,X,NEEDF,NF,F,NEEDG,LENG,G,CUSER,IUSER,
+                      RUSER)
    IMPLICIT NONE
*
* .. Scalar Arguments ..
    INTEGER LENG, N, NEEDF, NEEDG, NF, STATUS
*
* .. Array Arguments ..
    DOUBLE PRECISION F(NF), G(LENG), RUSER(*), X(N)
    INTEGER IUSER(*)
    CHARACTER*8 CUSER(*)
*
* .. Intrinsic Functions ..
    INTRINSIC SIN
*
* .. Executable Statements ..
    IF (NEEDF.GT.0) THEN
        F(1) = 1000.0D+0*SIN(-X(1)-0.25D+0) + 1000.0D+0*SIN(-X(2)
+                           -0.25D+0) - X(3)
        F(2) = 1000.0D+0*SIN(X(1)-0.25D+0) + 1000.0D+0*SIN(X(1)-X(2)
+                           -0.25D+0) - X(4)
    END IF

```

```

        F(3) = 1000.0D+0*SIN(X(2)-X(1)-0.25D+0) + 1000.0D+0*SIN(X(2)
+
        -0.25D+0)
        F(4) = -X(1) + X(2)
        F(5) = X(1) - X(2)
        F(6) = 1.0D-6*X(3)**3 + 2.0D-6*X(4)**3/3.0D+0 + 3*X(3) + 2*X(4)
    END IF
*
    END

```

9.2 Program Data

```

E04VJF Example Program Data
4       6           : Values of N and NF
-0.55D0   0.55D0   : Bounds on the variables, XLOW(i), XUPP(i), for i = 1 to N
-0.55D0   0.55D0
0.0D0    1200.0D0
0.0D0    1200.0D0

6           : Value of OBJROW
-894.8D0 -894.8D0 : Bounds on the functions, FLOW(i), FUPP(i), for i = 1 to NF
-894.8D0 -894.8D0
-1294.8D0 -1294.8D0
-0.55D0    1.0D25
-0.55D0    1.0D25
-1.0D25    1.0D25

```

9.3 Program Results

```

E04VJF Example Program Results

NEA (the number of non-zero entries in A) =  4
I      IAFUN(I)  JAVAR(I)      A(I)
----  -----
1          4          1      -1.0000E+00
2          5          1      1.0000E+00
3          4          2      1.0000E+00
4          5          2      -1.0000E+00

NEG (the number of non-zero entries in G) =  10
I      IGFUN(I)  JGVAR(I)
----  -----
1          1          1
2          2          1
3          3          1
4          1          2
5          2          2
6          3          2
7          6          3
8          6          4
9          1          3
10         2          4

Parameters
=====
Files
-----
Solution file.....      0      Old basis file .....      0      (Print file).....      6
Insert file.....      0      New basis file .....      0      (Summary file).....      0
Punch file.....      0      Backup basis file.....      0
Load file.....      0      Dump file.....      0

Frequencies
-----
Print frequency.....    100      Check frequency.....     60      Save new basis map.....     100
Summary frequency.....  100      Factorization frequency     50      Expand frequency..... 10000

QP subproblems
-----
QPsolver Cholesky.....
```

Scale tolerance.....	0.900	Minor feasibility tol..	1.00E-06	Iteration limit.....	10000																																																																																																																																																																																																																		
Scale option.....	0	Minor optimality tol..	1.00E-06	Minor print level.....	1																																																																																																																																																																																																																		
Crash tolerance.....	0.100	Pivot tolerance.....	1.11E-15	Partial price.....	1																																																																																																																																																																																																																		
Crash option.....	3	Elastic weight.....	1.00E+04	Prtl price section (A)	4																																																																																																																																																																																																																		
		New superbasics.....	99	Prtl price section (-I)	5																																																																																																																																																																																																																		
 The SQP Method																																																																																																																																																																																																																							

Minimize.....		Cold start.....		Proximal Point method..	1																																																																																																																																																																																																																		
Nonlinear objectiv vars	2	Objective Row.....	6	Function precision....	1.72E-13																																																																																																																																																																																																																		
Unbounded step size....	1.00E+20	Superbasics limit.....	4	Difference interval....	4.15E-07																																																																																																																																																																																																																		
Unbounded objective....	1.00E+15	Hessian dimension.....	4	Central difference int.	5.57E-05																																																																																																																																																																																																																		
Major step limit.....	2.00E+00	Nonderv. linesearch..		Derivative option.....	0																																																																																																																																																																																																																		
Major iterations limit.	1000	Linesearch tolerance...	0.90000	Verify level.....	0																																																																																																																																																																																																																		
Minor iterations limit.	500	Penalty parameter.....	0.00E+00	Major Print Level.....	1																																																																																																																																																																																																																		
		Major optimality tol...	2.00E-06																																																																																																																																																																																																																				
 Hessian Approximation																																																																																																																																																																																																																							

Full-Memory Hessian....		Hessian updates.....	99999999	Hessian frequency.....	99999999																																																																																																																																																																																																																		
				Hessian flush.....	99999999																																																																																																																																																																																																																		
 Nonlinear constraints																																																																																																																																																																																																																							

Nonlinear constraints..	3	Major feasibility tol..	1.00E-06	Violation limit.....	1.00E+06																																																																																																																																																																																																																		
Nonlinear Jacobian vars	4																																																																																																																																																																																																																						
 Miscellaneous																																																																																																																																																																																																																							

LU factor tolerance....	3.99	LU singularity tol....	1.05E-08	Timing level.....	0																																																																																																																																																																																																																		
LU update tolerance....	3.99	LU swap tolerance.....	1.03E-04	Debug level.....	0																																																																																																																																																																																																																		
LU partial pivoting...		eps (machine precision)	1.11E-16	System information.....	No																																																																																																																																																																																																																		
Nonlinear constraints	3	Linear constraints	2																																																																																																																																																																																																																				
Nonlinear variables	4	Linear variables	0																																																																																																																																																																																																																				
Jacobian variables	4	Objective variables	2																																																																																																																																																																																																																				
Total constraints	5	Total variables	4																																																																																																																																																																																																																				
The user has defined	0	out of	10	first derivatives																																																																																																																																																																																																																			
 <table border="0"> <thead> <tr><th>Itns</th><th>Major</th><th>Minors</th><th>Step</th><th>nCon</th><th>Feasible</th><th>Optimal</th><th>MeritFunction</th><th>L+U</th><th>BSwap</th><th>nS</th><th>condHz</th><th>Penalty</th></tr> </thead> <tbody> <tr><td>4</td><td>0</td><td>4</td><td></td><td>1</td><td>8.0E+02</td><td>1.0E+00</td><td>0.0000000E+00</td><td>14</td><td></td><td>1</td><td>3.0E+07</td><td>_ r</td></tr> <tr><td>6</td><td>1</td><td>2</td><td>1.2E-03</td><td>2</td><td>4.0E+02</td><td>1.0E+00</td><td>1.7331709E+06</td><td>13</td><td></td><td>1</td><td>1.3E+07</td><td>5.1E+00 _n rl</td></tr> <tr><td>7</td><td>2</td><td>1</td><td>1.3E-03</td><td>3</td><td>2.7E+02</td><td>5.5E-01</td><td>1.7301152E+06</td><td>13</td><td></td><td></td><td>5.1E+00 _s l</td></tr> <tr><td>7</td><td>3</td><td>0</td><td>7.5E-03</td><td>4</td><td>8.8E+01</td><td>5.4E-01</td><td>8.8193384E+05</td><td>13</td><td></td><td></td><td>2.8E+00 _ l</td></tr> <tr><td>7</td><td>4</td><td>0</td><td>2.3E-02</td><td>5</td><td>2.9E+01</td><td>5.3E-01</td><td>8.4262007E+05</td><td>13</td><td></td><td></td><td>2.8E+00 _ l</td></tr> <tr><td>7</td><td>5</td><td>0</td><td>6.9E-02</td><td>6</td><td>8.9E+00</td><td>5.2E-01</td><td>7.3075570E+05</td><td>13</td><td></td><td></td><td>2.8E+00 _ l</td></tr> <tr><td>8</td><td>6</td><td>1</td><td>2.2E-01</td><td>7</td><td>2.3E+00</td><td>8.0E+01</td><td>4.4817383E+05</td><td>13</td><td></td><td>1</td><td>1.2E+04</td><td>2.8E+00 _ l</td></tr> <tr><td>9</td><td>7</td><td>1</td><td>8.3E-01</td><td>8</td><td>1.7E-01</td><td>9.2E+00</td><td>2.4331006E+04</td><td>13</td><td></td><td>1</td><td>9.5E+03</td><td>2.8E+00 _ l</td></tr> <tr><td>10</td><td>8</td><td>1</td><td>1.0E+00</td><td>9</td><td>6.5E-03</td><td>4.0E+01</td><td>5.3126219E+03</td><td>13</td><td>1</td><td>1</td><td>1.3E+02</td><td>2.8E+00 _</td></tr> <tr><td>11</td><td>9</td><td>1</td><td>1.0E+00</td><td>10</td><td>4.6E-03</td><td>1.2E+01</td><td>5.1602362E+03</td><td>13</td><td></td><td></td><td>1</td><td>9.4E+01</td><td>2.8E+00 _</td></tr> <tr><td>12</td><td>10</td><td>1</td><td>1.0E+00</td><td>11</td><td>2.3E-04</td><td>6.2E-02</td><td>5.1265653E+03</td><td>13</td><td></td><td></td><td>1</td><td>9.6E+01</td><td>2.8E+00 _</td></tr> <tr><td>13</td><td>11</td><td>1</td><td>1.0E+00</td><td>12</td><td>(1.3E-08)</td><td>2.9E-04</td><td>5.1264981E+03</td><td>13</td><td></td><td></td><td>1</td><td>1.2E+02</td><td>2.8E+00 c</td></tr> <tr><td>14</td><td>11</td><td>2</td><td>1.0E+00</td><td>12</td><td>(1.3E-08)</td><td>2.7E-04</td><td>5.1264981E+03</td><td>13</td><td></td><td></td><td>1</td><td>1.2E+02</td><td>2.8E+00 c</td></tr> <tr><td>15</td><td>12</td><td>1</td><td>1.0E+00</td><td>13</td><td>(5.6E-13)</td><td>7.1E-05</td><td>5.1264981E+03</td><td>13</td><td></td><td></td><td>1</td><td>9.5E+01</td><td>2.8E+00 c</td></tr> <tr><td>16</td><td>13</td><td>1</td><td>1.0E+00</td><td>14</td><td>(1.8E-14)</td><td>(1.0E-09)</td><td>5.1264981E+03</td><td>13</td><td></td><td></td><td>1</td><td>9.5E+01</td><td>2.8E+00 c</td></tr> </tbody></table>						Itns	Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	L+U	BSwap	nS	condHz	Penalty	4	0	4		1	8.0E+02	1.0E+00	0.0000000E+00	14		1	3.0E+07	_ r	6	1	2	1.2E-03	2	4.0E+02	1.0E+00	1.7331709E+06	13		1	1.3E+07	5.1E+00 _n rl	7	2	1	1.3E-03	3	2.7E+02	5.5E-01	1.7301152E+06	13			5.1E+00 _s l	7	3	0	7.5E-03	4	8.8E+01	5.4E-01	8.8193384E+05	13			2.8E+00 _ l	7	4	0	2.3E-02	5	2.9E+01	5.3E-01	8.4262007E+05	13			2.8E+00 _ l	7	5	0	6.9E-02	6	8.9E+00	5.2E-01	7.3075570E+05	13			2.8E+00 _ l	8	6	1	2.2E-01	7	2.3E+00	8.0E+01	4.4817383E+05	13		1	1.2E+04	2.8E+00 _ l	9	7	1	8.3E-01	8	1.7E-01	9.2E+00	2.4331006E+04	13		1	9.5E+03	2.8E+00 _ l	10	8	1	1.0E+00	9	6.5E-03	4.0E+01	5.3126219E+03	13	1	1	1.3E+02	2.8E+00 _	11	9	1	1.0E+00	10	4.6E-03	1.2E+01	5.1602362E+03	13			1	9.4E+01	2.8E+00 _	12	10	1	1.0E+00	11	2.3E-04	6.2E-02	5.1265653E+03	13			1	9.6E+01	2.8E+00 _	13	11	1	1.0E+00	12	(1.3E-08)	2.9E-04	5.1264981E+03	13			1	1.2E+02	2.8E+00 c	14	11	2	1.0E+00	12	(1.3E-08)	2.7E-04	5.1264981E+03	13			1	1.2E+02	2.8E+00 c	15	12	1	1.0E+00	13	(5.6E-13)	7.1E-05	5.1264981E+03	13			1	9.5E+01	2.8E+00 c	16	13	1	1.0E+00	14	(1.8E-14)	(1.0E-09)	5.1264981E+03	13			1	9.5E+01	2.8E+00 c
Itns	Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	L+U	BSwap	nS	condHz	Penalty																																																																																																																																																																																																											
4	0	4		1	8.0E+02	1.0E+00	0.0000000E+00	14		1	3.0E+07	_ r																																																																																																																																																																																																											
6	1	2	1.2E-03	2	4.0E+02	1.0E+00	1.7331709E+06	13		1	1.3E+07	5.1E+00 _n rl																																																																																																																																																																																																											
7	2	1	1.3E-03	3	2.7E+02	5.5E-01	1.7301152E+06	13			5.1E+00 _s l																																																																																																																																																																																																												
7	3	0	7.5E-03	4	8.8E+01	5.4E-01	8.8193384E+05	13			2.8E+00 _ l																																																																																																																																																																																																												
7	4	0	2.3E-02	5	2.9E+01	5.3E-01	8.4262007E+05	13			2.8E+00 _ l																																																																																																																																																																																																												
7	5	0	6.9E-02	6	8.9E+00	5.2E-01	7.3075570E+05	13			2.8E+00 _ l																																																																																																																																																																																																												
8	6	1	2.2E-01	7	2.3E+00	8.0E+01	4.4817383E+05	13		1	1.2E+04	2.8E+00 _ l																																																																																																																																																																																																											
9	7	1	8.3E-01	8	1.7E-01	9.2E+00	2.4331006E+04	13		1	9.5E+03	2.8E+00 _ l																																																																																																																																																																																																											
10	8	1	1.0E+00	9	6.5E-03	4.0E+01	5.3126219E+03	13	1	1	1.3E+02	2.8E+00 _																																																																																																																																																																																																											
11	9	1	1.0E+00	10	4.6E-03	1.2E+01	5.1602362E+03	13			1	9.4E+01	2.8E+00 _																																																																																																																																																																																																										
12	10	1	1.0E+00	11	2.3E-04	6.2E-02	5.1265653E+03	13			1	9.6E+01	2.8E+00 _																																																																																																																																																																																																										
13	11	1	1.0E+00	12	(1.3E-08)	2.9E-04	5.1264981E+03	13			1	1.2E+02	2.8E+00 c																																																																																																																																																																																																										
14	11	2	1.0E+00	12	(1.3E-08)	2.7E-04	5.1264981E+03	13			1	1.2E+02	2.8E+00 c																																																																																																																																																																																																										
15	12	1	1.0E+00	13	(5.6E-13)	7.1E-05	5.1264981E+03	13			1	9.5E+01	2.8E+00 c																																																																																																																																																																																																										
16	13	1	1.0E+00	14	(1.8E-14)	(1.0E-09)	5.1264981E+03	13			1	9.5E+01	2.8E+00 c																																																																																																																																																																																																										
E04VHF EXIT	0	-- finished successfully																																																																																																																																																																																																																					
E04VHF INFO	1	-- optimality conditions satisfied																																																																																																																																																																																																																					
Problem name	E04VJFE																																																																																																																																																																																																																						
No. of iterations	16	Objective value	5.1264981096E+03																																																																																																																																																																																																																				
No. of major iterations	13	Linear objective	0.0000000000E+00																																																																																																																																																																																																																				
Penalty parameter	2.780E+00	Nonlinear objective	5.1264981096E+03																																																																																																																																																																																																																				
No. of calls to funobj	105	No. of calls to funcon	105																																																																																																																																																																																																																				
Calls with modes 1,2 (known g)	14	Calls with modes 1,2 (known g)	14																																																																																																																																																																																																																				
Calls for forward differencing	48	Calls for forward differencing	48																																																																																																																																																																																																																				
Calls for central differencing	24	Calls for central differencing	24																																																																																																																																																																																																																				
No. of superbasics	1	No. of basic nonlinear	3																																																																																																																																																																																																																				
No. of degenerate steps	0	Percentage	0.00																																																																																																																																																																																																																				
Max x	2 1.0E+03	Max pi	3 5.5E+00																																																																																																																																																																																																																				

```

Max Primal infeas      0 0.0E+00  Max Dual infeas      1 1.0E-08
Nonlinear constraint violn  1.5E-11

Name          E04VJFE          Objective Value  5.1264981096E+03
Status        Optimal Soln    Iteration       16  Superbasics     1

Objective      (Min)
RHS
Ranges
Bounds

Section 1 - Rows

Number ...Row.. State ...Activity... Slack Activity ...Lower Limit. ...Upper Limit. .Dual Activity ..i
      5   r     1   EQ    -894.80000      0.00000    -894.80000    -894.80000    -4.38698     1
      6   r     2   EQ    -894.80000      0.00000    -894.80000    -894.80000    -4.10563     2
      7   r     3   EQ   -1294.80000      0.00000   -1294.80000   -1294.80000   -5.46328     3
      8   r     4   BS    -0.51511      0.03489    -0.55000      None       .           4
      9   r     5   BS     0.51511      1.06511    -0.55000      None       .           5

Section 2 - Columns

Number .Column. State ...Activity... .Obj Gradient. ...Lower Limit. ...Upper Limit. Reduced Gradnt m+j
      1   x     1   BS      0.11888      .      -0.55000      0.55000      0.00000     6
      2   x     2   BS     -0.39623      .      -0.55000      0.55000      0.00000     7
      3   x     3   SBS    679.94532     4.38698      .      1200.00000      0.00000     8
      4   x     4   BS    1026.06713     4.10563      .      1200.00000      0.00000     9

On exit from E04VHF, IFAIL =      0
Final objective value =      5126.5
Optimal X =      0.12      -0.40      679.95    1026.07

```
