NAG Fortran Library Routine Document

E02AKF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

E02AKF evaluates a polynomial from its Chebyshev-series representation, allowing an arbitrary index increment for accessing the array of coefficients.

2 Specification

```
SUBROUTINE E02AKF(NP1, XMIN, XMAX, A, IA1, LA, X, RESULT, IFAIL)
INTEGER            NP1, IA1, LA, IFAIL
real               XMIN, XMAX, A(LA), X, RESULT
```

3 Description

If supplied with the coefficients a_i, for $i = 0, 1, \ldots, n$, of a polynomial $p(\bar{x})$ of degree n, where

$$p(\bar{x}) = \tfrac{1}{2}a_0 + a_1T_1(\bar{x}) + \cdots + a_nT_n(\bar{x}),$$

this routine returns the value of $p(\bar{x})$ at a user-specified value of the variable x. Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. It is assumed that the independent variable $\bar{x}$ in the interval $[-1, +1]$ was obtained from the user's original variable x in the interval $[x_{\mathrm{min}}, x_{\mathrm{max}}]$ by the linear transformation

$$\bar{x} = \frac{2x - (x_{\mathrm{max}} + x_{\mathrm{min}})}{x_{\mathrm{max}} - x_{\mathrm{min}}}.$$

The coefficients a_i may be supplied in the array A, with any increment between the indices of array elements which contain successive coefficients. This enables the routine to be used in surface fitting and other applications, in which the array might have two or more dimensions.

The method employed is based upon the three-term recurrence relation due to Clenshaw (see Clenshaw (1955)), with modifications due to Reinsch and Gentleman (see Gentleman (1969)). For further details of the algorithm and its use see Cox (1973) and Cox and Hayes (1973).

4 References

Clenshaw C W (1955) A note on the summation of Chebyshev-series *Math. Tables Aids Comput.* **9** 118–120

Cox M G (1973) A data-fitting package for the non-specialist user *NPL Report NAC 40* National Physical Laboratory

Cox M G and Hayes J G (1973) Curve fitting: a guide and suite of algorithms for the non-specialist user *NPL Report NAC26* National Physical Laboratory

Gentleman W M (1969) An error analysis of Goertzel's (Watt's) method for computing Fourier coefficients *Comput. J.* **12** 160–165

5 Parameters

1: NP1 – INTEGER *Input*

On entry: $n + 1$, where n is the degree of the given polynomial $p(\bar{x})$.

Constraint: NP1 ≥ 1.

2: XMIN – ***real*** *Input*
3: XMAX – ***real*** *Input*

On entry: the lower and upper end-points respectively of the interval $[x_{\min}, x_{\max}]$. The Chebyshev-series representation is in terms of the normalised variable $\bar{x}$, where

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}.$$

Constraint: XMIN < XMAX.

4: A(LA) – ***real*** array *Input*

On entry: the Chebyshev coefficients of the polynomial $p(\bar{x})$. Specifically, element $1 + i \times$ IA1 must contain the coefficient a_i, for $i = 0, 1, \ldots, n$. Only these $n + 1$ elements will be accessed.

5: IA1 – INTEGER *Input*

On entry: the index increment of A. Most frequently, the Chebyshev coefficients are stored in adjacent elements of A, and IA1 must be set to 1. However, if, for example, they are stored in $\mathrm{A}(1), \mathrm{A}(4), \mathrm{A}(7), \ldots$, then the value of IA1 must be 3.

Constraint: IA1 ≥ 1.

6: LA – INTEGER *Input*

On entry: the dimension of the array A as declared in the (sub)program from which E02AKF is called.

Constraint: LA $\geq$ (NP1 $-$ 1) $\times$ IA1 $+$ 1.

7: X – ***real*** *Input*

On entry: the argument x at which the polynomial is to be evaluated.

Constraint: XMIN $\leq$ X $\leq$ XMAX.

8: RESULT – ***real*** *Output*

On exit: the value of the polynomial $p(\bar{x})$.

9: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, NP1 < 1,
or IA1 < 1,
or LA $\leq$ (NP1 $-$ 1) $\times$ IA1,
or XMIN $\geq$ XMAX.

IFAIL = 2

X does not satisfy the restriction XMIN $\leq$ X $\leq$ XMAX.

7 Accuracy

The rounding errors are such that the computed value of the polynomial is exact for a slightly perturbed set of coefficients $a_i + \delta a_i$. The ratio of the sum of the absolute values of the δa_i to the sum of the absolute values of the a_i is less than a small multiple of $(n+1)\times$***machine precision***.

8 Further Comments

The time taken by the routine is approximately proportional to $n+1$.

9 Example

Suppose a polynomial has been computed in Chebyshev-series form to fit data over the interval $[-0.5, 2.5]$. The following program evaluates the polynomial at 4 equally spaced points over the interval. (For the purposes of this example, XMIN, XMAX and the Chebyshev coefficients are supplied in DATA statements. Normally a program would first read in or generate data and compute the fitted polynomial.)

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     E02AKF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NP1, LA
      PARAMETER        (NP1=7,LA=NP1)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*     .. Local Scalars ..
      real             P, X, XMAX, XMIN
      INTEGER          I, IFAIL, M
*     .. Local Arrays ..
      real             A(LA)
*     .. External Subroutines ..
      EXTERNAL         E02AKF
*     .. Intrinsic Functions ..
      INTRINSIC        real
*     .. Data statements ..
      DATA             XMIN, XMAX/-0.5e0, 2.5e0/
      DATA             (A(I),I=1,NP1)/2.53213e0, 1.13032e0, 0.27150e0,
     +                 0.04434e0, 0.00547e0, 0.00054e0, 0.00004e0/
*     .. Executable Statements ..
      WRITE (NOUT,*) 'E02AKF Example Program Results'
      WRITE (NOUT,*)
      WRITE (NOUT,*) '   I   Argument  Value of polynomial'
      M = 4
      DO 20 I = 1, M
         X = (XMIN*real(M-I)+XMAX*real(I-1))/real(M-1)
         IFAIL = 0
*
         CALL E02AKF(NP1,XMIN,XMAX,A,1,LA,X,P,IFAIL)
*
         WRITE (NOUT,99999) I, X, P
   20 CONTINUE
      STOP
*
99999 FORMAT (1X,I4,F10.4,4X,F9.4)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 E02AKF Example Program Results

    I   Argument  Value of polynomial
    1   -0.5000       0.3679
    2    0.5000       0.7165
    3    1.5000       1.3956
    4    2.5000       2.7183
```