NAG Fortran Library Routine Document

D01GZF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

D01GZF calculates the optimal coefficients, for use by D01GCF and D01GDF, when the number of points is the product of two primes.

2 Specification

```
SUBROUTINE D01GZF(NDIM, NP1, NP2, VK, IFAIL)
INTEGER          NDIM, NP1, NP2, IFAIL
real             VK(NDIM)
```

3 Description

Korobov (1963) gives a procedure for calculating optimal coefficients for p-point integration over the n-cube $[0,1]^n$, when the number of points is

$$p = p_1 p_2 \qquad (1)$$

where p_1 and p_2 are distinct prime numbers.

The advantage of this procedure is that if p_1 is chosen to be the nearest prime integer to p_2^2, then the number of elementary operations required to compute the rule is of the order of $p^{4/3}$ which grows less rapidly than the number of operations required by D01GYF. The associated error is likely to be larger although it may be the only practical alternative for high values of p.

4 References

Korobov N M (1963) *Number Theoretic Methods in Approximate Analysis* Fizmatgiz, Moscow

5 Parameters

1: NDIM – INTEGER *Input*

On entry: the number of dimensions of the integral, n.

Constraint: NDIM ≥ 1.

2: NP1 – INTEGER *Input*

On entry: the larger prime factor p_1 of the number of points in the integration rule.

Constraint: NP1 must be a prime number ≥ 5.

3: NP2 – INTEGER *Input*

On entry: the smaller prime factor p_2 of the number of points in the integration rule. For maximum efficiency, p_2^2 should be close to p_1.

Constraint: NP2 must be a prime number such that NP1 $>$ NP2 ≥ 2.

4: VK(NDIM) – ***real*** array *Output*

On exit: the n optimal coefficients.

5: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, NDIM < 1.

IFAIL $= 2$

On entry, NP1 < 5,
or NP2 < 2,
or NP1 $\leq$ NP2.

IFAIL $= 3$

The value NP1 $\times$ NP2 exceeds the largest integer representable on the machine, and hence the optimal coefficients could not be used in a valid call of D01GCF.

IFAIL $= 4$

On entry, NP1 is not a prime number.

IFAIL $= 5$

On entry, NP2 is not a prime number.

IFAIL $= 6$

The precision of the machine is insufficient to perform the computation exactly. Try smaller values of NP1 or NP2, or use an implementation with higher precision.

7 Accuracy

The optimal coefficients are returned as exact integers (though stored in a ***real*** array).

8 Further Comments

The time taken by the routine grows at least as fast as $(p_1p_2)^{4/3}$. (See Section 3.)

9 Example

This example program calculates the Korobov optimal coefficients where the number of dimensons is 4 and the number of points is the product of the two prime numbers, 89 and 11.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     D01GZF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NDIM
      PARAMETER        (NDIM=4)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, NP1, NP2
*     .. Local Arrays ..
      real             VK(NDIM)
*     .. External Subroutines ..
      EXTERNAL         D01GZF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'D01GZF Example Program Results'
      NP1 = 89
      NP2 = 11
      WRITE (NOUT,*)
      WRITE (NOUT,99999) 'NDIM =', NDIM, ' NP1 =', NP1, ' NP2 =', NP2
      IFAIL = 0
*
      CALL D01GZF(NDIM,NP1,NP2,VK,IFAIL)
*
      WRITE (NOUT,*)
      WRITE (NOUT,99998) 'Coefficients =', (VK(I),I=1,NDIM)
      STOP
*
99999 FORMAT (1X,A,I3,A,I6,A,I6)
99998 FORMAT (1X,A,4F6.0)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 D01GZF Example Program Results

 NDIM =  4 NP1 =    89 NP2 =    11

 Coefficients =    1.  102.  614.  951.
```