

NAG Fortran Library Routine Document

C06FCF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

C06FCF calculates the discrete Fourier transform of a sequence of n complex data values (using a work array for extra speed).

2 Specification

```
SUBROUTINE C06FCF(X, Y, N, WORK, IFAIL)
  INTEGER          N, IFAIL
  real            X(N), Y(N), WORK(N)
```

3 Description

Given a sequence of n complex data values z_j , for $j = 0, 1, \dots, n-1$, this routine calculates their discrete Fourier transform defined by:

$$\hat{z}_k = a_k + ib_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \times \exp\left(-i \frac{2\pi jk}{n}\right), \quad k = 0, 1, \dots, n-1.$$

(Note the scale factor of $\frac{1}{\sqrt{n}}$ in this definition.)

To compute the inverse discrete Fourier transform defined by:

$$\hat{w}_k = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} z_j \times \exp\left(+i \frac{2\pi jk}{n}\right),$$

this routine should be preceded and followed by calls of C06GCF to form the complex conjugates of the z_j and the \hat{z}_k .

The routine uses the fast Fourier transform (FFT) algorithm in Brigham (1974). There are some restrictions on the value of n (see Section 5).

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice-Hall

5 Parameters

1: X(N) – **real** array *Input/Output*

On entry: if X is declared with bounds (0 : N – 1) in the (sub)program from which C06FCF is called, then X(j) must contain x_j , the real part of z_j , for $j = 0, 1, \dots, n-1$.

On exit: the real parts a_k of the components of the discrete Fourier transform. If X is declared with bounds (0 : N – 1) in the (sub)program from which C06FCF is called, then for $0 \leq k \leq n-1$, a_k is contained in X(k).

2: Y(N) – **real** array *Input/Output*

On entry: if Y is declared with bounds (0 : N – 1) in the (sub)program from which C06FCF is called, then Y(j) must contain y_j , the imaginary part of z_j , for $j = 0, 1, \dots, n-1$.

On exit: the imaginary parts b_k of the components of the discrete Fourier transform. If Y is declared with bounds $(0 : N - 1)$ in the (sub)program from which C06FCF is called, then for $0 \leq k \leq n - 1$, b_k is contained in $Y(k)$.

3: N – INTEGER *Input*

On entry: the number of data values, n . The largest prime factor of N must not exceed 19, and the total number of prime factors of N, counting repetitions, must not exceed 20.

Constraint: $N > 1$.

4: WORK(N) – *real* array *Workspace*

5: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

At least one of the prime factors of N is greater than 19.

IFAIL = 2

N has more than 20 prime factors.

IFAIL = 3

$N \leq 1$.

IFAIL = 4

7 Accuracy

Some indication of accuracy can be obtained by performing a subsequent inverse transform and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Further Comments

The time taken by the routine is approximately proportional to $n \times \log n$, but also depends on the factorization of n . The routine is somewhat faster than average if the only prime factors of n are 2, 3 or 5; and fastest of all if n is a power of 2.

9 Example

This program reads in a sequence of complex data values and prints their discrete Fourier transform (as computed by C06FCF).

It then performs an inverse transform, using C06GCF and C06FCF, and prints the sequence so obtained alongside the original data values.

9.1 Program Text

Note: the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*      C06FCF Example Program Text
*      Mark 14 Revised.  NAG Copyright 1989.
*      .. Parameters ..
      INTEGER          NMAX
      PARAMETER        (NMAX=20)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*      .. Local Scalars ..
      INTEGER          IFAIL, J, N
*      .. Local Arrays ..
real                WORK(NMAX), X(0:NMAX-1), XX(0:NMAX-1),
+                    Y(0:NMAX-1), YY(0:NMAX-1)
*      .. External Subroutines ..
      EXTERNAL         C06FCF, C06GCF
*      .. Executable Statements ..
      WRITE (NOUT,*) 'C06FCF Example Program Results'
*      Skip heading in data file
      READ (NIN,*)
20  READ (NIN,*,END=100) N
      IF (N.GT.1 .AND. N.LE.NMAX) THEN
          DO 40 J = 0, N - 1
              READ (NIN,*) X(J), Y(J)
              XX(J) = X(J)
              YY(J) = Y(J)
40      CONTINUE
          IFAIL = 0

*
          CALL C06FCF(X,Y,N,WORK,IFAIL)

*
          WRITE (NOUT,*)
          WRITE (NOUT,*) 'Components of discrete Fourier transform'
          WRITE (NOUT,*)
          WRITE (NOUT,*) '          Real          Imag'
          WRITE (NOUT,*)
          DO 60 J = 0, N - 1
              WRITE (NOUT,99999) J, X(J), Y(J)
60      CONTINUE

*
          CALL C06GCF(Y,N,IFAIL)
          CALL C06FCF(X,Y,N,WORK,IFAIL)
          CALL C06GCF(Y,N,IFAIL)

*
          WRITE (NOUT,*)
          WRITE (NOUT,*)
          +      'Original sequence as restored by inverse transform'
          WRITE (NOUT,*)
          WRITE (NOUT,*) '          Original          Restored'
          WRITE (NOUT,*)
          +      '          Real          Imag          Real          Imag'
          WRITE (NOUT,*)
          DO 80 J = 0, N - 1
              WRITE (NOUT,99999) J, XX(J), YY(J), X(J), Y(J)
80      CONTINUE
          GO TO 20
          ELSE
              WRITE (NOUT,*) 'Invalid value of N'
          END IF
100 STOP

*
99999 FORMAT (1X,I5,2F10.5,5X,2F10.5)
      END
```

9.2 Program Data

C06FCF Example Program Data

```

7
0.34907 -0.37168
0.54890 -0.35669
0.74776 -0.31175
0.94459 -0.23702
1.13850 -0.13274
1.32850  0.00074
1.51370  0.16298

```

9.3 Program Results

C06FCF Example Program Results

Components of discrete Fourier transform

	Real	Imag
0	2.48361	-0.47100
1	-0.55180	0.49684
2	-0.36711	0.09756
3	-0.28767	-0.05865
4	-0.22506	-0.17477
5	-0.14825	-0.30840
6	0.01983	-0.56496

Original sequence as restored by inverse transform

	Original		Restored	
	Real	Imag	Real	Imag
0	0.34907	-0.37168	0.34907	-0.37168
1	0.54890	-0.35669	0.54890	-0.35669
2	0.74776	-0.31175	0.74776	-0.31175
3	0.94459	-0.23702	0.94459	-0.23702
4	1.13850	-0.13274	1.13850	-0.13274
5	1.32850	0.00074	1.32850	0.00074
6	1.51370	0.16298	1.51370	0.16298
