

# STRUCTURAL MECHANICS MODULE

REFERENCE GUIDE

VERSION 3.5

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*Structural Mechanics Module Reference Guide*

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# Introduction

The Structural Mechanics Module 3.5 is an optional package that extends the COMSOL Multiphysics modeling environment with customized user interfaces and functionality optimized for structural analysis. Like all modules in the COMSOL family, it provides a library of prewritten ready-to-run models that make it quicker and easier to analyze discipline-specific problems.

This particular module solves problems in the fields of structural and solid mechanics, adding special elements such as beams, plates, and shells. It provides static, eigenfrequency, time-dependent, quasi-static transient, parametric, linear buckling, and frequency response analysis capabilities. You can use both linear and nonlinear material models such as elasto-plastic, viscoelastic, and hyperelastic models and include large deformation effects as well as contact and friction in an analysis. Material models can be isotropic, orthotropic, or fully anisotropic. Define loads, constraints, and material models in local, user-defined coordinate systems or in a global coordinate system. Piezoelectric materials can be analyzed with the constitutive relations on either stress-charge or strain-charge form.

All application modes in this module are fully multiphysics enabled, making it possible to couple to any other physics application mode in COMSOL Multiphysics or the other modules. Coupling structural analysis with thermal analysis is one example of multiphysics easily implemented with the Structural Mechanics

Module. Piezoelectric materials, coupling the electric field and strain in both directions are fully supported inside the module through special application modes solving for both the electric potential and displacement. Structural mechanics couplings are common in simulations done with COMSOL Multiphysics and occur in interaction with fluid flow (FSI), chemical reactions, acoustics, electric fields, magnetic fields, and optical wave propagation.

The underlying equations for structural mechanics are automatically available in all of the application modes—a feature unique to COMSOL Multiphysics. This makes nonstandard modeling easily accessible. The Structural Mechanics Module also features extensible material and beam cross-section libraries.

The documentation set for the Structural Mechanics Module consists of the *Structural Mechanics Module User's Guide*, the *Structural Mechanics Module Model Library*, and this *Structural Mechanics Module Reference Guide*. All books are available in PDF and HTML versions from the COMSOL Help Desk. This book contains reference information about application mode variables, command-line programming, and command-line functions that are specific to the Structural Mechanics Module (for example, shape-function classes for special element types).

### *Typographical Conventions*

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All COMSOL manuals use a set of consistent typographical conventions that should make it easy for you to follow the discussion, realize what you can expect to see on the screen, and know which data you must enter into various data-entry fields. In particular, you should be aware of these conventions:

- A **boldface** font of the shown size and style indicates that the given word(s) appear exactly that way on the COMSOL graphical user interface (for toolbar buttons in the corresponding tooltip). For instance, we often refer to the **Model Navigator**, which is the window that appears when you start a new modeling session in COMSOL; the corresponding window on the screen has the title **Model Navigator**. As another example, the instructions might say to click the **Multiphysics** button, and the boldface font indicates that you can expect to see a button with that exact label on the COMSOL user interface.
- The names of other items on the graphical user interface that do not have direct labels contain a leading uppercase letter. For instance, we often refer to the Draw toolbar; this vertical bar containing many icons appears on the left side of the user interface during geometry modeling. However, nowhere on the screen will you see

the term “Draw” referring to this toolbar (if it were on the screen, we would print it in this manual as the **Draw** menu).

- The symbol > indicates a menu item or an item in a folder in the **Model Navigator**. For example, **Physics>Equation System>Subdomain Settings** is equivalent to: On the **Physics** menu, point to **Equation System** and then click **Subdomain Settings**. **COMSOL Multiphysics>Heat Transfer>Conduction** means: Open the **COMSOL Multiphysics** folder, open the **Heat Transfer** folder, and select **Conduction**.
- A **Code** (monospace) font indicates keyboard entries in the user interface. You might see an instruction such as “Type 1.25 in the **Current density** edit field.” The monospace font also indicates COMSOL Script codes.
- An *italic* font indicates the introduction of important terminology. Expect to find an explanation in the same paragraph or in the Glossary. The names of books in the COMSOL documentation set also appear using an italic font.



## Application Mode Reference

This chapter lists the application mode variables you can access in the Structural Mechanics Module's application modes.

# Application Mode Variables

A large number of variables are available for use in expressions and during postprocessing. This chapter lists the variables defined for each application mode. In addition to the variables listed herein, you always have access to variables related to the geometry and the mesh, for example.

The application mode variable tables are organized as follows:

- The Name column lists the names of the variables that you can use in the equations or for postprocessing. Almost all variables, such as stresses and strains, are also available as the amplitude and phase of those variables by appending `_amp` or `_ph` to the variable name. Exceptions are variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`. A single index  $i$  on the displacement,  $u_i$ , means that  $u_i$  runs over the available global displacements, for example  $(u, v, w)$  in 3D. A single index on other names, for example  $s_i$ , means that  $i$  runs over the global space variables  $(x, y, z)$ . A double index  $s_{ij}$  means that  $ij$  runs over the combination of the space variables  $(xy, yz, xz)$ . Exceptions to these conventions are noted in the tables. For example,  $s_i$  means the principle stresses when  $i$  runs over  $(1, 2, 3)$ . For elasto-plastic materials the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined: the normally defined variable and the Gauss-point evaluated variable. Notationally, the latter are distinguished by an added suffix `Gp` to the variable name, for example, `sxGp` instead of `sx`. It is only possible to use the Gauss-point evaluated variables for postprocessing.

In a COMSOL Multiphysics model, the variable names get an underscore plus the application mode name appended to the names listed in the tables. For example, the default name of the Plane Stress application mode is `smps`. With this name the variable for the von Mises effective stress is `mises_smps`.

- The Symbol column lists the symbol notation for each variable.
- In the Analysis column you can see the availability of variables for the different analysis types. The following abbreviations are used:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Parametric	P

ANALYSIS	ABBREVIATION
Transient	T
Eigenfrequency	E
Damped eigenfrequency	D

- The Domain column lists whether variables are available on subdomains (S), boundaries (B), edges (E), points (P), or all domains (All).
- In the Description column you can find a short description for each variable.
- Where applicable, the Expression column lists the expression used for determining each variable.

# Continuum Application Modes

## *Solid, Stress-Strain*

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In addition to the variables in Table 2-1, almost all application-mode parameters are available as variables. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

Table 2-1 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  (`ui`) refers to the global displacements ( $u, v, w$ ).

For elasto-plastic materials the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined: the normally defined variable and the Gauss-point evaluated variable. Notationally, the latter are distinguished by an added suffix `Gp` to the variable name, for example, `sxGp` instead of `sx`. It is only possible to use the Gauss-point evaluated variables for postprocessing.

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<code>ui</code>	$u_i$	All	All	$x_i$ displacement	$u_i$
<code>uit</code>	$u_{it}$	T	All	$x_i$ velocity	$u_i$
<code>ui_amp</code>	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
<code>ui_ph</code>	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
<code>ui_t</code>	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
<code>ui_t_amp</code>	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
<code>ui_t_ph</code>	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
<code>ui_tt</code>	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i_{tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_i_{tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
p	p	All	All	Pressure	p
p_amp	$p_{amp}$	F	All	Pressure amplitude	p
p_ph	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
$\epsilon_i, \epsilon_{ij}$	$\epsilon_i, \epsilon_{ij}$	All	S	Strain, global coord. system	Engineering or Green strain for small and large deformations, respectively
$\epsilon_{pi}, \epsilon_{pij}$	$\epsilon_{pi}, \epsilon_{pij}$	S T	S	Plastic strain, global coord. system	
epe	$\epsilon_{pe}$	S T	S	Effective plastic strain	
$\epsilon_{il}, \epsilon_{ijl}$	$\epsilon_{il}, \epsilon_{ijl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$\epsilon_{it}, \epsilon_{ijt}$	$\epsilon_{it}, \epsilon_{ijt}$	F T	S	Velocity strain, global coord system	Engineering or Green strain time derivative for small and large deformations, respectively
$\epsilon_{itl}, \epsilon_{ijtl}$	$\epsilon_{itl}, \epsilon_{ijtl}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
$\sigma_i, \sigma_{ij}$	$\sigma_i, \tau_{ij}$	All	S	Cauchy stress, global coord. system	Defined differently depending on coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{il}, s_{ijl}$	$\sigma_i, \tau_{ij}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending on coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$s_{i\_t}, s_{ij\_t}$	$\sigma_{it}, \tau_{ijt}$	F T	S	Time derivative of Cauchy stress, global coord. system	
$s_{il\_t}, s_{ijl\_t}$	$\sigma_{ilt}, \tau_{ijlt}$	F T	S	Time derivative of Cauchy stress, user-defined local coord. system	
$S_i, S_{ij}$	$S_i, S_{ij}$	All	S	Second Piola Kirchhoff stress, global coord. system	
$S_{il}, S_{ijl}$	$S_{il}, S_{ijl}$	All	S	Second Piola Kirchhoff stress, user-defined local coord. system	
$s_{i\_t}, s_{ij\_t}$	$S_{it}, S_{ijt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	
$s_{il\_t}$	$S_{ilt}, S_{ijlt}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined local coord. system	
$P_i, P_{ij}$	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1, 2, 3$	
$e_i$	$\varepsilon_i$	All	S	Principal strains, $i=1, 2, 3$	
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i, j=1, 2, 3$	
$e_{ixj}$	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i, j=1, 2, 3$	

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
evol	$\varepsilon_{vol}$	All	All	Volumetric strain	Defined differently for small and large deformations
sq <i>i</i> , sq <i>ij</i>	$s_{qi}, s_{qij}$	All	S	Viscoelastic stress	Defined for viscoelastic materials: $s_{qi} = \sum_m 2G_m q_{im}$
E	$E$	All	All	Young's modulus	See "Elastic Moduli" on page 175 of the <i>Structural Mechanics Module User's Guide</i> .
nu	$\nu$	All	All	Poisson's ratio	
K	$K$	All	All	Bulk modulus	
G	$G$	All	All	Shear modulus	
lambLame	$\lambda$	All	All	Lamé constant $\lambda$	
muLame	$\mu$	All	All	Lamé constant $\mu$	
F <sub>ij</sub>	$F_{ij}, i,j=1, 2, 3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
c <sub>ij</sub>	$c_{ij}, i,j=1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
invF <sub>ij</sub>	$invF_{ij}, i,j=1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
detF	$detF$	All	All	Determinant of deformation gradient	$detF$
J	$J$	All	All	Volume ratio	$detF$
J <sub>el</sub>	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
I <sub>1</sub>	$I_1$	All	All	First invariant of the right Cauchy-Green strain tensor	$trace(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
I <sub>2</sub>	$I_2$	All	All	Second invariant of the right Cauchy-Green strain tensor	$\frac{1}{2}(I_1^2 - trace(C^2))$
I <sub>3</sub>	$I_3$	All	All	Third invariant of the right Cauchy-Green strain tensor	$J_{el}^2$

TABLE 2-I: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
II1	$\bar{I}_1$	All	All	First modified invariant of the right Cauchy-Green strain tensor	$I_1 J_{\text{el}}^{-\frac{2}{3}} = I_1 \bar{I}_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	All	Second modified invariant of the right Cauchy-Green strain tensor	$I_2 J_{\text{el}}^{-\frac{4}{3}} = I_1 \bar{I}_3^{-\frac{2}{3}}$
II3	$\bar{I}_3$	All	All	Third modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_3 = 1$
I1e	$I_1^\varepsilon$	All	All	First invariant of the Green strain tensor	$I_1^\varepsilon = \frac{1}{2}(I_1 - 3)$
I2e	$I_2^\varepsilon$	All	All	Second invariant of the Green strain tensor	$I_2^\varepsilon = \frac{1}{4}(I_2 - 2I_1 + 3)$
I3e	$I_3^\varepsilon$	All	All	Third invariant of the Green strain tensor	$I_3^\varepsilon = \frac{1}{8}(I_3 - I_2 + I_1 - 1)$
II1e	$\bar{I}_1^\varepsilon$	All	All	First modified invariant of the Green strain tensor	$\bar{I}_1^\varepsilon = \frac{1}{2}(\bar{I}_1 - 3)$
II2e	$\bar{I}_2^\varepsilon$	All	All	Second modified invariant of the Green strain tensor	$\bar{I}_2^\varepsilon = \frac{1}{4}((\bar{I}_2 - 3) - 2(\bar{I}_1 - 3))$
II3e	$\bar{I}_3^\varepsilon$	All	All	Third modified invariant of the Green strain tensor	$\bar{I}_3^\varepsilon = \frac{1}{8}((\bar{I}_1 - 3) - (\bar{I}_2 - 3))$
tresca	$\sigma_{\text{tresca}}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{\text{mises}}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending on material model and mixed or displacement formulation
Ent	$S_{\text{elast}}$	All	All	Entropy per unit volume	Defined only for small deformations and either no damping or loss factor damping. See definition in the theory section

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Qdamp	$Q_d$	F	All	Heat associated with mechanical losses in material	Defined only for viscoelastic materials and for other materials with either no damping or loss factor damping
$G_m$	$G_m$	All	S	Shear modulus, branch $m$	Defined for viscoelastic materials from the table in the <b>Subdomain Settings</b> dialog box
$\tau_m$	$\tau_m$	All	S	Relaxation time, branch $m$	
$G_0$	$G_0$	All	S	Instantaneous shear modulus	For viscoelastic materials: $G + \sum_m G_m$
Gstor	$G'$	D F	S	Dynamic storage modulus	For viscoelastic materials: $G + \sum_m G_m \frac{(\omega\tau_m)^2}{(1+(\omega\tau_m)^2)}$
Gloss	$G''$	D F	S	Dynamic loss modulus	For viscoelastic materials: $\sum_m G_m \frac{\omega\tau_m}{(1+(\omega\tau_m)^2)}$
ShiftWLF	$a_T$	All	S	WLF time shift factor	Defined for viscoelastic materials: $\log(a_T) = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}$
$e_{iig}, e_{ijig}$	$\varepsilon_{iig}, \varepsilon_{ijig}$	All	S	Initial strains in global coord. system	Defined differently depending on the material coordinate system
$e_{iil}, e_{ijil}$	$\varepsilon_{iil}, \varepsilon_{ijil}$	All	S	Initial strains in user-defined coord. system	
$s_{iig}, s_{ijig}$	$\sigma_{iig}, \tau_{ijig}$	All	S	Initial stresses in global coord. system.	
$s_{iil}, s_{ijil}$	$\sigma_{iil}, \tau_{ijil}$	All	S	Initial stresses in user-defined coord. system	
$F_{ig}$	$F_{ig}$	All	All	Body load, face load, edge load, point load, in global $x_i$ direction	Defined differently depending on force definition

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\text{RF}_i$	$\text{RF}_i$	S F P T	All	Reaction force in global $x_i$ direction on subdomain, face, edge, or point. Should only be used for integration, as they are defined only in the node points.	$\text{reacf}(u_i)$
$\text{RM}_i$	$\text{RM}_i$	S F P T	S B E	Reaction moment in global $x_i$ direction with respect to reference point on subdomain, face, or point. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times \text{RF}$
$F_{tij}$	$F_{tij}$	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i \mathbf{T}_{xj}$
$w_{cn\_cpi}$	$w_{cn}$	S P	B	Contact help variable for contact pair $i$	$\text{nojac}(T_{np}) - T_n$
$w_{ctxj\_cpi}$	$w_{ctj}$	P	B	Contact help variable for contact pair $i$	See definition in the theory section
$\text{slip}_{xj\_cpi}$	$\text{slip}_{xj}$	P	B	Slip vector $x_j$ dir. reference frame, contact pair $i$	$\text{map}(x_j) + x_j^m \text{ old}$
$\text{slip\_cpi}$	slip	P	B	Slip vector magnitude reference frame, contact pair $i$	$\sqrt{\sum_j (\text{slip}_{xj})^2}$
$\text{slipd}_{xrj\_cpi}$	$\text{slip}_{xrj}$	P	B	Slip vector $x_{rj}$ dir. deformed frame, contact pair $i$	$\sum_k \text{map}(F_{tij}) \text{slip}_{xj}$
$\text{slipd\_cpi}$	slipd	P	B	Slip vector magnitude deformed frame, contact pair $i$	$\sqrt{\sum_j (\text{slipd}_{xrj})^2}$

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Tnp_cpi	$T_{np}$	S P	B	Penalized contact pressure, contact pair $i$	See definition in the theory section
Ttxj_cpi	$T_{tpj}$	P	B	Penalized friction traction $x_j$ dir., contact pair $i$	See definition in the theory section
Tttrialxj_cpi	$T_{ttrialj}$	P	B	Trial friction force $x_j$ dir., contact pair $i$	See definition in the theory section
vslipxrj_cpi	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair $i$	$\frac{\text{slipd}_{xrj}}{t - t_{\text{old}}}$
vslip_cpi	$v_s$	P	B	Slip velocity magnitude, contact pair $i$	$\sqrt{\sum_j \text{vslip}_{xrj}^2}$
mu_cpi	$\mu$	S P	B	Frictional coefficient, contact pair $i$	See definition in the theory section
Ttcrit_cpi	$\mu$	P	B	Maximum friction traction, contact pair $i$	See definition in the theory section
gap_cpi	$g$	S P	B	Gap distance including offsets, contact pair $i$	$\text{Geomgap}_{\text{cpi}} - \text{offset}_{\text{cpi}} - \text{map}(\text{offset}_{\text{cpi}})$
contact_cpi	contact	S P	B	In contact variable, contact pair $i$	Defined differently depending on the pair setting
friction_cpi	friction	S P	B	Enabling friction variable, contact pair $i$	contact_cpi_old
PMLxi	$\text{PML}x_i$	F	S	PML coordinate $x_i$ , Cartesian PML	$\begin{aligned} & \text{sign}(x_i - X_{i0} + \text{eps})  x_i - X_{i0} + \text{eps} ^n \\ & \times L_{x_i}(1-i)/dx_i^n \end{aligned}$ $n \equiv \text{PML scaling exponent}$
rx	$r_x$	F	S	$r$ vector in PML cylinder, $x$ -coord., cylindrical PML	$(y_{\text{axis}}^2 + z_{\text{axis}}^2)(x - x_0) -$ $(y_{\text{axis}}(y - y_0) + z_{\text{axis}}(z - z_0))x_{\text{axis}}$
ry	$r_y$	F	S	$r$ vector in PML cylinder, $y$ -coord., cylindrical PML	$(z_{\text{axis}}^2 + x_{\text{axis}}^2)(y - y_0) -$ $(z_{\text{axis}}(z - z_0) + x_{\text{axis}}(x - x_0))y_{\text{axis}}$

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
rz	$r_z$	F	S	$r$ vector in PML cylinder, $z$ -coord., cylindrical PML	$(x_{\text{axis}}^2 + y_{\text{axis}}^2)(z - z_0) - (x_{\text{axis}}(x - x_0) + y_{\text{axis}}(y - y_0))z_{\text{axis}}$
normr	normr	F	S	$r$ vector in PML cylinder, norm, cylindrical PML	$(rx^2 + ry^2 + rz^2)^{1/2}$
R	$R$	F	S	Scaled radial coordinate, cylindrical PML	$R_0 + (\text{normr} - R_0)^n L_r (1-i)/dr^n$ $n \equiv \text{PML scaling exponent}$
PMLxi	$\text{PML}x_i$	F	S	PML coordinate $x_i$ , cylindrical PML	$x_i + (-1 + R/\text{normr})rx_i + (\mathbf{r}_{\text{axis}} \cdot (\mathbf{x} - \mathbf{x}_0)) /  \mathbf{r}_{\text{axis}}  - Z_0)^n L_z (1-i)/dz^n - \mathbf{r}_{\text{axis}} \cdot (\mathbf{x} - \mathbf{x}_0)x_{i\text{axis}} /  \mathbf{r}_{\text{axis}} ^2$
R	$R$	F	S	Scaled radial coordinate, spherical PML	$R_0 + (\Delta_0 - R_0)^n (1-i)L_r/dr^n$ $\Delta_0 \equiv [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}$ $n \equiv \text{PML scaling exponent}$
PMLxi	$\text{PML}x_i$	F	S	PML coordinate $x_i$ , spherical PML	$R(x_i - x_{0i})/\Delta_0$
Jxixj	$\mathbf{J}_{x_i x_j}$	F	S	PML transformation matrix, element $x_i x_j$	$\frac{\partial}{\partial x_j} \text{PML}x_i$
invJxixj		F	S	PML inverse transformation matrix, element $x_i x_j$	$(\mathbf{J}^{-1})_{x_i x_j}$
uiPMLxj		F	S	PML $x_j$ derivative of $x_i$ displacement	$\sum_k \frac{\partial u_i}{\partial x_k} (\mathbf{J}^{-1})_{x_k x_j}$
detJ	$ \mathbf{J} $	F	S	Determinant of PML transformation matrix	$\det(\mathbf{J})$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), edg (edge), bnd (boundary), and sub (subdomain) refer to the different domain levels.

TABLE 2-2: SOLID APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$\text{RF}_{itot}$	$\text{RF}_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
$F_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction
$\text{RM}_{itot}$	$\text{RM}_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point
$M_{itot}$	$M_{itot}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point
$\text{RF}_{itotdom}$	$\text{RF}_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt, edg, bnd, and sub)
$F_{itotdom}$	$F_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt, edg, bnd, and sub)
$\text{RM}_{itotdom}$	$\text{RM}_{itotdom}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, edg, bnd, and sub)
$M_{itotdom}$	$M_{itotdom}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, edg, bnd, and sub)

### Plane Stress

In addition to the variables listed in Table 2-3, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

- $\text{sx\_amp}$  is the amplitude of the normal stress in the  $x$  direction
- $\text{ex\_ph}$  is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added `Gp` to the variable name. Example, `sxGp` instead of `sx`. The Gauss point evaluated variables can only be used for postprocessing.

Table 2-3 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements  $(u, v)$ .

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
$p$	$p$	All	All	Pressure	$p$
$p_{amp}$	$p_{amp}$	F	All	Pressure amplitude	$ p $
$p_{ph}$	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
$\text{disp}$	$\text{disp}$	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\epsilon_i, \epsilon_z, \epsilon_{xy}$	$\epsilon_i, \epsilon_z, \epsilon_{xy}$	All	S	Strain global system	Engineering or Green strain for small and large deformations, respectively. $\epsilon_z$ defined differently if loss factor damping is used.
$\epsilon_{pi}, \epsilon_{pz}, \epsilon_{pxy}$	$\epsilon_{pi}, \epsilon_{pz}, \epsilon_{pxy}$	S T	S	Plastic strain global system	
$\epsilon_{pe}$	$\epsilon_{pe}$	S T	S	Effective plastic strain	
$\epsilon_{il}, \epsilon_{xyl}$	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$\epsilon_{it}, \epsilon_{zt}, \epsilon_{xyt}$	$\epsilon_{it}, \epsilon_{zt}, \epsilon_{xyt}$	F T	S	Velocity strain, global coord. system	Defined differently depending on small or large deformation and analysis type
$\epsilon_{ilt}, \epsilon_{xylt}$	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
$\sigma_i, \sigma_{xy}$	$\sigma_i, \tau_{xy}$	All	S	Cauchy stress, global coord. system	Defined differently depending on material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$\sigma_{il}, \sigma_{xyl}$	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending on material model, and if loss factor damping is used
$\sigma_{it}, \sigma_{xyt}$	$\sigma_{it}, \tau_{xyt}$	F T	S	Time derivative of Cauchy stress, global coord. system	Defined differently depending on material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$\sigma_{ilt}, \sigma_{xylt}$	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	
$S_i, S_{xy}$	$S_i, S_{xy}$	All	S	Second Piola Kirchhoff stress, global coord. system	
$S_{il}, S_{xyl}$	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending on material model, and if loss factor damping is used

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_i_t, S_{xyt}$	$S_{it}, S_{xyt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	Defined differently depending on material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il\_t}, S_{xyl\_t}$	$S_{it}, S_{xy}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined coord. system	
$P_i, P_{ij}$	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
$\sigma_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
$\epsilon_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	
$\sigma_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$\epsilon_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
evol	$\epsilon_{vol}$	All	All	Volumetric strain	Defined differently depending on small or large displacement
$s_{qi}, s_{qij}$	$s_{qi}, s_{qij}$	All	S	Viscoelastic stress	Defined for viscoelastic materials: $s_{qi} = \sum_m 2G_m q_{im}$
E	$E$	All	All	Young's modulus	See "Elastic Moduli" on page 175 of the <i>Structural Mechanics Module User's Guide</i> .
nu	$\nu$	All	All	Poisson's ratio	
K	$K$	All	All	Bulk modulus	
G	$G$	All	All	Shear modulus	
lambLame	$\lambda$	All	All	Lamé constant $\lambda$	
muLame	$\mu$	All	All	Lamé constant $\mu$	
$F_{ij}$	$F_{ij}, i,j=1, 2, 3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \bar{\mathbf{X}}}$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$c_{ij}$	$c_{ij}, i, j = 1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
$\det F$	$\det F$	All	All	Determinant of deformation gradient	$\det F$
$\text{inv}F_{ij}$	$\text{inv}F_{ij}, i, j = 1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
$J$	$J$	All	All	Volume ratio	$\det F$
$J_{\text{el}}$	$J_{\text{el}}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
$I_1$	$I_1$	All	All	First strain invariant of the right Cauchy-Green strain tensor	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
$I_2$	$I_2$	All	All	Second strain invariant of the right Cauchy-Green strain tensor	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
$I_3$	$I_3$	All	All	Third strain invariant of the right Cauchy-Green strain tensor	$J_{\text{el}}^2$
$\bar{I}_1$	$\bar{I}_1$	All	All	First modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_1 J_{\text{el}}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
$\bar{I}_2$	$\bar{I}_2$	All	All	Second modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_2 J_{\text{el}}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
$\bar{I}_3$	$\bar{I}_3$	All	All	Third modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_3 = 1$
$I_1^\varepsilon$	$I_1^\varepsilon$	All	All	First invariant of the Green strain tensor	$I_1^\varepsilon = \frac{1}{2}(I_1 - 3)$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
I2e	$I_2^\varepsilon$	All	All	Second invariant of the Green strain tensor	$I_2^\varepsilon = \frac{1}{4}(I_2 - 2I_1 + 3)$
I3e	$I_3^\varepsilon$	All	All	Third invariant of the Green strain tensor	$I_3^\varepsilon = \frac{1}{8}(I_3 - I_2 + I_1 - 1)$
II1e	$\bar{I}_1^\varepsilon$	All	All	First modified invariant of the Green strain tensor	$\bar{I}_1^\varepsilon = \frac{1}{2}(\bar{I}_1 - 3)$
II2e	$\bar{I}_2^\varepsilon$	All	All	Second modified invariant of the Green strain tensor	$\bar{I}_2^\varepsilon = \frac{1}{4}((\bar{I}_2 - 3) - 2(\bar{I}_1 - 3))$
II3e	$\bar{I}_3^\varepsilon$	All	All	Third modified invariant of the Green strain tensor	$\bar{I}_3^\varepsilon = \frac{1}{8}((\bar{I}_1 - 3) - (\bar{I}_2 - 3))$
tresca	$\sigma_{\text{tresca}}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{\text{mises}}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending on material model and if mixed or displacement formulation
Ent	$S_{\text{elast}}$	All	All	Entropy per unit volume	Defined only for small deformations and either no damping or loss factor damping. See definition in the theory section
Qdamp	$Q_d$	F	All	Heat associated with mechanical losses in material	Defined only for viscoelastic materials and for other materials with either no damping or loss factor damping
$G_m$	$G_m$	All	S	Shear modulus, branch $m$	Defined for viscoelastic materials from the table in the <b>Subdomain Settings</b> dialog box
$\tau_m$	$\tau_m$	All	S	Relaxation time, branch $m$	
$G_0$	$G_0$	All	S	Instantaneous shear modulus	For viscoelastic materials: $G + \sum_m G_m$
Gstor	$G'$	D F	S	Dynamic storage modulus	For viscoelastic materials: $G + \sum_m G_m \frac{(\omega \tau_m)^2}{(1 + (\omega \tau_m)^2)}$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Gloss	$G''$	D F	S	Dynamic loss modulus	For viscoelastic materials: $\sum_m G_m \frac{\omega\tau_m}{(1 + (\omega\tau_m)^2)}$
ShiftWLF	$a_T$	All	S	WLF time shift factor	Defined for viscoelastic materials: $\log(a_T) = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}$
Tai	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	Defined differently depending on small or large deformation
eiig, ezig, exyig	$\epsilon_{iig}, \epsilon_{zig}, \epsilon_{xyig}$	All	S	Initial strains in global coord. system.	Defined differently depending on the material coordinate system
eiil, exyil	$\epsilon_{iil}, \epsilon_{xyil}$	All	S	Initial strains in user-defined coord. system	
siig, szig, sxyig	$\sigma_{iig}, \sigma_{zig}, \tau_{xyig}$	All	S	Initial stresses in global coord. system.	
siil, sxyil	$\sigma_{iil}, \tau_{xyil}$	All	S	Initial stresses in user-defined coord. system	
Fig	$F_{ig}$	All	S	Point, edge, or body load, in global $x_i$ direction	Defined differently depending on force definition
RF <sub>i</sub>	$RF_i$	S F P T	All	Reaction force in global $x_i$ direction on subdomain, boundary, or point. Should only be used for integration, as they are defined only in the node points.	$\text{reacf}(u_i)$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\text{RM}_i$	$\text{RM}_i$	S F P T	S B	Reaction moment in global $x_i$ direction with respect to reference point on subdomain or boundary. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times \text{RF}$
$F_{tij}$	$F_{tij}$	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i \mathbf{T} \mathbf{x}_j$
$w_{cn\_cp\ i}$	$w_{cn}$	S P	B	Contact help variable for contact pair $i$	$\text{nojac}(T_{np}) - T_n$
$w_{ctxj\_cp\ i}$	$w_{ctj}$	P	B	Contact help variable for contact pair $i$	See definition in the theory section
$\text{slip}_{xj\_cp\ i}$	$\text{slip}_{xj}$	P	B	Slip vector $x_j$ dir. reference frame, contact pair $i$	$\text{map}(x_j) + x_j^m \text{ old}$
$\text{slip\_cp\ i}$	slip	P	B	Slip vector magnitude reference frame, contact pair $i$	$\sqrt{\sum_j \text{slip}_{xj}^2}$
$\text{slipd}_{xrj\_cp\ i}$	$\text{slip}_{xrj}$	P	B	Slip vector $xrj$ dir. deformed frame, contact pair $i$	$\sum_j \text{map}(F_{tij}) \text{slip}_{xj}$
$\text{slipd\_cp\ i}$	slipd	P	B	Slip vector magnitude deformed frame, contact pair $i$	$\sqrt{\sum_j \text{slipd}_{xrj}^2}$
$T_{np\_cp\ i}$	$T_{np}$	S P	B	Penalized contact pressure, contact pair $i$	See definition in the theory section

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Ttxpj_cpi	$T_{tpj}$	P	B	Penalized friction force $x_j$ dir., contact pair $i$	See definition in the theory section
Tttrialxj_cpi	$T_{ttrialj}$	P	B	Trial friction force $x_j$ dir., contact pair $i$	See definition in the theory section
vslipxrj_cpi	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair $i$	$\frac{\text{slip}_{xri}}{t - t_{\text{old}}}$
vslip_cpi	$v_s$	P	B	Slip velocity, contact pair $i$	$\sqrt{\sum_j (\text{vslip}_{xrij})^2}$
mu_cpi	$\mu$	S P	B	Frictional coefficient, contact pair $i$	See definition in the theory section
Ttcrit_cpi	$\mu$	P	B	Maximum frictional traction, contact pair $i$	See definition in the theory section
gap_cpi	$g$	S P	B	Gap distance including offsets, contact pair $i$	$\text{Geomgap}_{\text{cpi}} - \text{offset}_{\text{cpi}} - \text{map}(\text{offset}_{\text{cpi}})$
contact_cpi	contact	S P	B	In contact variable, contact pair $i$	Defined differently depending on the pair setting
friction_cpi	friction	S P	B	Enabling friction variable, contact pair $i$	$\text{contact}_{\text{cpi\_old}}$
PMLxi	$\text{PML}_{xi}$	F	S	PML coordinate $x_i$ , Cartesian PML	$\text{sign}(x_i - X_{i0} + \text{eps})  x_i - X_{i0} + \text{eps} ^n \times L_{x_i}(1-i)/dx_i^n$ $n \equiv \text{PML scaling exponent}$
R	$R$	F	S	Scaled radial coordinate, cylindrical PML	$R_0 + (\delta_0 - R_0)^n L_r(1-i)/dr^n$ $n \equiv \text{PML scaling exponent}$
PMLxi	$\text{PML}_{xi}$	F	S	PML coordinate $x_i$ , cylindrical PML	$R(x_i - x_{0i})/\delta_0$ . $\delta_0 \equiv [(x - x_0)^2 + (y - y_0)^2]^{1/2}$
Jxixj	$J_{x_ix_j}$	F	S	PML transformation matrix, element $xx$	$\frac{\partial}{\partial x_j} \text{PML}_{xi}$

TABLE 2-3: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
invJ $x_i x_j$		F	S	PML inverse transformation matrix, element $x_i x_j$	$(\mathbf{J}^{-1})_{x_i x_j}$
detJ	$\mathbf{J}$	F	S	Determinant of PML transformation matrix	$\det(\mathbf{J})$
$u_i$ PML $x_j$		F	S	PML $x_j$ derivative of $x_i$ displacement	$\sum_k \frac{\partial u_i}{\partial x_k} (\mathbf{J}^{-1})_{x_k x_j}$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), bnd (boundary), and sub (subdomain) refer to the different domain levels.

TABLE 2-4: PLANE STRESS APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RF $_{itot}$	RF $_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
F $_{itot}$	F $_{itot}$	S F P T	Total applied force global $x_i$ -direction
RM $_{itot}$	RM $_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point
M $_{itot}$	M $_{itot}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point
RF $_{itotdom}$	RF $_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)
F $_{itotdom}$	F $_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)
RM $_{itotdom}$	RM $_{itotdom}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)
M $_{itotdom}$	M $_{itotdom}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)

## Plane Strain

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In addition to the variables in Table 2-5, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added `Gp` to the variable name. Example, `sxGp` instead of `sx`. It is only possible to use the Gauss point evaluated variables for postprocessing.

Table 2-5 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  ( $ui$ ) refers to the global displacements  $(u, v)$ .

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<code>ui</code>	$u_i$	All	All	$x_i$ displacement	$u_i$
<code>uit</code>	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
<code>u_amp, v_amp</code>	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
<code>ui_ph</code>	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
<code>ui_t</code>	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
<code>ui_t_amp</code>	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
<code>ui_t_ph</code>	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
<code>ui_tt</code>	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
<code>ui_tt_amp</code>	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<i>ui_tt_ph</i>	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
p	$p$	All	All	Pressure	$p$
p_amp	$p_{amp}$	F	All	Pressure amplitude	$ p $
p_ph	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
$\epsilon_i, \epsilon_{xy}$	$\epsilon_i, \epsilon_{xy}$	All	S	Strain, global coord. system	Engineering or Green strain for small and large deformations, respectively
$\epsilon_{pi}, \epsilon_{pxy}$	$\epsilon_{pi}, \epsilon_{pxy}$	S T	S	Plastic strain, global coord. system	
epe	$\epsilon_{pe}$	S T	S	Effective plastic strain, global coord. system	
$\epsilon_{il}, \epsilon_{xyl}$	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_i T_{\text{coord}}$
$\epsilon_{it}, \epsilon_{xyt}$	$\epsilon_{it}, \epsilon_{xyt}$	F T	S	Velocity strain, global coord. system	Defined differently depending on small or large deformation and analysis type
$\epsilon_{ilt}, \epsilon_{xylt}$	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_i T_{\text{coord}}$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_i, s_z, s_{xy}$	$\sigma_i, \sigma_z, \tau_{xy}$	All	S	Cauchy stress, global coord. system	Defined differently depending on material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il}, s_{xyl}$	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	
$s_{i\_t}, s_{z\_t}, s_{xy\_t}$	$\sigma_{it}, \sigma_z, \tau_{xyt}$	F T	S	Time derivative of Cauchy stress, global coord. system	
$s_{il\_t}, s_{xyl\_t}$	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	
$S_i, S_z, S_{xy}$	$S_i, S_z, S_{xy}$	All	S	Second Piola Kirchhoff stress, global coord. system	
$S_{il}, S_{xyl}$	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	
$S_{i\_t}, S_{z\_t}, S_{xy\_t}$	$S_{it}, S_{zt}, S_{xyt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	
$S_{il\_t}, S_{xyl\_t}$	$S_{ilt}, S_{xylt}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined coord. system	
$P_i, P_z, P_{xy}$	$P_i, P_z, P_{xy}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
$e_i$	$\varepsilon_i$	All	S	Principal strains, $i=1,2,3$	
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$e_{ixj}$	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
evol	$\varepsilon_{vol}$	All	All	Volumetric strain	Defined differently for small and large displacement
$s_{qi}, s_{qij}$	$s_{qi}, s_{qij}$	All	S	Viscoelastic stress	Defined for viscoelastic materials: $s_{qi} = \sum_m 2G_m q_{im}$
E	$E$	All	All	Young's modulus	See "Elastic Moduli" on page 175 of the <i>Structural Mechanics Module User's Guide</i> .
nu	$\nu$	All	All	Poisson's ratio	
K	$K$	All	All	Bulk modulus	
G	$G$	All	All	Shear modulus	
lambLame	$\lambda$	All	All	Lamé constant $\lambda$	
muLame	$\mu$	All	All	Lamé constant $\mu$	
$F_{ij}$	$F_{ij}$ , $i,j=1, 2, 3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$c_{ij}$	$c_{ij}$ , $i,j=1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
detF	$\det F$	All	All	Determinant of deformation gradient	$\det F$
invFij	$\text{inv}F_{ij}$ , $i,j=1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
J	$J$	All	All	Volume ratio	$\det F$
$J_{el}$	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
$I_1$	$I_1$	All	All	First invariant of the right Cauchy-Green strain tensor	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
$I_2$	$I_2$	All	All	Second invariant of the right Cauchy-Green strain tensor	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
$I_3$	$I_3$	All	All	Third invariant of the right Cauchy-Green strain tensor	$J_{el}^2$
$II_1$	$\bar{I}_1$	All	All	First modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_1 J_{el}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
$II_2$	$\bar{I}_2$	All	All	Second modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_2 J_{el}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
$II_3$	$\bar{I}_3$	All	All	Third modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_3 = 1$
$I_{1e}$	$I_1^\varepsilon$	All	All	First invariant of the Green strain tensor	$I_1^\varepsilon = \frac{1}{2}(I_1 - 3)$
$I_{2e}$	$I_2^\varepsilon$	All	All	Second invariant of the Green strain tensor	$I_2^\varepsilon = \frac{1}{4}(I_2 - 2I_1 + 3)$
$I_{3e}$	$I_3^\varepsilon$	All	All	Third invariant of the Green strain tensor	$I_3^\varepsilon = \frac{1}{8}(I_3 - I_2 + I_1 - 1)$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
II1e	$\bar{I}_1^\varepsilon$	All	All	First modified invariant of the Green strain tensor	$\bar{I}_1^\varepsilon = \frac{1}{2}(\bar{I}_1 - 3)$
II2e	$\bar{I}_2^\varepsilon$	All	All	Second modified invariant of the Green strain tensor	$\bar{I}_2^\varepsilon = \frac{1}{4}((\bar{I}_2 - 3) - 2(\bar{I}_1 - 3))$
II3e	$\bar{I}_3^\varepsilon$	All	All	Third modified invariant of the Green strain tensor	$\bar{I}_3^\varepsilon = \frac{1}{8}((\bar{I}_1 - 3) - (\bar{I}_2 - 3))$
tresca	$\sigma_{\text{tresca}}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{\text{mises}}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending on material model and if mixed or displacement formulation
Ent	$S_{\text{elast}}$	All	All	Entropy per unit volume	Defined only for small deformations and either no damping or loss factor damping. See definition in the theory section
Qdamp	$Q_d$	F	All	Heat associated with mechanical losses in material	Defined only for viscoelastic materials and for other materials with either no damping or loss factor damping
Gm	$G_m$	All	S	Shear modulus, branch $m$	Defined for viscoelastic materials from the table in the <b>Subdomain Settings</b> dialog box
taum	$\tau_m$	All	S	Relaxation time, branch $m$	
G0	$G_0$	All	S	Instantaneous shear modulus	For viscoelastic materials: $G + \sum_m G_m$
Gstor	$G'$	D F	S	Dynamic storage modulus	For viscoelastic materials: $G + \sum_m G_m \frac{(\omega \tau_m)^2}{(1 + (\omega \tau_m)^2)}$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Gloss	$G''$	D F	S	Dynamic loss modulus	For viscoelastic materials: $\sum_m G_m \frac{\omega \tau_m}{(1 + (\omega \tau_m)^2)}$
ShiftWLF	$a_T$	All	S	WLF time shift factor	Defined for viscoelastic materials: $\log(a_T) = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}$
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in $x_i$ direction	Defined differently depending on the force definition
e <sub>iig</sub> , e <sub>zig</sub> , e <sub>xyig</sub>	$\epsilon_{iig}$ , $\epsilon_{zig}$ , $\epsilon_{xyig}$	All	S	Initial strains in global coord. system	Defined differently depending on the material coordinate system
e <sub>iil</sub> , e <sub>xyil</sub>	$\epsilon_{iil}$ , $\epsilon_{xyil}$	All	S	Initial strains in user-defined coord. system	
s <sub>iig</sub> , s <sub>zig</sub> , s <sub>xyig</sub>	$\sigma_{iig}$ , $\sigma_{zig}$ , $\tau_{xyig}$	All	S	Initial stresses in global coord. system	
s <sub>iil</sub> , s <sub>xyil</sub>	$\sigma_{iil}$ , $\tau_{xyil}$	All	S	Initial stresses in user-defined coord. system	
F <sub>ig</sub>	$F_{ig}$	All	S	Point, Edge, Body load in global $x_i$ direction	Defined differently depending on the force definition
RF <sub>i</sub>	RF <sub>i</sub>	S F P T	All	Reaction force in global $x_i$ direction on subdomain, boundary, or point. Should only be used for integration, as they are defined only in the node points.	reacff( $u_i$ )

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$RM_i$	$RM_i$	S F P T	S B	Reaction moment in global $x_i$ direction with respect to reference point on subdomain or boundary. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF$
$F_{tij}$	$F_{tij}$	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i T x_j$
$wcn\_cp_i$	$w_{cn}$	SP	B	Contact help variable for contact pair $i$	$\text{nojac}(T_{np}) - T_n$
$wctxj\_cp_i$	$w_{ctj}$	P	B	Contact help variable for contact pair $i$	See definition in the theory section
$\text{slip}_{xj\_cp_i}$	$\text{slip}_{xj}$	P	B	Slip vector $x_j$ dir. reference frame, contact pair $i$	$\text{map}(x_j) + x_j^m_{\text{old}}$
$\text{slip\_cp}_i$	slip	P	B	Slip vector magnitude reference frame, contact pair $i$	$\sqrt{\sum_j \text{slip}_{xj}^2}$
$\text{slipd}_{xrj\_cp_i}$	$\text{slip}_{xrj}$	P	B	Slip vector $x_{rj}$ dir. deformed frame, contact pair $i$	$\sum_j \text{map}(F_{tij}) \text{slip}_{xj}$
$\text{slipd\_cp}_i$	slipd	P	B	Slip vector magnitude deformed frame, contact pair $i$	$\sqrt{\sum_j (\text{slipd}_{xrj})^2}$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Tnp_cpi	$T_{np}$	SP	B	Penalized contact pressure, contact pair $i$	See definition in the theory section
Ttxpj_cpi	$T_{tpj}$	P	B	Penalized friction force $x_j$ dir., contact pair $i$	See definition in the theory section
Tttrialxj_cpi	$T_{ttrialj}$	P	B	Trial friction force $x_j$ dir., contact pair $i$	See definition in the theory section
vslipxrj_cpi	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair $i$	$\frac{\text{slipd}_{xrj}}{t - t_{\text{old}}}$
vslip_cpi	$v_s$	P	B	Slip velocity, contact pair $i$	$\sqrt{\sum_j (\text{vslip}_{xrj})^2}$
mu_cpi	$\mu$	SP	B	Frictional coefficient, contact pair $i$	See definition in the theory section
Ttcrit_cpi	$\mu$	P	B	Maximum friction traction, contact pair $i$	See definition in the theory section
gap_cpi	$g$	SP	B	Gap distance including offsets, contact pair $i$	$\text{Geomgap}_{\text{cpi}} - \text{offset}_{\text{cpi}} - \text{map}(\text{offset}_{\text{cpi}})$
contact_cpi	contact	SP	B	In contact variable, contact pair $i$	Defined differently depending on the pair setting
friction_cpi	friction	SP	B	Enabling friction variable, contact pair $i$	contact_cpi_old
PMLxi	$\text{PML}_{xi}$	F	S	PML coordinate $x_i$ , Cartesian PML	$\begin{aligned} & \text{sign}(x_i - X_{i0} + \text{eps})  x_i - X_{i0} + \text{eps} ^n \\ & \times L_{x_i}(1-i)/dx_i^n \end{aligned}$ $n \equiv \text{PML scaling exponent}$

TABLE 2-5: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
R	$R$	F	S	Scaled radial coordinate, cylindrical PML	$R_0 + (\delta_0 - R_0)^n L_r (1 - i) / dr^n$ $n \equiv \text{PML scaling exponent}$
PML $x_i$	$\text{PML}x_i$	F	S	PML coordinate $x_i$ , cylindrical PML	$R(x_i - x_{0i})/\delta_0$ , $\delta_0 \equiv [(x - x_0)^2 + (y - y_0)^2]^{1/2}$
$J_{x_i x_j}$	$J_{x_i x_j}$	F	S	PML transformation matrix, element $xx$	$\frac{\partial}{\partial x_j} \text{PML}x_i$
inv $J_{x_i x_j}$		F	S	PML inverse transformation matrix, element $x_i x_j$	$(J^{-1})_{x_i x_j}$
det $J$	$ J $	F	S	Determinant of PML transformation matrix	$\det(J)$
$u_i \text{PML}x_j$		F	S	PML $x_j$ derivative of $x_i$ displacement	$\sum_k \frac{\partial u_i}{\partial x_k} (J^{-1})_{x_k x_j}$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), bnd (boundary), and sub (subdomain) refer to the different domain levels.

TABLE 2-6: PLANE STRAIN APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RF $_{itot}$	$RF_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
F $_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction
RM $_{itot}$	$RM_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point
M $_{itot}$	$M_{itot}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point
RF $_{itotdom}$	$RF_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)

TABLE 2-6: PLANE STRAIN APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
F <sub>itotdom</sub>	$F_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)
R <sub>Mitotdom</sub>	$RM_{itotdom}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)
M <sub>itotdom</sub>	$M_{itotdom}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)

### Axial Symmetry, Stress-Strain

In addition to the variables in Table 2-7, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

- $s_x$ \_amp is the amplitude of the normal stress in the  $x$  direction
- $e_x$ \_ph is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added Gp to the variable name. Example,  $s_x$ Gp instead of  $s_x$ . It is only possible to use the gauss point evaluated variables for postprocessing.

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uor	uor	All	All	$r$ displacement divided by $r$	uor
uaxi	uaxi	All	All	$r$ displacement	uor· $r$
w	w	All	All	$z$ displacement	w

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uort	$u_{or_t}$	T	All	$r$ velocity divided by $r$	$u_{or_t}$
uaxi_t	$u_{axi_t}$	T	All	$r$ velocity	$u_{or_t} \cdot r$
w_t	$w_t$	T	All	$z$ velocity	$w_t$
uaxi_amp	$u_{axi\_amp}$	F	All	$r$ displacement amplitude	$ u_{axi} $
w_amp	$w_{amp}$	F	All	$z$ displacement amplitude	$ w $
uaxi_ph	$u_{axi\_ph}$	F	All	$r$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_{axi}), 2\pi)$
w_ph	$w_{ph}$	F	All	$z$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(w), 2\pi)$
uaxi_t	$u_{axi_t}$	F	All	$r$ velocity	$j\omega u_{axi}$
w_t	$w_t$	F	All	$z$ velocity	$j\omega w$
uaxi_t_amp	$u_{axi\_t\_amp}$	F	All	$r$ velocity amplitude	$\omega u_{axi\_amp}$
w_t_amp	$w_{tamp}$	F	All	$z$ velocity amplitude	$\omega w_{amp}$
uaxi_t_ph	$u_{axi\_t\_ph}$	F	All	$r$ velocity phase	$\text{mod}(u_{axi\_ph} + 90^\circ, 360^\circ)$
w_t_ph	$w_{tph}$	F	All	$z$ velocity phase	$\text{mod}(w_{ph} + 90^\circ, 360^\circ)$
uaxi_tt	$u_{axi\_tt}$	F	All	$r$ acceleration	$-\omega^2 u_{axi}$
w_tt	$w_{tt}$	F	All	$z$ acceleration	$-\omega^2 w$
uaxi_tt_amp	$u_{axi\_tamp}$	F	All	$r$ acceleration amplitude	$\omega^2 u_{axi\_amp}$
w_tt_amp	$w_{tamp}$	F	All	$z$ acceleration amplitude	$\omega^2 w_{amp}$
uaxi_tt_ph	$u_{axi\_tph}$	F	All	$r$ acceleration phase	$\text{mod}(u_{axi\_ph} + 180^\circ, 360^\circ)$
w_tt_ph	$w_{tph}$	F	All	$z$ acceleration phase	$\text{mod}(w_{ph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{u_{axi}^2 + w^2}$
p	$p$	All	All	Pressure	$p$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
p_amp	$p_{amp}$	F	All	Pressure amplitude	$ p $
p_ph	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
er, ez, ephi, erz	$\epsilon_r, \epsilon_z, \epsilon_\phi,$ $\epsilon_{rz}$	All	S	Strain, global coord. system	Engineering or Green strain for small and large deformations, respectively
epr, epz, epphi, eprz	$\epsilon_{pr}, \epsilon_{pz},$ $\epsilon_{p\phi}, \epsilon_{prz}$	S T	S	Plastic strain, global coord. system	
epe	$\epsilon_{pe}$	S T	S	Plastic strain, global coord. system	
eil, exyl	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strains, user-defined coord. system	Defined differently depending on material model and coordinate system
er_t, ez_t, ephi_t, erz_t	$\epsilon_{rt}, \epsilon_{zt}, \epsilon_{\phi t},$ $\epsilon_{rzt}$	F T	S	Velocity strain, global coord. system	Defined differently depending on small or large displacement
eil_t, exyl_t	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sr, sphi, sz, srz	$\sigma_r, \sigma_\phi, \sigma_z,$ $\tau_{rz}$	All	S	Cauchy stress, global coord. system	Defined differently depending on material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
sil, sxyl	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	
sr_t, sphi_t, sz_t, srz_t	$\sigma_{rt}, \sigma_{\phi t},$ $\sigma_{zt}, \sigma_{rzt}$	F T	S	Time derivative of Cauchy stress, global coord. system	
sil_t, sxyl_t	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	
Sr, Sphi, Sz, Srz	$S_r, S_\phi, S_z,$ $S_{rz}$	All	S	Second Piola Kirchhoff stress, global coord. system	
Sil, Sxyl	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	
Sr_t, Sphi_t, Sz_t, Srz_t	$S_{rt}, S_{\phi t},$ $S_{zt}, S_{rzt}$	T	S	Time der. of second Piola Kirchhoff stress, global coord. system	
Sil_t, Sxyl_t	$S_{ilt}, S_{xylt}$	T	S	Time der. of second Piola Kirchhoff stress, user-defined coord. system	
Pi, Pij	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
si	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\epsilon_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$\epsilon_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
evol	$\epsilon_{vol}$	All	All	Volumetric strain	Defined differently for small and large displacement
$s_{qi}$ , $s_{qij}$	$s_{qi}$ , $s_{qij}$	All	S	Viscoelastic stress	Defined for viscoelastic materials: $s_{qi} = \sum_m 2G_m q_{im}$
E	$E$	All	All	Young's modulus	See "Elastic Moduli" on page 175 of the <i>Structural Mechanics Module User's Guide</i> .
nu	$\nu$	All	All	Poisson's ratio	
K	$K$	All	All	Bulk modulus	
G	$G$	All	All	Shear modulus	
lambLame	$\lambda$	All	All	Lamé constant $\lambda$	
muLame	$\mu$	All	All	Lamé constant $\mu$	
$F_{ij}$	$F_{ij}$ , $i,j=1, 2, 3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$c_{ij}$	$c_{ij}$ , $i,j=1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
detF	$\det F$	All	All	Determinant of deformation gradient	$\det F$
invFij	$\text{inv}F_{ij}$ , $i,j=1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
J	$J$	All	All	Volume ratio	$\det F$
Jel	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
I1	$I_1$	All	S	First invariant of the right Cauchy-Green strain tensor	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
I2	$I_2$	All	S	Second invariant of the right Cauchy-Green strain tensor	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
I3	$I_3$	All	S	Third invariant of the right Cauchy-Green strain tensor	$J_{\text{el}}^2$
II1	$\bar{I}_1$	All	S	First modified invariant of the right Cauchy-Green strain tensor	$I_1 J_{\text{el}}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	S	Second modified invariant of the right Cauchy-Green strain tensor	$I_2 J_{\text{el}}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
II3	$\bar{I}_3$	All	S	Third modified invariant of the right Cauchy-Green strain tensor	$\bar{I}_3 = 1$
I1e	$I_1^\varepsilon$	All	S	First invariant of the Green strain tensor	$I_1^\varepsilon = \frac{1}{2}(I_1 - 3)$
I2e	$I_2^\varepsilon$	All	S	Second invariant of the Green strain tensor	$I_2^\varepsilon = \frac{1}{4}(I_2 - 2I_1 + 3)$
I3e	$I_3^\varepsilon$	All	S	Third invariant of the Green strain tensor	$I_3^\varepsilon = \frac{1}{8}(I_3 - I_2 + I_1 - 1)$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
II1e	$\bar{I}_1^\varepsilon$	All	S	First modified invariant of the Green strain tensor	$\bar{I}_1^\varepsilon = \frac{1}{2}(\bar{I}_1 - 3)$
II2e	$\bar{I}_2^\varepsilon$	All	S	Second modified invariant of the Green strain tensor	$\bar{I}_2^\varepsilon = \frac{1}{4}((\bar{I}_2 - 3) - 2(\bar{I}_1 - 3))$
II3e	$\bar{I}_3^\varepsilon$	All	S	Third modified invariant of the Green strain tensor	$\bar{I}_3^\varepsilon = \frac{1}{8}((\bar{I}_1 - 3) - (\bar{I}_2 - 3))$
tresca	$\sigma_{\text{tresca}}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{\text{mises}}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending on material model and if mixed or displacement formulation
Ent	$S_{\text{elast}}$	All	S	Entropy per unit volume	Defined only for small deformations and either no damping or loss factor damping. See definition in the theory section
Qdamp	$Q_d$	F	S	Heat associated with mechanical losses in material	Defined only for viscoelastic materials and for other materials with either no damping or loss factor damping
$G_m$	$G_m$	All	S	Shear modulus, branch $m$	Defined for viscoelastic materials from the table in the <b>Subdomain Settings</b> dialog box
$\tau_m$	$\tau_m$	All	S	Relaxation time, branch $m$	
$G_0$	$G_0$	All	S	Instantaneous shear modulus	For viscoelastic materials: $G + \sum_m G_m$
Gstor	$G'$	D F	S	Dynamic storage modulus	For viscoelastic materials: $G + \sum_m G_m \frac{(\omega \tau_m)^2}{(1 + (\omega \tau_m)^2)}$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Gloss	$G''$	D F	S	Dynamic loss modulus	For viscoelastic materials: $\sum_m G_m \frac{\omega \tau_m}{(1 + (\omega \tau_m)^2)}$
ShiftWLF	$a_T$	All	S	WLF time shift factor	Defined for viscoelastic materials: $\log(a_T) = \frac{-C_1(T - T_0)}{C_2 + (T - T_0)}$
Tar, Taz	$T_a, T_z$	All	B	Surface traction (force/area) in $r$ and $z$ directions	Defined differently depending on small or large deformation
erig, ephiig, ezig, erzig	$\varepsilon_{rig}, \varepsilon_{\varphi ig}, \varepsilon_{zig}, \varepsilon_{xyig}$	All	S	Initial strains in global coord. system	Defined differently depending on the material coordinate system
eiil, exyil	$\varepsilon_{iil}, \varepsilon_{xyil}$	All	S	Initial strains in user-defined coord. system	
srig, sphiiig, szig, srzig	$\sigma_{rig}, \sigma_{\varphi ig}, \sigma_{zig}, \tau_{xyig}$	All	S	Initial stresses in global coord. system	
sril, sxyil	$\sigma_{iil}, \tau_{xyil}$	All	S	Initial stresses in user-defined coord. system	
Frg, Fzg	$F_{rg}, F_{zg}$	All	All	Body, edge, point load in global $r$ and $z$ directions	Defined differently depending on force definition
RFr	$RF_r$	S F P T	All	Reaction force in global $x_i$ direction on subdomain boundary, or point. Should only be used for integration, as they are defined only in the node points.	$2reacf(uor)\pi/r$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
RFz	$RF_r$	S F P T	All	Reaction force in global $x_i$ direction on subdomain, boundary, or point. Should only be used for integration, as they are defined only in the node points.	$2\text{reacf}(w)\pi$
PMLr	$\text{PML}_r$	F	S	PML coordinate $r$ , cylindrical PML	$R_0 +  r - R_0 + \text{eps} ^n L_r(1 - i)/dr^n$ $n \equiv \text{PML scaling exponent}$
PMLz	$\text{PML}_z$	F	S	PML coordinate $z$ , cylindrical PML	$\text{sign}(z - z_0) z - Z_0 + \text{eps} ^n L_z(1 - i)/dz^n$
R	$R$	F	S	Scaled radial coordinate, spherical PML	$R_0 +  \Delta_0 - R_0 ^n L_r(1 - i)/dr^n$ $\Delta_0 \equiv [(x - x_0)^2 + (y - y_0)^2]^{1/2}$ $n \equiv \text{PML scaling exponent}$
PMLr	$\text{PML}_r$	F	S	PML coordinate $r$ , spherical PML	$Rr/\delta_0$ $\delta_0 \equiv [r^2 + (z - z_0)^2]^{1/2}$
PMLz	$\text{PML}_z$	F	S	PML coordinate $z$ , spherical PML	$(z - z_0)R/\delta_0$
$J_{xixj}$	$\mathbf{J}_{ij}$	F	S	PML transformation matrix, element $ij; x_i, x_j = r, z$	$\frac{\partial}{\partial x_j} \text{PML}_x_i$
$\text{inv}J_{xixj}$		F	S	PML inverse transformation matrix, element $ij; x_i, x_j = r, z$	$(\mathbf{J}^{-1})_{x_i x_j}$
detJ	$ \mathbf{J} $	F	S	Determinant of PML transformation matrix	$\det(\mathbf{J})$
PMLuor	$\text{PML}_{uor}$	F	S	$r$ displacement divided by $\text{PML}_r$	$u_{or} \cdot r / \text{PML}_r$

TABLE 2-7: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
PMLuorr	PMLuor <sub>r</sub>	F	S	$r$ derivative of PMLuor	$\left(\left(\frac{\partial}{\partial r} u_{or}\right) r + u_{or} + \frac{u_{or} \cdot r \cdot J_{rr}}{PMLr}\right) / (PMLr)$
PMLuorPMLr		F	S	PML $r$ derivative of PMLuor	PMLuor <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rr</sub> + uor <sub>z</sub> · r( $\mathbf{J}^{-1}$ ) <sub>rz</sub> / PML $r$
PMLuorPMLz		F	S	PML $z$ derivative of PMLuor	PMLuor <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rz</sub> + uor <sub>z</sub> · r( $\mathbf{J}^{-1}$ ) <sub>zz</sub> / PML $r$
uaxiPMLr		F	S	PML $r$ derivative of $r$ displacement	u <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rr</sub> + u <sub>z</sub> ( $\mathbf{J}^{-1}$ ) <sub>rz</sub>
uaxiPMLz		F	S	PML $z$ derivative of $r$ displacement	u <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rz</sub> + u <sub>z</sub> ( $\mathbf{J}^{-1}$ ) <sub>zz</sub>
wPMLr		F	S	PML $r$ derivative of $z$ displacement	w <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rr</sub> + w <sub>z</sub> ( $\mathbf{J}^{-1}$ ) <sub>rz</sub>
wPMLz		F	S	PML $z$ derivative of $z$ displacement	w <sub>r</sub> ( $\mathbf{J}^{-1}$ ) <sub>rz</sub> + w <sub>z</sub> ( $\mathbf{J}^{-1}$ ) <sub>zz</sub>

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), bnd (boundary), and sub (subdomain) refer to the different domain levels.

TABLE 2-8: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RF <sub>itot</sub>	RF <sub>itot</sub>	S F P T	Total reaction force in global $x_i$ -direction
F <sub>itot</sub>	F <sub>itot</sub>	S F P T	Total applied force global $x_i$ -direction
RF <sub>itotdom</sub>	RF <sub>itotdom</sub>	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)
F <sub>itotdom</sub>	F <sub>itotdom</sub>	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt, bnd, and sub)

# Plates, Beams, Trusses, and Shells

## *Mindlin Plate*

In addition to the variables listed below, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp`, the amplitude of the normal stress in the  $x$  direction
- `ex_ph`, the phase of the normal strain in the  $x$  direction

The exception being variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, `s1`, etc.

The table uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
w	$w$	All	All	$z$ displacement	$w$
$\text{thx}_i$	$\theta_{xi}$	All	All	$x_i$ rotation	$\theta_{xi}$
$w_t$	$w_t$	T	All	$z$ velocity	$w_t$
$\text{thx}_i t$	$\theta_{xit}$	T	All	$x_i$ angular velocity	$\theta_{xit}$
w_amp	$w_{\text{amp}}$	F	All	$z$ displacement amplitude	$ w $
$\text{thx}_i \text{amp}$	$\theta_{xi\text{amp}}$	F	All	$x_i$ rotation amplitude	$ \theta_{xi} $

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
w_ph	$w_{ph}$	F	All	$z$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(w), 2\pi)$
thxi_ph	$\theta_{xiph}$	F	All	$x_i$ rotation phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(\theta_{xi}), 2\pi)$
w_t	$w_t$	F	All	$z$ velocity	$j\omega w$
thxi_t	$\theta_{xit}$	F	All	$x_i$ angular velocity	$j\omega \theta_{xi}$
w_t_amp	$w_{tamp}$	F	All	$z$ velocity amplitude	$\omega w_{amp}$
thxi_t_amp	$\theta_{xitamp}$	F	All	$x_i$ angular velocity amplitude	$\omega \theta_{xiamp}$
w_t_ph	$w_{itph}$	F	All	$z$ velocity phase	$\text{mod}(w_{ph} + 90^\circ, 360^\circ)$
thxi_t_ph	$\theta_{xitph}$	F	All	$x_i$ angular velocity phase	$\text{mod}(\theta_{xiph} + 90^\circ, 360^\circ)$
w_tt	$w_{tt}$	F	All	$z$ acceleration	$-\omega^2 w$
thxi_tt	$\theta_{xitt}$	F	All	$x_i$ angular acceleration	$-\omega^2 \theta_{xi}$
w_tt_amp	$w_{ttamp}$	F	All	$z$ acceleration amplitude	$\omega^2 w_{amp}$
thxi_tt_amp	$\theta_{xitamp}$	F	All	$x_i$ angular acceleration amplitude	$\omega^2 \theta_{xiamp}$
w_tt_ph	$w_{ttph}$	F	All	$z$ acceleration phase	$\text{mod}(w_{ph} + 180^\circ, 360^\circ)$
thxi_tt_ph	$\theta_{xitph}$	F	All	$x_i$ angular acceleration phase	$\text{mod}(\theta_{xiph} + 180^\circ, 360^\circ)$
totrot	totrot	All	All	Total rotation	$\sqrt{\sum_i (\text{real}(\theta_{xi}))^2}$
postheight	$z$	All	S	Postprocessing height for stress and strain evaluation	Dependent on the settings on the postprocessing page

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ex	$\epsilon_x$	All	S	$\epsilon_x$ normal strain global coord. system	$z \frac{\partial \theta_y}{\partial x}$
ey	$\epsilon_y$	All	S	$\epsilon_y$ normal strain global coord. system	$-z \frac{\partial \theta_x}{\partial y}$
exy	$\epsilon_{xy}$	All	S	$\epsilon_{xy}$ shear strain global coord. system	$\frac{z}{2} \left( \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \right)$
eyz	$\epsilon_{yz}$	All	S	$\epsilon_{yz}$ shear strain global coord. system	$\frac{5}{8} \gamma_{yz} \left( 1 - \frac{4z^2}{th^2} \right)$
exz	$\epsilon_{xz}$	All	S	$\epsilon_{xz}$ shear strain global coord. system	$\frac{5}{8} \gamma_{xz} \left( 1 - \frac{4z^2}{th^2} \right)$
exl	$\epsilon_{xl}$	All	S	$\epsilon_x$ normal strain local coord. system	$z \left( \frac{\partial \theta_y}{\partial x} \right)_1$
eyl	$\epsilon_{yl}$	All	S	$\epsilon_y$ normal strain local coord. system	$-z \left( \frac{\partial \theta_x}{\partial y} \right)_1$
exyl	$\epsilon_{xyl}$	All	S	$\epsilon_{xy}$ shear strain local coord. system	$\frac{z}{2} \left( \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \right)_1$
eyzl	$\epsilon_{yzl}$	All	S	$\epsilon_{yz}$ shear strain global coord. system	$\frac{5}{8} \gamma_{yzl} \left( 1 - \frac{4z^2}{th^2} \right)$
exzl	$\epsilon_{xzl}$	All	S	$\epsilon_{xz}$ shear strain global coord. system	$\frac{5}{8} \gamma_{xzl} \left( 1 - \frac{4z^2}{th^2} \right)$

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Mx <sub>ip</sub>	$M_{xip}$	All	S	$M_{xip}$ plate bending moment global system	If material is defined in global coord. sys. $\frac{(\text{th})^3}{12} \left[ D_p \left( \Theta - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ With loss factor damping in global coord. sys. $\frac{(\text{th})^3}{12} \left[ D_p \left( (1+j\eta)\Theta - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ If material is defined in user def. coord. sys. $T_{\text{coord}} M_{pi} T_{\text{coord}}^T$
Mxy <sub>p</sub>	$M_{xyp}$	All	S	$M_{xyp}$ plate torsional moment global system	If material is defined in global coord. sys. $\frac{(\text{th})^3}{12} \left[ D_p \left( \Theta - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ With loss factor damping in global coord. sys. $\frac{(\text{th})^3}{12} \left[ D_p \left( (1+j\eta)\Theta - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ If material is defined in user def. coord. sys. $T_{\text{coord}} M_{pi} T_{\text{coord}}^T$
Mx <sub>ipl</sub>	$M_{xipl}$	All	S	$M_{xip}$ plate bending moment local system	$\frac{(\text{th})^3}{12} \left[ D_p \left( \Theta_l - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ With loss factor damping $\frac{(\text{th})^3}{12} \left[ D_p \left( (1+j\eta)\Theta_l - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$
Mxy <sub>pl</sub>	$M_{xypl}$	All	S	$M_{xyp}$ plate torsional moment local system	$\frac{(\text{th})^3}{12} \left[ D_p \left( \Theta_l - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$ With loss factor damping $\frac{(\text{th})^3}{12} \left[ D_p \left( (1+j\eta)\Theta_l - \frac{\alpha_{\text{vec}} \Delta T}{\text{th}} - \Theta_i \right) \right] + M_{pi}$

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Q <sub>xip</sub>	Q <sub>xip</sub>	All	S	Q <sub>xip</sub> plate shear force global system	If material is defined in global coord. sys. $D_s \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} - \begin{bmatrix} \gamma_{yzi} \\ \gamma_{xzi} \end{bmatrix} + Q_{pi}$ With loss factor damping in global coord. sys. $D_s \begin{bmatrix} (1+j\eta) \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} - \begin{bmatrix} \gamma_{yzi} \\ \gamma_{xzi} \end{bmatrix} + Q_{pi}$ If material is defined in user def. coord. sys. $\begin{bmatrix} Q_{xp} \\ Q_{yp} \end{bmatrix} = T \begin{bmatrix} Q_{xp} \\ Q_{yp} \end{bmatrix}_1$
Q <sub>xipl</sub>	Q <sub>xip</sub>	All	S	Q <sub>xip</sub> plate shear force local system	th · D <sub>s</sub> $\begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}_1 - \begin{bmatrix} \gamma_{yzi} \\ \gamma_{xzi} \end{bmatrix} + Q_{pi}$ With loss factor damping th · D <sub>s</sub> $\begin{bmatrix} (1+j\eta) \gamma_{yz} \\ \gamma_{xz} \end{bmatrix}_1 - \begin{bmatrix} \gamma_{yzi} \\ \gamma_{xzi} \end{bmatrix} + Q_{pi}$
s <sub>x</sub>	σ <sub>x</sub>	All	S	σ <sub>x</sub> normal stress global coord. system	$12 \frac{M_{xp}}{th^3} z$
s <sub>y</sub>	σ <sub>y</sub>	All	S	σ <sub>y</sub> normal stress global coord. system	$12 \frac{M_{yp}}{th^3} z$
s <sub>xy</sub>	τ <sub>xy</sub>	All	S	τ <sub>xy</sub> shear stress global coord. system	$12 \frac{M_{xyp}}{th^3} z$
s <sub>xl</sub>	σ <sub>xl</sub>	All	S	σ <sub>x</sub> normal stress local coord. system	$12 \frac{M_{xpl}}{th^3} z$

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
syl	$\sigma_y$	All	S	$\sigma_y$ normal stress local coord. system	$12 \frac{M_{ypl}}{th^3} z$
sxyl	$\tau_{xy}$	All	S	$\tau_{xy}$ shear stress global coord. system	$12 \frac{M_{xpl}}{th^3} z$
$\sigma_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
$\epsilon_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	
$\sigma_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$\epsilon_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
Fzg	$F_{zg}$	All	S	Body, edge, point load, in global $z$ dir.	Defined differently depending on how the force is defined
RFz	$RF_z$	S F P T	All	Reaction force in global $z$ direction on subdomain, boundary, or point. Should only be used for integration, as they are defined only in the node points.	reacf( $w$ )

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\text{RM}_i$	$\text{RM}_i$	S F P T	B	Reaction moment in global $x_i$ direction with respect to reference point on boundary. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times \mathbf{RF} + \text{reacf}(\theta_i) + \text{reacf}(\theta_n)n_{\theta ni}$
$\text{RM}_i$	$\text{RM}_i$	S F P T	S	Reaction moment in global $x_i$ direction with respect to reference point on subdomain. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times \mathbf{RF} + \text{reacf}(\theta_i)$
$\text{RM}_i$	$\text{RM}_i$	S F P T	P	Reaction torque in global $x_i$ direction	$\text{reacf}(\theta_i)$

TABLE 2-9: MINDLIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
M <sub>xig</sub>	$M_{xig}$	All	S	Body, edge, point moment, in global $x_i$ dir.	Defined differently depending on how the moment is defined
W <sub>s</sub>	$W_s$	All	S	Strain energy density	If global coordinate system $\frac{1}{2} \left( \frac{\partial \theta_y}{\partial x} M_{xp} - \frac{\partial \theta_x}{\partial y} M_{yp} + \left( \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \right) M_{xyp} \right)$ $\frac{1}{2} (\gamma_{yz} Q_{yp} + \gamma_{xz} Q_{xp})$ If other coordinate system $\frac{1}{2} \left( \frac{\partial \theta_{y1}}{\partial x} M_{xlp} - \frac{\partial \theta_{x1}}{\partial y} M_{ylp} + \left( \frac{\partial \theta_{y1}}{\partial y} - \frac{\partial \theta_{x1}}{\partial x} \right) M_{xylp} \right)$ $\frac{1}{2} (\gamma_{yz1} Q_{yp} + \gamma_{xz1} Q_{lp})$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), bnd (boundary), and sub (subdomain) refer to the different domain levels.

TABLE 2-10: MINDLIN APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RF <sub>ztot</sub>	$RF_{ztot}$	S F P T	Total reaction force in global $z$ -direction
F <sub>ztot</sub>	$F_{ztot}$	S F P T	Total applied force global $z$ -direction
RM <sub>itot</sub>	$RM_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction relative reference point
M <sub>itot</sub>	$M_{itot}$	S F P T	Total applied moment in global $x_i$ -direction relative reference point
RF <sub>ztotdom</sub>	$RF_{ztotdom}$	S F P T	Total reaction force in global $z$ -direction for all different domain levels (pnt, bnd, and sub)
F <sub>ztotdom</sub>	$F_{ztotdom}$	S F P T	Total applied force in global $z$ -direction for all different domain levels (pnt, bnd, and sub)

TABLE 2-10: MINDLIN APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RM <sub>i</sub> totdom	RM <sub>i</sub> totdom	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)
M <sub>i</sub> totdom	M <sub>i</sub> totdom	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, bnd, and sub)

### In-Plane Euler Beam

In addition to the variables listed below almost all application mode parameters are available as variables. Some variables are different for different analyses, which is seen in the Analysis column. For frequency response analysis, a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

- M\_amp, the amplitude of the bending moment
- sn\_ph, the phase of the axial stress

The exception to this scheme consists of variables defined using a nonlinear operator such as snmax, snmin, disp, etc.

The table below uses a convention where the index  $i$  (or  $i$ ) runs over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  ( $ui$ ) refers to the global displacements ( $u$ ,  $v$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E

TABLE 2-11: IN-PLANE EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ui	$u_i$	All	All	$x_i$ displacement	$u_i$
th	$\theta$	All	All	$z$ rotation	$\theta$

TABLE 2-II: IN-PLANE EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$uit$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$tht$	$\theta_t$	T	All	$z$ angular velocity	$\dot{\theta}_t$
$ui\_amp$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$th\_amp$	$\theta_{amp}$	F	All	$z$ rotation amplitude	$ \theta $
$ui\_ph$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$th\_ph$	$\theta_{ph}$	F	All	$z$ rotation phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(\theta), 2\pi)$
$ui\_t$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$th\_t$	$\theta_t$	F	All	$z$ angular velocity	$j\omega\theta$
$ui\_t\_amp$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$th\_t\_amp$	$\theta_{tamp}$	F	All	$z$ angular velocity amplitude	$\omega\theta_{amp}$
$ui\_t\_ph$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$th\_t\_ph$	$\theta_{tph}$	F	All	$z$ angular velocity phase	$\text{mod}(\theta_{ph} + 90^\circ, 360^\circ)$
$ui\_tt$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$th\_tt$	$\theta_{tt}$	F	All	$z$ angular acceleration	$-\omega^2\theta$
$ui\_tt\_amp$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$th\_tt\_amp$	$\theta_{ttamp}$	F	All	$z$ angular acceleration amplitude	$\omega^2\theta_{amp}$

TABLE 2-11: IN-PLANE EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ui_tt_ph	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
th_tt_ph	$\theta_{ittph}$	F	All	$z$ angular acceleration phase	$\text{mod}(\theta_{ph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
N	$N$	All	B	Axial force	$EA \left[ \left( \frac{\partial u_{\text{axi}}}{\partial s} - \left( \frac{\partial u_{\text{axi}}}{\partial s} \right)_i \right) - \alpha(T - T_{\text{ref}}) \right] + N_i$ <p>With loss factor damping in frequency response analysis</p> $EA \left[ \left( (1 + j\eta) \frac{\partial u_{\text{axi}}}{\partial s} - \left( \frac{\partial u_{\text{axi}}}{\partial s} \right)_i \right) - \alpha(T - T_{\text{ref}}) \right] + N_i$
M	$M$	All	B	Moment	$-EI \left[ \frac{\partial \theta}{\partial s} - \left( \frac{\partial \theta}{\partial s} \right)_i - \alpha \frac{\Delta T}{h} \right] + M_i$ <p>With loss factor damping in frequency response analysis</p> $-EI \left[ (1 + j\eta) \frac{\partial \theta}{\partial s} - \left( \frac{\partial \theta}{\partial s} \right)_i - \alpha \frac{\Delta T}{h} \right] + M_i$
T	$T$	All	B	Shear force	$EI_{yy} \frac{\partial^2 \theta}{\partial s^2}$ <p>With loss factor damping in frequency response analysis</p> $EI_{yy} (1 + j\eta) \frac{\partial^2 \theta}{\partial s^2}$
sn	$\sigma_n$	All	B	Axial stress	$E \frac{\partial u_{\text{axi}}}{\partial s}$
en	$\epsilon_n$	All	B	Axial strain	$\frac{\partial u_{\text{axi}}}{\partial s}$
sbttop	$\sigma_{btop}$	All	B	Bending stress at top of section	$\frac{M h_z}{2 I_{yy}}$

TABLE 2-II: IN-PLANE EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sbbot	$\sigma_{\text{bbot}}$	All	B	Bending stress at bottom of section	$-\frac{M h_z}{2 I_{yy}}$
snmax	$\sigma_{n\text{max}}$	All	B	Max normal stress	$\max(\text{real}(\sigma_n + \sigma_{\text{btop}}), \text{real}(\sigma_n + \sigma_{\text{bbot}}))$
snmin	$\sigma_{n\text{min}}$	All	B	Min normal stress	$\min(\text{real}(\sigma_n + \sigma_{\text{btop}}), \text{real}(\sigma_n + \sigma_{\text{bbot}}))$
N_t	$N_t$	All	B	Time derivative of axial force	$EA \left[ \left( \frac{\partial u_{\text{taxi}}}{\partial s} \right) \right]$ With loss factor damping in frequency response analysis $j\omega EA \left[ (1 + j\eta) \frac{\partial u_{\text{axi}}}{\partial s} \right]$
M_t	$M_t$	All	B	Time derivative of moment	$-EI_{yy} \frac{\partial \theta_t}{\partial s}$ With loss factor damping in frequency response analysis $-j\omega EI_{yy} (1 + j\eta) \frac{\partial \theta}{\partial s}$
F <sub>ig</sub>	$F_{ig}$	All	B P	Edge, point load in global $x_i$ direction	Defined differently depending on how the force is defined and the analysis type used
RF <sub>i</sub>	$RF_i$	S F P T	All	Reaction force in global $x_i$ direction on boundary or point. Should only be used for integration, as they are defined only in the node points.	reacf( $u_i$ )

TABLE 2-11: IN-PLANE EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
RMz	$RM_z$	S F P T	B	Reaction moment in $z$ direction with respect to reference point on boundary. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF + reacf(\theta)$
RMz	$RM_z$	S F P T	P	Reaction torque in $z$ direction	$reacf(\theta)$
Mzg	$M_{zg}$	All	B P	Edge or point moment in $z$ direction	Defined differently depending on how the moment is defined and the analysis type used
Ws	$W_s$	All	B	Strain energy density	$-\frac{1}{2}(\theta_s M - u v t s N)$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point) and bnd (boundary) refer to the different domain levels.

TABLE 2-12: IN-PLANE EULER BEAM APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$RF_{itot}$	$RF_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
$F_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction
$RM_{ztot}$	$RM_{ztot}$	S F P T	Total reaction moment in global $z$ -direction with respect to reference point
$M_{ztot}$	$M_{ztot}$	S F P T	Total applied moment in global $z$ -direction with respect to reference point
$RF_{itotdom}$	$RF_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt and bnd)

TABLE 2-12: IN-PLANE EULER BEAM APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$F_{itotdom}$	$F_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt and bnd)
$RM_{ztotdom}$	$RM_{ztotdom}$	S F P T	Total reaction moment in global $z$ -direction with respect to reference point for all different domain levels (pnt and bnd)
$M_{ztotdom}$	$M_{ztotdom}$	S F P T	Total applied moment in global $z$ -direction with respect to reference point for all different domain levels (pnt and bnd)

### *3D Euler Beam*

In addition to the variables listed below almost all application mode parameters are available as variables. Some variables are different for different analyses, which is seen in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

- $My1\_amp$ , the amplitude of the bending moment in the local  $y$  direction
- $sn\_ph$ , the phase of the axial stress

The exception to this scheme consists of variables defined using a nonlinear operator such as  $snmax$ ,  $snmin$ ,  $disp$ , and so on.

The table below uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$\theta_i$	$\theta_i$	All	All	$x_i$ rotation	$\theta_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$\theta_{it}$	$\theta_{it}$	T	All	$x_i$ angular velocity	$\theta_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$\theta_{i\_amp}$	$\theta_{iamp}$	F	All	$x_i$ rotation amplitude	$ \theta_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$\theta_{i\_ph}$	$\theta_{iph}$	F	All	$x_i$ rotation phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(\theta_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$\theta_{i\_t}$	$\theta_{it}$	F	All	$x_i$ angular velocity	$j\omega\theta_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$\theta_{i\_t\_amp}$	$\theta_{itamp}$	F	All	$x_i$ angular velocity amplitude	$\omega\theta_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
th <sub>i</sub> _t_ph	$\theta_{itph}$	F	All	$x_i$ angular velocity phase	$\text{mod}(\theta_{iph} + 90^\circ, 360^\circ)$
u <sub>i</sub> _tt	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
th <sub>i</sub> _tt	$\theta_{itt}$	F	All	$x_i$ angular acceleration	$-\omega^2 \theta_i$
u <sub>i</sub> _tt_amp	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
th <sub>i</sub> _tt_amp	$\theta_{ittamp}$	F	All	$x_i$ angular acceleration amplitude	$\omega^2 \theta_{iamp}$
u <sub>i</sub> _tt_ph	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
th <sub>i</sub> _tt_ph	$\theta_{ittph}$	F	All	$x_i$ angular acceleration phase	$\text{mod}(\theta_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
totrot	totrot	All	All	Total rotation	$\sqrt{\sum_i (\text{real}(\theta_i))^2}$
N	$N$	All	E	Axial force	$EA \left[ \left( \frac{\partial u_{\text{axi}}}{\partial s} - \left( \frac{\partial u_{\text{axi}}}{\partial s} \right)_i \right) - \alpha(T - T_{\text{ref}}) \right] + N_i$ With loss factor damping in frequency response analysis $EA \left[ \left( (1 + j\eta) \frac{\partial u_{\text{axi}}}{\partial s} - \left( \frac{\partial u_{\text{axi}}}{\partial s} \right)_i \right) - \alpha(T - T_{\text{ref}}) \right] + N_i$
Mx1	$M_{xl}$	All	E	Torsional moment local $x$ direction	$\frac{E}{2(1+\nu)} J \left[ \frac{\partial \theta_{xl}}{\partial s} - \left( \frac{\partial \theta_{xl}}{\partial s} \right)_i \right] + M_{xi}$ With loss factor damping in frequency response analysis $\frac{E}{2(1+\nu)} J \left[ (1 + j\eta) \frac{\partial \theta_{xl}}{\partial s} - \left( \frac{\partial \theta_{xl}}{\partial s} \right)_i \right] + M_{xi}$

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
My1	$M_{x1}$	All	E	Bending moment local $y$ direction	$EI_{yy} \left[ \frac{\partial \theta_{y1}}{\partial s} - \left( \frac{\partial \theta_{y1}}{\partial s} \right)_i - \alpha \frac{\Delta T_z}{h_z} \right] + M_{yi}$ With loss factor damping in frequency response analysis $EI_{yy} \left[ (1+j\eta) \frac{\partial \theta_{y1}}{\partial s} - \left( \frac{\partial \theta_{y1}}{\partial s} \right)_i - \alpha \frac{\Delta T_z}{h_z} \right] + M_{yi}$
Mz1	$M_{x1}$	All	E	Bending moment local $z$ direction	$EI_{zz} \left[ \frac{\partial \theta_{z1}}{\partial s} - \left( \frac{\partial \theta_{z1}}{\partial s} \right)_i - \alpha \frac{\Delta T_y}{h_y} \right] + M_{zi}$ With loss factor damping in frequency response analysis $EI_{zz} \left[ (1+j\eta) \frac{\partial \theta_{z1}}{\partial s} - \left( \frac{\partial \theta_{z1}}{\partial s} \right)_i - \alpha \frac{\Delta T_y}{h_y} \right] + M_{zi}$
Ty1	$T_y$	All	E	Shear force local $y$ direction	$-EI_{zz} \frac{\partial^2 \theta_{z1}}{\partial s^2}$ With loss factor damping in frequency response analysis $-EI_{zz} (1+j\eta) \frac{\partial^2 \theta_{z1}}{\partial s^2}$
Tz1	$T_z$	All	E	Shear force local $z$ direction	$EI_{yy} \frac{\partial^2 \theta_{y1}}{\partial s^2}$ With loss factor damping in frequency response analysis $EI_{yy} (1+j\eta) \frac{\partial^2 \theta_{y1}}{\partial s^2}$
sn	$\sigma_n$	All	E	Axial stress	$E \frac{\partial u_{axi}}{\partial s}$
en	$\epsilon_n$	All	E	Axial strain	$\frac{\partial u_{axi}}{\partial s}$
sbytop	$\sigma_{bytop}$	All	E	Bending stress at $y$ top fiber	$-\frac{M_{z1} h_y}{2I_{zz}}$

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sbybot	$\sigma_{bybot}$	All	E	Bending stress at $y$ bottom fiber	$\frac{M_z h_y}{2I_{zz}}$
sbztop	$\sigma_{bztop}$	All	E	Bending stress at $z$ top fiber	$\frac{M_y h_z}{2I_{yy}}$
sbzbot	$\sigma_{bzbot}$	All	E	Bending stress at $z$ bottom fiber	$-\frac{M_y h_z}{2I_{yy}}$
snmaxy	$\sigma_{nmaxy}$	All	E	Max normal stress $y$ fiber	$\max(\text{real}(\sigma_n + \sigma_{bytop}), \text{real}(\sigma_n + \sigma_{bybot}))$
snminy	$\sigma_{nminy}$	All	E	Min normal stress $y$ fiber	$\min(\text{real}(\sigma_n + \sigma_{bytop}), \text{real}(\sigma_n + \sigma_{bybot}))$
snmaxz	$\sigma_{nmaxz}$	All	E	Max normal stress $z$ fiber	$\max(\text{real}(\sigma_n + \sigma_{bztop}), \text{real}(\sigma_n + \sigma_{bzbot}))$
snminz	$\sigma_{nminz}$	All	E	Min normal stress $z$ fiber	$\min(\text{real}(\sigma_n + \sigma_{bztop}), \text{real}(\sigma_n + \sigma_{bzbot}))$
N_t	$N_t$	All	E	Time derivative of axial force	$EA\left[\left(\frac{\partial u_{taxi}}{\partial s}\right)\right]$ With loss factor damping in frequency response analysis $EA\left[(1+j\eta)j\omega\frac{\partial u_{axi}}{\partial s}\right]$
Mx1_t	$M_{xlt}$	All	E	Time derivative of torsional moment local $x$ direction	$\frac{E}{2(1+\nu)}J\frac{\partial \theta_{xlt}}{\partial s}$ With loss factor damping in frequency response analysis $\frac{E}{2(1+\nu)}J(1+j\eta)j\omega\frac{\partial \theta_{x1}}{\partial s}$
My1_t	$M_{ylt}$	All	E	Time derivative of bending moment local $y$ direction	$EI_{yy}\frac{\partial \theta_{ylt}}{\partial s}$ With loss factor damping in frequency response analysis $EI_{yy}(1+j\eta)j\omega\frac{\partial \theta_{y1}}{\partial s}$

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Mz1_t	$M_{zlt}$	All	E	Time derivative of bending moment local $z$ direction	$EI_{zz} \frac{\partial \theta_{zlt}}{\partial s}$ $EI_{zz}(1+j\eta)j\omega \frac{\partial \theta_{z1}}{\partial s}$
Fig	$F_{ig}$	All	E	Edge, point load in global $x_i$ direction	Different depending on how the force is defined and the analysis type used
RF $_i$	RF $_i$	S F P T	All	Reaction force in global $x_i$ direction on edge or point. Should only be used for integration, as they are defined only in the node points.	reacf( $u_i$ )
RM $_i$	RM $_i$	S F P T	E	Reaction moment in global $x_i$ direction with respect to reference point on edges. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF + reacf(\theta_i)$
RM $_i$	RM $_i$	S F P T	P	Reaction torque in global $x_i$ direction	reacf( $\theta_i$ )

TABLE 2-13: 3D EULER BEAM APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Mig	$M_{ig}$	All	E	Edge or point moment in global $x_i$ direction	Different depending on how the moment is defined and the analysis type used
Filocal	$F_{ilocal}$	All	E P	Edge or point load in local $x_i$ direction	Different depending on how the moment is defined and the analysis type used
Milocal	$M_{ilocal}$	All	E P	Edge or point moment in local $x_i$ direction	Different depending on how the force is defined and the analysis type used
Ws	$W_s$	All	E	Strain energy density	$\frac{1}{2}(\theta_{xs}M_{x1} + \theta_{ys}M_{y1} + \theta_{zs}M_{z1} + uvwtsN)$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point) and edg (edge) refer to the different domain levels.

TABLE 2-14: 3D EULER BEAM APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
RF <sub>itot</sub>	RF <sub>itot</sub>	S F P T	Total reaction force in global $x_i$ -direction
F <sub>itot</sub>	F <sub>itot</sub>	S F P T	Total applied force global $x_i$ -direction
RM <sub>itot</sub>	RM <sub>itot</sub>	S F P T	Total reaction moment relative reference point in global $x_i$ -direction
M <sub>itot</sub>	M <sub>itot</sub>	S F P T	Total applied moment relative reference point in global $x_i$ -direction
RF <sub>itotdom</sub>	RF <sub>itotdom</sub>	S F P T	Total reaction force for all different domain levels (pnt and edg) in global $x_i$ -direction
F <sub>itotdom</sub>	F <sub>itotdom</sub>	S F P T	Total applied force for all different domain levels (pnt and edg) in global $x_i$ -direction
RM <sub>itotdom</sub>	RM <sub>itotdom</sub>	S F P T	Total reaction moment relative reference point for all different domain levels (pnt and edg) in global $x_i$ -direction
M <sub>itotdom</sub>	M <sub>itotdom</sub>	S F P T	Total applied moment relative reference point for all different domain levels (pnt and edg) in global $x_i$ -direction

## In-Plane Truss

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A large number of variables are available for use in expressions and for postprocessing purposes. almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `en_amp` is the amplitude of the axial strain
- `sn_ph` is the phase of the axial stress

Table 2-15 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  (`ui`) refers to the global displacements ( $u, v$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E

TABLE 2-15: IN PLANE TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<code>ui</code>	$u_i$	All	All	$x_i$ displacement	$u_i$
<code>uit</code>	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
<code>ui_amp</code>	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
<code>ui_ph</code>	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
<code>ui_t</code>	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
<code>ui_t_amp</code>	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
<code>ui_t_ph</code>	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
<code>ui_tt</code>	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$

TABLE 2-15: IN PLANE TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ui_tt_amp	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
ui_tt_ph	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
xn	$x_n$	All	B	Parameter along edge only used for linear constraint	$\frac{x(x_2 - x_1) + y(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$
exixjT	$\epsilon_{xixjT}$	All	B	Tangential strain tensor	If large deformation $\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T \right)$ else $\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T \right)$
en	$\epsilon_n$	All	B	Axial strain	$t_x(\epsilon_x T t_x + \epsilon_{xy} T t_y) + t_y(\epsilon_{xy} T t_x + \epsilon_y T t_y)$
sn	$\sigma_n$	All	B	Axial stress	$E(\epsilon_n - \alpha(T - T_{\text{ref}}) - \epsilon_{ni}) + \sigma_{ni}$
exixjT_t	$\epsilon_{xixjTt}$	T	B	Tangential strain rate tensor	If large deformation $\frac{1}{2} \left( \frac{\partial ut_i}{\partial x_j} \Big _T + \frac{\partial ut_j}{\partial x_i} \Big _T + \frac{\partial ut_k}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial ut_k}{\partial x_j} \Big _T \right)$ else $\frac{1}{2} \left( \frac{\partial ut_i}{\partial x_j} \Big _T + \frac{\partial ut_j}{\partial x_i} \Big _T \right)$
exixjT_t	$\epsilon_{xixjTt}$	F	B	Tangential strain rate tensor	$\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T \right) j \omega$

TABLE 2-15: IN PLANE TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
exixjT_b	$\epsilon_{xixjTb}$	Buckling	B	Tangential strain buckling tensor	$\frac{1}{2} \left( \frac{\partial u_t}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial u_t}{\partial x_j} \Big _T \right)$
en_t	$\epsilon_{nt}$	F T	B	Axial strain rate	$t_x(\epsilon_{xTt} t_x + \epsilon_{xyTt} t_y) + t_y(\epsilon_{xyTt} t_x + \epsilon_{yTt} t_y)$
en_b	$\epsilon_{nb}$	F T	B	Axial buckling strain	$t_x(\epsilon_{xTb} t_x + \epsilon_{xyTb} t_y) + t_y(\epsilon_{xyTb} t_x + \epsilon_{yTb} t_y)$
sn_t	$\sigma_{nt}$	F T	B	Axial stress rate	$E\epsilon_{nt}$
N	$N$	All	B	Axial force	$A\sigma_n$
Fig	$F_{ig}$	S T E	B P	Edge, point load in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$
Fig	$F_{ig}$	F	B P	Edge, point load in global $x_i$ direction	If global coordinate system $F_{ig} = F_i F_{i\text{Amp}} e^{jF_{iPh}\frac{\pi}{180}}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x F_{x\text{Amp}} e^{jF_{xiPh}\frac{\pi}{180}} \\ F_y F_{y\text{Amp}} e^{jF_{yPh}\frac{\pi}{180}} \end{bmatrix}$

TABLE 2-15: IN PLANE TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$RF_i$	$RF_i$	S F P T	All	Reaction force in global $x_i$ direction on edge or point. Should only be used for integration, as they are defined only in the node points.	$reacf(u_i)$
$RM_z$	$RM_z$	S F P T	All	Reaction moment in global $z$ direction with respect to reference point on boundary. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF$
$Ws$	$W_s$	All	B	Strain energy density	$\frac{A}{2}(\epsilon_n \sigma_n)$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point) and bnd (boundary) refer to the different domain levels.

TABLE 2-16: IN-PLANE TRUSS APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$RF_{itot}$	$RF_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
$F_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction
$RM_{ztot}$	$RM_{ztot}$	S F P T	Total reaction moment in global $z$ -direction with respect to reference point

TABLE 2-16: IN-PLANE TRUSS APPLICATION MODE GLOBAL VARIABLES

NAME	S Y M B O L	A N A L Y S I S	D E S C R I P T I O N
Mztot	$M_{z\text{tot}}$	S F P T	Total applied moment in global $z$ -direction with respect to reference point
RF $_{i\text{totdom}}$	$\text{RF}_{i\text{totdom}}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt and bnd)
F $_{i\text{totdom}}$	$F_{i\text{totdom}}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt and bnd)
RM $_{z\text{totdom}}$	$\text{RM}_{z\text{totdom}}$	S F P T	Total reaction moment in global $z$ -direction with respect to reference point for all different domain levels (pnt and bnd)
M $_{z\text{totdom}}$	$M_{z\text{totdom}}$	S F P T	Total applied moment in global $z$ -direction with respect to reference point for all different domain levels (pnt and bnd)

### 3D Truss

In addition to the variables listed below almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

`en_amp` is the amplitude of the bending moment in the local  $y$  direction

- `sn_ph` is the phase of the axial stress

Table 2-17 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F

ANALYSIS	ABBREVIATION
Transient	T
Eigenfrequency	E

TABLE 2-17: 3D TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{itpph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
xn	$x_n$	All	E	Parameter along edge only used for linear constraint	$\frac{x(x_2 - x_1) + y(y_2 - y_1) + z(z_2 - z_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

TABLE 2-17: 3D TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ex <sub>ixjT</sub>	$\varepsilon_{xixjT}$	All	E	Tangential strain tensor	If large deformation $\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T \right)$ else $\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T \right)$
en	$\varepsilon_n$	All	E	Axial strain	$t_x(\varepsilon_{xTt_x} + \varepsilon_{xyTt_y} + \varepsilon_{xzTt_z})$ $+ t_y(\varepsilon_{xyTt_x} + \varepsilon_{yTt_y} + \varepsilon_{yzTt_z})$ $+ t_z(\varepsilon_{xzTt_x} + \varepsilon_{yzTt_y} + \varepsilon_{zTt_z})$
sn	$\sigma_n$	All	E	Axial stress	$E(\varepsilon_n - \alpha(T - T_{ref}) - \varepsilon_{ni}) + \sigma_{ni}$
ex <sub>ixjT_t</sub>	$\varepsilon_{xixjTt}$	T	E	Tangential strain rate tensor	If large deformation $\frac{1}{2} \left( \frac{\partial ut_i}{\partial x_j} \Big _T + \frac{\partial ut_j}{\partial x_i} \Big _T + \frac{\partial ut_k}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial ut_k}{\partial x_j} \Big _T \right)$ else $\frac{1}{2} \left( \frac{\partial ut_i}{\partial x_j} \Big _T + \frac{\partial ut_j}{\partial x_i} \Big _T \right)$
ex <sub>ixjT_t</sub>	$\varepsilon_{xixjTt}$	F	E	Tangential strain rate tensor	$\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} \Big _T + \frac{\partial u_j}{\partial x_i} \Big _T \right) j\omega$
ex <sub>ixjT_b</sub>	$\varepsilon_{xixjTb}$	Buckling	E	Tangential strain buckling tensor	$\frac{1}{2} \left( \frac{\partial ut_k}{\partial x_i} \Big _T \cdot \frac{\partial u_k}{\partial x_j} \Big _T + \frac{\partial u_k}{\partial x_i} \Big _T \cdot \frac{\partial ut_k}{\partial x_j} \Big _T \right)$
en_t	$\varepsilon_{nt}$	F T	E	Axial strain rate	$t_x(\varepsilon_{xTt}t_x + \varepsilon_{xyTt}t_y + \varepsilon_{xzTt}t_z)$ $+ t_y(\varepsilon_{xyTt}t_x + \varepsilon_{yTt}t_y + \varepsilon_{yzTt}t_z)$ $+ t_z(\varepsilon_{xzTt}t_x + \varepsilon_{yzTt}t_y + \varepsilon_{zTt}t_z)$
en_b	$\varepsilon_{nb}$	F T	E	Axial buckling strain	$t_x(\varepsilon_{xTb}t_x + \varepsilon_{xyTb}t_y + \varepsilon_{xzTb}t_z)$ $+ t_y(\varepsilon_{xyTb}t_x + \varepsilon_{yTb}t_y + \varepsilon_{yzTb}t_z)$ $+ t_z(\varepsilon_{xzTb}t_x + \varepsilon_{yzTb}t_y + \varepsilon_{zTb}t_z)$

TABLE 2-17: 3D TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sn_t	$\sigma_{nt}$	F T	E	Axial stress rate	$E\epsilon_{nt}$
N	$N$	All	E	Axial force	$A\sigma_n$
Fig	$F_{ig}$	S T E	E P	Edge, point load in global $x_i$ direction	<p>If global coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$ <p>If other coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$
Fig	$F_{ig}$	F	E/P	Edge, point load in global $x_i$ direction	<p>If global coordinate system</p> $F_{ig} = F_i F_{i\text{Amp}} e^{jF_{iPh} \frac{\pi}{180}}$ <p>If other coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x F_{x\text{Amp}} e^{jF_{xiPh} \frac{\pi}{180}} \\ F_y F_{y\text{Amp}} e^{jF_{yiPh} \frac{\pi}{180}} \\ F_z F_{z\text{Amp}} e^{jF_{ziPh} \frac{\pi}{180}} \end{bmatrix}$

TABLE 2-17: 3D TRUSS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$RF_i$	$RF_i$	S F P T	All	Reaction force in global $x_i$ direction on edge or point. Should only be used for integration, as they are defined only in the node points.	$reacf(u_i)$
$RM_i$	$RM_i$	S F P T	E	Reaction moment in global $x_i$ direction with respect to reference point on edge. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF$
$Ws$	$W_s$	All	E	Strain energy density	$\frac{A}{2}(\epsilon_n \sigma_n)$

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point) and edg (edge) refer to the different domain levels.

TABLE 2-18: 3D TRUSS APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$RF_{itot}$	$RF_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
$F_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction
$RM_{itot}$	$RM_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point

TABLE 2-18: 3D TRUSS APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
M <sub>itot</sub>	$M_{itot}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point
RF <sub>itotdom</sub>	$RF_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt and edg)
F <sub>itotdom</sub>	$F_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt and edg)
RM <sub>itotdom</sub>	$RM_{itotdom}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt and edg)
M <sub>itotdom</sub>	$M_{itotdom}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt and edg)

### *Shell*

In addition to the variables listed below almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append \_amp or \_ph to the variable name. For example:

- sx\_amp, the amplitude of the normal stress in the  $x$  direction.
- ex\_ph, the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as mises, disp, Tresca, s1, and so on.

The table below uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E

TABLE 2-19: SHELL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$\theta_i$	$\theta_i$	All	All	$x_i$ rotation	$\theta_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$\theta_{it}$	$\theta_{it}$	T	All	$x_i$ angular velocity	$\theta_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$\theta_{i\_amp}$	$\theta_{iamp}$	F	All	$x_i$ rotation amplitude	$ \theta_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$\theta_{i\_ph}$	$\theta_{iph}$	F	All	$x_i$ rotation phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(\theta_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$\theta_{i\_t}$	$\theta_{it}$	F	All	$x_i$ angular velocity	$j\omega\theta_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$\theta_{i\_t\_amp}$	$\theta_{itamp}$	F	All	$x_i$ angular velocity amplitude	$\omega\theta_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$

TABLE 2-19: SHELL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
th <i>i</i> _t_ph	$\theta_{iph}$	F	All	$x_i$ angular velocity phase	$\text{mod}(\theta_{iph} + 90^\circ, 360^\circ)$
ui_tt	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
th <i>i</i> _tt	$\theta_{itt}$	F	All	$x_i$ angular acceleration	$-\omega^2 \theta_i$
ui_tt_amp	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
th <i>i</i> _tt_amp	$\theta_{ittamp}$	F	All	$x_i$ angular acceleration amplitude	$\omega^2 \theta_{iamp}$
ui_tt_ph	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
th <i>i</i> _tt_ph	$\theta_{ittph}$	F	All	$x_i$ angular acceleration phase	$\text{mod}(\theta_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
totrot	totrot	All	All	Total rotation	$\sqrt{\sum_i (\text{real}(\theta_i))^2}$
postheight	$z$	All	B	Postprocessing height for stress and strain evaluation	Dependent on the settings on the postprocessing page
si	$\sigma_i$	All	B	$\sigma_i$ normal stress global coord. system	$D(\epsilon - \alpha_{\text{vec}}(T - T_{\text{ref}}))$ With loss factor damping in frequency response analysis $D((1 + j\eta)\epsilon - \alpha_{\text{vec}}(T - T_{\text{ref}}))$
sij	$\tau_{ij}$	All	B	$\tau_{ij}$ shear stress global coord. system	$D(\epsilon - \alpha_{\text{vec}}(T - T_{\text{ref}}))$ With loss factor damping in frequency response analysis $D((1 + j\eta)\epsilon - \alpha_{\text{vec}}(T - T_{\text{ref}}))$

TABLE 2-19: SHELL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sil	$\sigma_i$	All	B	$\sigma_i$ normal stress shell local coord. system	$D(\varepsilon_l - \alpha_{\text{vec}}(T - T_{\text{ref}}))$ With loss factor damping in frequency response analysis $D((1 + j\eta)\varepsilon_l - \alpha_{\text{vec}}(T - T_{\text{ref}}))$
sijl	$\tau_{ij}$	All	B	$\tau_{ij}$ shear stress shell local coord. system	$D(\varepsilon_l - \alpha_{\text{vec}}(T - T_{\text{ref}}))$ With loss factor damping in frequency response analysis $D((1 + j\eta)\varepsilon_l - \alpha_{\text{vec}}(T - T_{\text{ref}}))$
si	$\sigma_i$	All	B	Principal stresses, $i=1, 2, 3$	
ei	$\varepsilon_i$	All	B	$\varepsilon_i$ normal strain global system	Defined by the elshell_arg2 element
eij	$\varepsilon_{ij}$	All	B	$\varepsilon_{ij}$ shear strain global coord. system	Defined by the elshell_arg2 element
eil	$\varepsilon_{il}$	All	B	$\varepsilon_{il}$ normal strain user defined coord. system	Defined by the elshell_arg2 element
eijl	$\varepsilon_{ijl}$	All	B	$\varepsilon_{ijl}$ shear strain user defined coord. system	Defined by the elshell_arg2 element
ei	$\varepsilon_i$	All	B	Principal strains, $i=1, 2, 3$	
si <sub>xj</sub>	$\sigma_{ixj}$	All	B	Principal stress directions $i, j=1, 2, 3$	
ei <sub>xj</sub>	$\varepsilon_{ixj}$	All	B	Principal strain directions $i, j=1, 2, 3$	
tresca	$\sigma_{\text{tresca}}$	All	B	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{\text{mises}}$	All	B	von Mises stress	

TABLE 2-19: SHELL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$N_{xil}$	$N_{xil}$	All	B	Local in-plane normal force $x_i$ dir.	Defined by the elshell_arg2 element
$N_{xy1}$	$N_{xy1}$	All	B	Local in-plane shear force	Defined by the elshell_arg2 element
$M_{xil}$	$M_{xil}$	All	B	Local bending moment $x_i$ direction	Defined by the elshell_arg2 element
$M_{xy1}$	$M_{xy1}$	All	B	Local torsion moment	Defined by the elshell_arg2 element
$Q_{xil}$	$Q_{xil}$	All	B	Local out of-plane shear force $x_i$ direction	Defined by the elshell_arg2 element
$e_{xilxj}$	$e_{xilxj}$	All	B	Local shell coordinate system base vectors	Defined by the elshell_arg2 element
$F_{ig}$	$F_{ig}$	All	B E P	Body, edge, point load in global $x_i$ direction	Defined differently depending on how the force is defined and the analysis type used
$RF_i$	$RF_i$	S F P T	All	Reaction force in global $x_i$ direction on boundary, edge, or point. Should only be used for integration, as they are defined only in the node points.	$\text{reacf}(u_i)$

TABLE 2-19: SHELL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$RM_i$	$RM_i$	S F P T	B E	Reaction moment in global $x_i$ direction with respect to reference point on boundary or edge. Should only be used for integration, as they are defined only in the node points.	$(\mathbf{r} - \mathbf{r}_0) \times RF + reacf(\theta_i)$
$RM_i$	$RM_i$	S F P T	P	Reaction torque in global $x_i$ direction	$reacf(\theta_i)$
$M_{ig}$	$M_{ig}$	All	B E P	Body, edge, or point moment in global $x_i$ direction	Defined differently depending on how the moment is defined and the analysis type used
Filocal	$F_{ilocal}$	All	B E P	Body, edge, or point load in local $x_i$ direction	Defined differently depending on how the force is defined and the analysis type used
Milocal	$M_{ilocal}$	All	B E P	Body, edge, or point moment in local $x_i$ direction	Defined differently depending on how the moment is defined and the analysis type used

In addition to the domain variables, some global variables available from the **Global Variables Plot** dialog box are defined. In the table below, pnt (point), edg (edge), and bnd (boundary) refer to the different domain levels.

TABLE 2-20: SHELL APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$RF_{itot}$	$RF_{itot}$	S F P T	Total reaction force in global $x_i$ -direction
$F_{itot}$	$F_{itot}$	S F P T	Total applied force global $x_i$ -direction

TABLE 2-20: SHELL APPLICATION MODE GLOBAL VARIABLES

NAME	SYMBOL	ANALYSIS	DESCRIPTION
$\text{RM}_{itot}$	$\text{RM}_{itot}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point
$M_{itot}$	$M_{itot}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point
$\text{RF}_{itotdom}$	$\text{RF}_{itotdom}$	S F P T	Total reaction force in global $x_i$ -direction for all different domain levels (pnt, edg, and bnd)
$F_{itotdom}$	$F_{itotdom}$	S F P T	Total applied force in global $x_i$ -direction for all different domain levels (pnt, edg, and bnd)
$\text{RM}_{itotdom}$	$\text{RM}_{itotdom}$	S F P T	Total reaction moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, edg, and bnd)
$M_{itotdom}$	$M_{itotdom}$	S F P T	Total applied moment in global $x_i$ -direction with respect to reference point for all different domain levels (pnt, edg, and bnd)

# Piezoelectrical Application Modes

In addition to the variables listed below, almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` represents the amplitude of the normal stress in the  $x$  direction
- `ex_ph` represents the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

The tables use a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$ ,  $y$ , and  $z$ . In particular,  $u_i$  ( $ui$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Transient	T
Eigenfrequency	E
Damped eigenfrequency	D

Table 2-21 lists the variables common to all piezoelectrical application modes, while the subsequent tables list the remaining variables by application mode.

## Common Piezoelectrical Application Mode Variables

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TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$e_i$	$\epsilon_i$	All	S	$\epsilon_i$ normal strain global coord. system	Engineering strain $\frac{\partial u_i}{\partial x_i}$  or Green strain $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i} \cdot \frac{\partial u_k}{\partial x_j}\right)$  for small and large deformations, respectively
$e_{ij}$	$\epsilon_{ij}$	All	S	$\epsilon_{ij}$ shear strain global coord. system	Engineering strain $\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$  or Green strain (see $e_i$ ) for small and large deformations, respectively
$e_{il}$	$\epsilon_{il}$	All	S	$\epsilon_{il}$ normal strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$e_{ijl}$	$\epsilon_{ijl}$	All	S	$\epsilon_{ijl}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$e_{i\_t}, e_{ij\_t}$	$\epsilon_{it}$	T	S	$\epsilon_{it}$ normal and $\epsilon_{ijt}$ shear velocity strain, global system	Engineering or Green strain time derivative for small and large deformations, respectively
$e_{il\_t}$	$\epsilon_{ilt}$	F T	S	$\epsilon_{ilt}$ normal velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
$e_{ijl\_t}$	$\epsilon_{ijlt}$	F T	S	$\epsilon_{ijlt}$ shear velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_i, s_{ij}$	$\sigma_i, \tau_{ij}$	All	S	$\sigma_i$ normal stress, $\tau_{ij}$ shear stress, global coord. system	If material defined in global coord. sys.: $c_E \varepsilon - e^T \mathbf{E}$ or $D\varepsilon$  With loss factor damping in frequency response analysis: $(1 + j\eta)c_E \varepsilon - e^T \mathbf{E}$ or $(1 + j\eta)D\varepsilon$  If material defined in user-def. coord. sys.: $T_{\text{coord}} \sigma_l T_{\text{coord}}^T$ $J^{-1} F S F^T$ if large def.
$s_{il}, s_{ijl}$	$\sigma_{il}, \tau_{ijl}$	All	S	$\sigma_i$ normal stress, $\tau_{ij}$ shear stress, user-defined local coord. system	$c_E \varepsilon_l - e^T \mathbf{E}_l$ or $D\varepsilon_l$  With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_l - e^T \mathbf{E}_l$ or $(1 + j\eta)D\varepsilon_l$
$s_{i\_t}, s_{ij\_t}$	$\sigma_{it}, \tau_{ijt}$	F T	S	$\sigma_{it}$ time derivative of normal stress, $\tau_{ijt}$ time derivative of shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D\varepsilon_t$  With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D\varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_l T_{\text{coord}}^T$
$s_{il\_t}, s_{ijl\_t}$	$\sigma_{ilt}, \tau_{ijlt}$	F T	S	$\sigma_{ilt}$ time derivative of normal stress, user-defined local coord. system	$c_E \varepsilon_{lt}$ or $D\varepsilon_{lt}$  With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_l$ or $(1 + j\eta)j\omega D\varepsilon_l$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$S_i, S_{ij}$	$S_i, S_{ij}$	All	S	Second Piola Kirchhoff stress, global coord. system.	If material defined in global coord. sys., with Green strain: $c_E \epsilon - e^T \mathbf{E}_m$ or $D\epsilon$
$S_{il}, S_{ijl}$	$S_{il}, S_{ijl}$	All	S	Second Piola Kirchhoff stress, user-defined local coord. system	With Green strain: $c_E \epsilon_l - e^T \mathbf{E}_{ml}$ or $D\epsilon_l$
$S_{i\_t}, S_{ij\_t}$	$S_{it}, S_{ijt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	
$S_{il\_t}, S_{ijl\_t}$	$S_{ilt}, S_{ijlt}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined local coord. system	
$cE_{ij}$	$c_E$	All	S	Stiffness matrix components, $i,j = 1, \dots, 6$	Components of the $c_E$ matrix if material in stress-charge form. With isotropic loss and anisotropic loss the real data is multiplied elementwise with the loss factor matrix $(1 + j\eta_{cE})$ . $s_E^{-1}$ , if the material is specified on strain-charge form, calculated by a special inverting-matrices element.
$sE_{ij}$	$s_E$	All	S	Compliance matrix components, $i,j = 1, \dots, 6$	Components of the $s_E$ matrix if material in strain-charge form. With isotropic loss and anisotropic loss the real data is multiplied elementwise with the loss factor matrix $(1 - j\eta_{sE})$ .

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$e_{ij}$	$e$	All	S	Piezoelectric coupling matrix components, $i = 1, \dots, 3$ , $j = 1, \dots, 6$	Components of the $e$ matrix if material in stress-charge form. $ds_E^{-1}$ when material in strain-charge form.
$\epsilon_{Sij}$	$\epsilon_S$	All	S	Electric permittivity with strain field constant	If material defined on stress-charge form: $\epsilon_0 \epsilon_{rS}$ ; if material defined on strain-charge form: $\epsilon_0 \epsilon_{rT} - ds_E^{-1} d^T$ . With isotropic and anisotropic dielectric loss the real data is multiplied elementwise with the loss factor matrix $(1 - j\eta_{\epsilon S})$ .
$D_{ij}$	$D$	All	S	Stiffness matrix components, $i, j = 1, \dots, 6$	For isotropic and anisotropic materials. With isotropic loss and anisotropic loss the real data is multiplied elementwise with the loss factor matrix $(1 + j\eta_s)$ .
$\epsilon_{onij}$	$\epsilon_e$	All	S	Electric permittivity matrix components	$\epsilon_0 \epsilon_r$ for isotropic and anisotropic materials. With isotropic and anisotropic dielectric loss the real data is multiplied elementwise with the loss factor matrix $(1 - j\eta_e)$ .
$\sigma_{aij}$	$\sigma_e$	F	S	Electric conductivity matrix components	For isotropic and anisotropic materials
$\sigma_{Sij}$	$\sigma_S$	F	S	Electric conductivity matrix components at constant strain	If material defined on stress-charge form, values of the $\sigma_S$ matrix; if material defined on strain-charge form, values of the $\sigma_T$ matrix

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
eta_cEij	$\eta_{cE}$	F D	S	Anisotropic loss factor matrix for $c_E$ , $i,j = 1,...,6$	Components of the anisotropic loss factor matrix $\eta_{cE}$
eta_cE	$\eta_{cE}$	F D	S	Isotropic loss factor for $c_E$	Value of the isotropic loss factor $\eta_{cE}$
eta_sEij	$\eta_{sEij}$ , $i,j=1,...,6$	F D	S	Anisotropic loss factor matrix for $s_E$ , $i,j = 1,...,6$	Components of the anisotropic loss factor matrix $\eta_{sE}$
eta_sE	$\eta_{sE}$	F D	S	Isotropic loss factor for $s_E$	Value of the isotropic loss factor $\eta_{sE}$
eta_sij	$\eta_s$	F D	S	Anisotropic structural loss factor matrix, $i,j = 1,...,6$	Components of the anisotropic structural loss factor matrix $\eta_s$
eta_s	$\eta_s$	F D	S	Isotropic structural loss factor	Value of the isotropic loss factor $\eta_s$
eta_eij	$\eta_e$	F D	S	Anisotropic loss factor matrix for $e$ , $i = 1,...,3$ , $j = 1,...,6$	Components of the anisotropic loss factor matrix $\eta_e$
eta_e	$\eta_e$	F D	S	Isotropic loss factor for $e$	Value of the isotropic loss factor $\eta_e$
eta_dij	$\eta_d$	F D	S	Anisotropic loss factor matrix for $d$ , $i = 1,...,3$ , $j = 1,...,6$	Components of the anisotropic loss factor matrix $\eta_d$
eta_d	$\eta_d$	F D	S	Isotropic loss factor for $d$	Value of the isotropic loss factor $\eta_d$
eta_epsSij	$\eta_{\varepsilon S}$	F D	S	Anisotropic loss factor matrix for $\varepsilon_S$	Components of the anisotropic loss factor matrix $\eta_{\varepsilon S}$
eta_epsS	$\eta_{\varepsilon S}$	F D	S	Isotropic loss factor for $\varepsilon_S$	Value of the isotropic loss factor $\eta_{\varepsilon S}$
eta_epsTij	$\eta_{\varepsilon T}$	F D	S	Anisotropic loss factor matrix for $\varepsilon_T$	Components of the anisotropic loss factor matrix $\eta_{\varepsilon T}$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
eta_epsT	$\eta_{\varepsilon T}$	FD	S	Isotropic loss factor for $\varepsilon_T$	Value of the isotropic loss factor
eta_epsij	$\eta_{\varepsilon}$	FD	S	Anisotropic loss factor matrix for $\varepsilon_e$	Components of the anisotropic loss factor matrix $\eta_{\varepsilon}$
eta_eps	$\eta_{\varepsilon}$	FD	S	Isotropic loss factor for $\varepsilon_e$	Value of the isotropic loss factor $\eta_{\varepsilon}$
eson	eson	All	S	Electrical equation available	1 or 0, with assembly
Ei	$E_i$	All	S	Electric field	$-\left(\frac{\partial V}{\partial x_i}\right)$
					$F^T \mathbf{E}_m$ Large def., no ALE
Eil	$E_{il}$	All	S	Electric field, user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}$
Emi	$E_{mi}$	All	S	Electric field, in material orientation	$-\left(\frac{\partial V}{\partial x_i}\right)$ Large def., no ALE $F^T \mathbf{E}$ Large def., no ALE
Emil	$E_{mil}$	All	S	Electric field, in material orientation user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}_m$
Di	$D_i$	All	S	Electric displacement, $x_i$ component	If material defined in global coord. system: $e\varepsilon + \varepsilon_S \mathbf{E}$ or $\varepsilon_e \mathbf{E}$  If material defined in user-def. coord. sys.: $T_{\text{coord}} \mathbf{D}_1$ $\mathbf{D} = J^{-1} F \mathbf{D}_m$ if large def.
Dil	$D_{il}$	All	S	Electric displacement, $x_i$ component, local coord. sys.	$e\varepsilon_l + \varepsilon_S \mathbf{E}_l$ or $\varepsilon_e \mathbf{E}_l$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
D <sub>mi</sub>	D <sub>mi</sub>	All	S	Electric displacement in material orientation, $x_i$ component	$\mathbf{D}_m = \mathbf{P}_m + \epsilon_0 J C^{-1} \mathbf{E}_m$ if large deformations
D <sub>mil</sub>	D <sub>mil</sub>	All	S	Electric displacement in material orientation, $x_i$ component, local coord. sys.	
P <sub>mi</sub>	P <sub>mi</sub>	All	S	Electric polarization in material orientation, $x_i$ component	If large def., material defined in global coord. system: $e\epsilon + (\epsilon_S - \epsilon_0)\mathbf{E}_m$ or $(\epsilon_e - \epsilon_0)\mathbf{E}_m$
P <sub>mil</sub>	P <sub>mil</sub>	All	S	Electric polarization in material orientation, $x_i$ component, local coord. sys.	
J <sub>i</sub>	J <sub>i</sub>	T F D	S	Total current density, $x_i$ component	$J_{d,i} + J_{p,i}$ or $J_{d,i}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
J <sub>il</sub>	J <sub>il</sub>	T F	S	Total current density, $x_i$ component, local coord. sys.	$J_{d,il} + J_{p,il}$ or $J_{d,il}$
J <sub>d,i</sub>	J <sub>d,i</sub>	T	S	Displacement current density, $x_i$ component	$\frac{\partial D_i}{\partial t}$
		F D			$j\omega D_i$
J <sub>d,il</sub>	J <sub>d,il</sub>	T	S	Displacement current density, $x_i$ component, local coord. sys.	$\frac{\partial D_{il}}{\partial t}$
		F			$j\omega D_{il}$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$J_{dm,i}$	$J_{dm,i}$	T F D	S	Displacement current density, material orientation, $x_i$ component	Calculated from material electric displacement $D_{mi}$
$J_{dm,il}$	$J_{dm,il}$	T F	S	Displacement current density, material orientation, $x_i$ component, local coord. sys.	Calculated from material electric displacement $D_{mil}$
$J_{p,i}$	$J_{p,i}$	T F D	S	Potential current density, $x_i$ component	$\sigma_e \mathbf{E}$ If material defined in user-def. coordinate system $T_{\text{coord}} \mathbf{J}_l$
$J_{p,il}$	$J_{p,il}$	F	S	Potential current density, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_l$
$J_{pm,i}$	$J_{pm,i}$	T F D	S	Potential current density, material orientation, $x_i$ component	$\sigma_e \mathbf{E}_m$
$J_{pm,il}$	$J_{pm,il}$	F	S	Potential current density, material orientation, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_{ml}$
nD	nD	All	B	Surface charge density	$\mathbf{n}_{\text{up}} \cdot (\mathbf{D}_{\text{down}} - \mathbf{D}_{\text{up}})$
nJ	nJ	T F	B	Current density outflow	$\mathbf{n} \cdot \mathbf{J}^d$
nJs	nJs	F	B	Source current density	Only for unsymmetric electric currents. $\mathbf{n}_{\text{up}} \cdot (\mathbf{J}_{\text{down}} - \mathbf{J}_{\text{up}})$ or, with weak constraints, the Lagrange multiplier for V.

TABLE 2-2I: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
normD	$ \mathbf{D} $	All	S	Electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$
normE	$ \mathbf{E} $	All	S	Electric field, norm	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
Qe	$Q_e$	F D	S	Electric power dissipation density	$\text{real}(\mathbf{E} \cdot \text{conj}(\mathbf{J}_p + j\omega \mathbf{D})) / 2$
Qes	$Q_{es}$	F D	S	Power dissipation density	$Q_e + Q_s$
Qs	$Q_s$	F D	S	Structural power dissipation density	$\text{real}(\sigma \cdot \text{conj}(j\omega \epsilon)) / 2$
smon	smon	All	S	Structural equation available	1 or 0, with assembly
mises	$\sigma_{\text{mises}}$	All	S	von Mises stress	
tresca	$\sigma_{\text{tresca}}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
V	$V$	All	All	Electric potential	$V$
V_amp	$V_{\text{amp}}$	F	All	Electric potential amplitude	$ V $
V_ph	$V_{\text{ph}}$	F	All	Electric potential phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(V), 2\pi)$
$V_{il}$	$V_{il}$	All	S	Electric potential gradient, user-defined coord. system	$T_{\text{coord}}^T \nabla V$
We	$W_e$	All	S	Electric energy density	If material properties defined in global coord. system: $\mathbf{E} \cdot \mathbf{D} / 2$ , $\text{real}(\text{conj}(\mathbf{E}) \cdot \mathbf{D}) / 2$ in frequency response) If material properties defined in local user-defined coord. system: $\mathbf{E}_l \cdot \mathbf{D}_l / 2$ , $\text{real}(\text{conj}(\mathbf{E}_l) \cdot \mathbf{D}_l) / 2$ in frequency response)

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Wes	$W_{es}$	All		Total energy density	$W_e + W_s$ , $J W_e + W_s$ with ale
Ws	$W_s$	All	S	Strain energy density	If material properties defined in global coord. system: $\frac{\sigma \cdot \epsilon}{2}, \frac{1}{2} \text{real}(\sigma \cdot \text{conj}(\epsilon))$ in frequency response analysis. If material properties defined in local user-defined coord. system: $\frac{\sigma_l \cdot \epsilon_l}{2},$ $\frac{\text{real}(\sigma_l \cdot \text{conj}(\epsilon_l))}{2}$ in $+ \frac{\text{real}(\sigma_\phi \cdot \text{conj}(\epsilon_\phi))}{2}$ in frequency response analysis
$I_i$	$\mathbf{I}$	T F E D	S	Mechanical energy flux	$-\sigma \mathbf{v}$ (T) $-\sigma \text{conj}(\mathbf{v})$ (F E D)
$n\mathbf{I}$	$I_n$	T F E D	B	Normal component of mechanical energy flux	$\mathbf{n} \cdot \mathbf{I}$
$I_{av,i}$	$\mathbf{I}_{av}$	F E D	S	Mechanical energy flux, time average	$-0.5 \text{ real}(\sigma \text{conj}(\mathbf{v}))$
$n\mathbf{I}_{av}$	$I_{n,av}$	F E D	B	Normal component of mechanical energy flux, time average	$\mathbf{n} \cdot \mathbf{I}_{av}$
p	$p$	All	S	Pressure	$-(\sigma_x + \sigma_y + \sigma_z)/3$
$\text{curlU}_i$	$\text{curlU}$			Curl of displacement, $x_i$ -component	$\nabla \times \mathbf{u}$

TABLE 2-21: COMMON PIEZOELECTRICAL APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$F_{ij}$	$F_{ij},$ $i,j=1, 2, 3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$C_{ij}$	$C_{ij},$ $i,j=1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
$\text{inv}C_{ij}$	$C^{-1}_{ij},$ $i,j=1, 2, 3$	All	All	Inverse of Right Cauchy-Green symmetric tensor all components are defined	$F^{-1} F^{-T}$
$\text{inv}F_{ij}$	$\text{inv}F_{ij},$ $i,j=1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
$\det F$	$\det F$	All	All	Determinant of deformation gradient	$\det F$
$J$	$J$	All	All	Volume ratio	$\det F$
$nT_{E,\text{up}}$	$nT_{E,\text{up}}$	All	B	Electric Maxwell surface stress tensor, $x_i$ component, up side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{up}} \cdot \mathbf{D}_{\text{up}})\mathbf{n}_{\text{down}} + (\mathbf{n}_{\text{down}} \cdot \mathbf{D}_{\text{up}})\mathbf{E}_{\text{up}}$
$nT_{E,\text{down}}$	$nT_{E,\text{down}}$	All	B	Electric Maxwell surface stress tensor, $x_i$ component, down side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{down}} \cdot \mathbf{D}_{\text{down}})\mathbf{n}_{\text{up}} + (\mathbf{n}_{\text{up}} \cdot \mathbf{D}_{\text{down}})\mathbf{E}_{\text{down}}$

### Piezo Solid

TABLE 2-22: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$

TABLE 2-22: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i\text{_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_i\text{_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_i\text{_t}$	$u_{it}$	T F E D	All	$x_i$ velocity	$u_{it}$ (T) or $j\omega u_i$ (F E D)
$u_i\text{_t_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_i\text{_t_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_i\text{_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_i\text{_tt_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_i\text{_tt_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	Defined by the elpric element
$e_i$	$\varepsilon_i$	All	S	Principal strains, $i=1,2,3$	Defined by the elpric element
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	Defined by the elpric element
$e_{ixj}$	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	Defined by the elpric element

TABLE 2-22: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in x <sub>i</sub> direction	$\begin{bmatrix} \mathbf{Ta}_x \\ \mathbf{Ta}_y \\ \mathbf{Ta}_z \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$
F <sub>ig</sub>	F <sub>ig</sub>	All	All	Body load, face load, edge load, point load, in global x <sub>i</sub> direction	If global coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$

### Piezo Plane Stress

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TABLE 2-23: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
u <sub>i</sub>	u <sub>i</sub>	All	All	x <sub>i</sub> displacement	u <sub>i</sub>
u <sub>it</sub>	u <sub>it</sub>	T	All	x <sub>i</sub> velocity	u <sub>it</sub>
u <sub>i_amp</sub>	u <sub>iamp</sub>	F	All	x <sub>i</sub> displacement amplitude	u <sub>i</sub>
u <sub>i_ph</sub>	u <sub>i_ph</sub>	F	All	x <sub>i</sub> displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
u <sub>i_t</sub>	u <sub>it</sub>	T F E D	All	x <sub>i</sub> velocity	u <sub>it</sub> (T) or jωu <sub>i</sub> (F E D)
u <sub>i_t_amp</sub>	u <sub>itamp</sub>	F	All	x <sub>i</sub> velocity amplitude	ωu <sub>iamp</sub>
u <sub>i_t_ph</sub>	u <sub>itph</sub>	F	All	x <sub>i</sub> velocity phase	mod(u <sub>i_ph</sub> + 90°, 360°)
u <sub>i_tt</sub>	u <sub>itt</sub>	F	All	x <sub>i</sub> acceleration	-ω <sup>2</sup> u <sub>i</sub>
u <sub>i_tt_amp</sub>	u <sub>ittamp</sub>	F	All	x <sub>i</sub> acceleration amplitude	ω <sup>2</sup> u <sub>iamp</sub>

TABLE 2-23: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<i>ui_tt_ph</i>	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
ez	$\varepsilon_z$	All	S	$\varepsilon_z$ normal strain, out of the $xy$ -plane	$\frac{\left( \sum_j e_{j3} E_j - \sum_{k=1,2,4} (c_E)_{3k} \varepsilon_k \right)}{(c_E)_{33}} \text{ or}$ $- \frac{\sum_{k=1,2,4} (D)_{3k} \varepsilon_k}{(D)_{33}}$ <p>With loss factor damping in frequency response analysis</p> $\frac{\left( \sum_j e_{j3} E_j - \sum_{k=1,2,4} (1+j\eta)(c_E)_{3k} \varepsilon_k \right)}{(1+j\eta)(c_E)_{33}}$ $- \sum_{k=1,2,4} (1+j\eta)(c_E)_{3k} \varepsilon_k$ <p>or <math>\frac{(1+j\eta)(c_E)_{33}}{(1+j\eta)(c_E)_{33}}</math></p>
exy	$\varepsilon_{xy}$	All	S	$\varepsilon_{xy}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$
ez_t	$\varepsilon_z$	F T	S	$\varepsilon_z$ normal velocity strain out of the $xy$ -plane	$\frac{\left( - \sum_{k=1,2,4} (M)_{3k} \varepsilon_{kt} \right)}{(M)_{33}} \quad (M \text{ is } c_E \text{ or } D)$
exy_t	$\varepsilon_{xyt}$	T	S	$\varepsilon_{xyt}$ shear velocity strain global coord. system	$\frac{1}{2} \left( \frac{\partial u_t}{\partial y} + \frac{\partial v_t}{\partial x} \right)$

TABLE 2-23: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
exy_t	$\epsilon_{xyt}$	F	S	$\epsilon_{xyt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) j\omega$
exyl_t	$\epsilon_{xylt}$	F T	S	$\epsilon_{xylt}$ shear velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
s_i	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	Defined by the elpric element
e_i	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	Defined by the elpric element
s_ixj	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	Defined by the elpric element
e_ixj	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	Defined by the elpric element
Tai	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	$\begin{bmatrix} Ta_x \\ Ta_y \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$
Fig	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_z \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$

## Piezo Plane Strain

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TABLE 2-24: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	T F E D	All	$x_i$ velocity	$u_{it}$ (T) or $j\omega u_i$ (F E D)
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittp}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
$\epsilon_{xy}$	$\epsilon_{xy}$	All	S	$\epsilon_{xy}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$

TABLE 2-24: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sz	$\sigma_z$	All	S	$\sigma_z$ normal stress	If material defined in global coord. sys.  $\sum_k (c_E)_{3k} \varepsilon_k - \sum_j e_{j3} E_j, \text{ or } \sum_k (D)_{3k} \varepsilon_k$ With loss factor damping in frequency response analysis  $\sum_k (1+j\eta)(c_E)_{3k} \varepsilon_k - \sum_j e_{j3} E_j, \text{ or }$ $\sum_k (1+j\eta)(D)_{3k} \varepsilon_k$ If material defined in user-def. coord. sys.  $\sum_k (c_E)_{3k} (\varepsilon_1)_k - \sum_j e_{j3} (E_1)_j, \text{ or }$ $\sum_k (D)_{3k} (\varepsilon_1)_k$
sz_t	$\sigma_{zt}$	All	S	$\sigma_{zt}$ time derivative of normal stress	If material defined in global coord. sys.  $\sum_k (D)_{3k} (\varepsilon_t)_k \quad (M \text{ is } c_E \text{ or } D)$ With loss factor damping in frequency response analysis  $\sum_k (1+j\eta)(M)_{3k} j\omega \varepsilon_k \quad (M \text{ is } c_E \text{ or } D)$ If material defined in user-def. coord. sys.  $\sum_k (M)_{3k} (\varepsilon_{1t})_k \quad (M \text{ is } c_E \text{ or } D)$
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	Defined by the elpric element

TABLE 2-24: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$\epsilon_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	Defined by the elpric element
$\sigma_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	Defined by the elpric element
$\epsilon_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	Defined by the elpric element
$Ta_i$	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	$\begin{bmatrix} Ta_x \\ Ta_y \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$
$F_{ig}$	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$

### Piezo Axial Symmetry

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TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uor	uor	All	All	$r$ displacement divided by $r$	uor
uaxi	uaxi	All	All	$r$ displacement	uor $\cdot r$
w	w	All	All	$z$ displacement	w
uort	$uor_t$	T	All	$r$ velocity divided by $r$	$uor_t$
uaxi_t	$uaxi_t$	T F E D	All	$r$ velocity	$uor_t \cdot r$ (T) or $j\omega uor \cdot r$ (F E D)
w_t	$w_t$	T F E D	All	$z$ velocity	$w_t$ (T) or $j\omega w$ (F E D)

TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uaxi_amp	$u_{\text{axi}}_{\text{amp}}$	F	All	$r$ displacement amplitude	$ u_{\text{axi}} $
w_amp	$w_{\text{amp}}$	F	All	$z$ displacement amplitude	$ w $
uaxi_ph	$u_{\text{axi}}_{\text{ph}}$	F	All	$r$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_{\text{axi}}), 2\pi)$
w_ph	$w_{\text{ph}}$	F	All	$z$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(w), 2\pi)$
uaxi_t	$u_{\text{axi}}_t$	F	All	$r$ velocity	$j\omega u_{\text{axi}}$
w_t	$w_t$	F	All	$z$ velocity	$j\omega w$
uaxi_t_amp	$u_{\text{axi}}_{t\text{amp}}$	F	All	$r$ velocity amplitude	$\omega u_{\text{axi}}_{\text{amp}}$
w_t_amp	$w_{t\text{amp}}$	F	All	$z$ velocity amplitude	$\omega w_{\text{amp}}$
uaxi_t_ph	$u_{\text{axi}}_{t\text{ph}}$	F	All	$r$ velocity phase	$\text{mod}(u_{\text{axi}}_{\text{ph}} + 90^\circ, 360^\circ)$
w_t_ph	$w_{t\text{ph}}$	F	All	$z$ velocity phase	$\text{mod}(w_{\text{ph}} + 90^\circ, 360^\circ)$
uaxi_tt	$u_{\text{axi}}_{tt}$	F	All	$r$ acceleration	$-\omega^2 u_{\text{axi}}$
w_tt	$w_{tt}$	F	All	$z$ acceleration	$-\omega^2 w$
uaxi_tt_amp	$u_{\text{axi}}_{t\text{amp}}$	F	All	$r$ acceleration amplitude	$\omega^2 u_{\text{axi}}_{\text{amp}}$
w_tt_amp	$w_{t\text{amp}}$	F	All	$z$ acceleration amplitude	$\omega^2 w_{\text{amp}}$
uaxi_tt_ph	$u_{\text{axi}}_{t\text{ph}}$	F	All	$r$ acceleration phase	$\text{mod}(u_{\text{axi}}_{\text{ph}} + 180^\circ, 360^\circ)$
w_tt_ph	$w_{t\text{ph}}$	F	All	$z$ acceleration phase	$\text{mod}(w_{\text{ph}} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{u_{\text{axi}}^2 + w^2}$
er	$\epsilon_r$	All	S	$\epsilon_r$ normal strain, global system	$u_{\text{or}} + \frac{\partial}{\partial r}(u_{\text{or}}) \cdot r$
ez	$\epsilon_z$	All	S	$\epsilon_z$ normal strain, global system	$\frac{\partial w}{\partial z}$

TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
ephi	$\epsilon_\phi$	All	S	$\epsilon_\phi$ normal strain	$u_{\theta r}$
erz	$\epsilon_{rz}$	All	S	$\epsilon_{rz}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z} (u_{\theta r}) \cdot r + \frac{\partial w}{\partial r} \right)$
exl, eyl	$\epsilon_{xl}, \epsilon_{yl}$	All	S	$\epsilon_{xl}, \epsilon_{yl}$ normal strains, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
exyl	$\epsilon_{xyl}$	All	S	$\epsilon_{xy}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
er_t	$\epsilon_{rt}$	T	S	$\epsilon_{rt}$ velocity normal strain, global system	$u_{\theta r} t + \frac{\partial}{\partial r} (u_{\theta r} t) \cdot r$
er_t	$\epsilon_{rt}$	F	S	$\epsilon_{rt}$ velocity normal strain, global system	$j\omega \left( u_{\theta r} + \frac{\partial}{\partial r} u_{\theta r} \cdot r \right)$
ez_t	$\epsilon_{zt}$	T	S	$\epsilon_{zt}$ velocity normal strain, global system	$\frac{\partial w}{\partial z} t$
ez_t	$\epsilon_{zt}$	F	S	$\epsilon_{zt}$ velocity normal strain, global system	$j\omega \left( \frac{\partial w}{\partial z} \right)$
ephi_t	$\epsilon_{\phi t}$	T	S	$\epsilon_{\phi t}$ velocity normal strain	$u_{\theta r} t$
ephi_t	$\epsilon_{\phi t}$	F	S	$\epsilon_{\phi t}$ velocity normal strain	$j\omega u_{\theta r}$
erz_t	$\epsilon_{rzt}$	T	S	$\epsilon_{rzt}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z} (u_{\theta r} t) \cdot r + \frac{\partial w}{\partial r} t \right)$
erz_t	$\epsilon_{rzt}$	F	S	$\epsilon_{rzt}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z} (u_{\theta r}) \cdot r + \frac{\partial w}{\partial r} \right) j\omega$

TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
exl_t, eyl_t	$\varepsilon_{xlt}, \varepsilon_{ylt}$	F T	S	$\varepsilon_{xlt}, \varepsilon_{ylt}$ velocity normal strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
exyl_t	$\varepsilon_{xylt}$	F T	S	$\varepsilon_{xylt}$ velocity shear strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
sr, sz	$\sigma_r, \sigma_z$	All	S	$\sigma_r, \sigma_z$ normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^T \mathbf{E}$ or $D\varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta) c_E \varepsilon - e^T \mathbf{E}$ or $(1 + j\eta) D\varepsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_l T_{\text{coord}}^T$
sphi	$\sigma_\phi$	All	S	$\sigma_\phi$ normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^T \mathbf{E}$ or $D\varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta) c_E \varepsilon - e^T \mathbf{E}$ or $(1 + j\eta) D\varepsilon$ If material defined in user-def. coord. sys. $c_E \varepsilon_l - e^T \mathbf{E}_l$

TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
srz	$\tau_{rz}$	All	S	$\tau_{rz}$ shear stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon - e^T \mathbf{E}$ or $D\epsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \epsilon - e^T \mathbf{E}$ or $(1 + j\eta)D\epsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{lt} T_{\text{coord}}^T$
sr_t, sz_t	$\sigma_{rt}, \sigma_{zt}$	F T	S	$\sigma_{rt}, \sigma_{zt}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon_t$ or $D\epsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \epsilon_t$ or $(1 + j\eta)j\omega D\epsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{lt} T_{\text{coord}}^T$
sphi_t	$\sigma_{\phi t}$	F T	S	$\sigma_{\phi t}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon_t$ or $D\epsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \epsilon_t$ or $(1 + j\eta)j\omega D\epsilon$ If material defined in user-def. coord. sys. $c_E \epsilon_{lt}$
s_i	$\sigma_i$	All	S	Principal stresses, $i = 1, 2, 3$	Defined by the elpric element
e_i	$\epsilon_i$	All	S	Principal strains, $i = 1, 2, 3$	Defined by the elpric element

TABLE 2-25: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sixj	$\sigma_{ixj}$	All	S	Principal stress directions, $i, j = 1, 2, 3$	Defined by the elpric element
eixj	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i, j = 1, 2, 3$	Defined by the elpric element
Tai	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	$\begin{bmatrix} Ta_r \\ Ta_z \end{bmatrix} = \begin{bmatrix} \sigma_r & \tau_{rz} \\ \tau_{rz} & \sigma_z \end{bmatrix} \begin{bmatrix} n_r \\ n_z \end{bmatrix}$
Fig	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{rg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} F_r \\ F_z \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_r \\ F_z \end{bmatrix}$

## Programming Reference

The *Structural Mechanics Module User's Guide* and *Structural Mechanics Module Model Library* focus on describing and exemplifying how to solve problems with the Structural Mechanics Module using COMSOL Multiphysics' graphical user interface. Although this user interface provides a convenient environment for modeling many problems, it can sometimes be useful to work with a programming tool.

This chapter provides details about the fields in the application mode structure for the structural and piezoelectric application modes.

# Application Structure Overview

This section contains a summary of the application structure, and how application objects are used for a convenient transformation of application mode data to PDE and boundary coefficients. For details on specific functions, see the “Command Reference” chapter of the *COMSOL Multiphysics Reference Guide*.

## *The Application Structure Fields*

---

The process of performing a simulation using the application modes available through the Structural Mechanics Module includes the correct setup of the application structure. The application structure contains the necessary information for the model setup in several fields. This section describes the application structure in the context of the Structural Mechanics Module. See also the section “Application Structures” on page 54 in the *COMSOL Multiphysics Scripting Guide*. Most fields have corresponding entries in the FEM structure, described in the section “Specifying a Model” on page 3 in the *COMSOL Multiphysics Scripting Guide*. The following table gives an overview of the fields in the application structure.

FIELD	DESCRIPTION
appl.mode	Application mode class
appl.dim	Cell array of dependent variable names
appl.sdim	Cell array of spatial coordinates
appl.border	Assembly on interior boundaries; turn on/off assembly on interior boundaries
appl.name	Application mode name
appl.var	Cell array or structure with application-specific scalar variables.
appl.assign	Assigned variable names
appl.assignsuffix	Suffix to append to all application mode variable names
appl.equ	Structure containing domain properties
appl.bnd	Structure containing boundary conditions
appl.edg	Structure containing edge conditions
appl.pnt	Structure containing point conditions
appl.prop	Application mode specific properties

Most of these fields have default values and need not be specified when solving a problem using the programming language. The function `multiphysics` transforms the application structure data to the FEM structure to generate the complete set of equations; see the entry for `multiphysics` on page 350 of the *COMSOL Multiphysics Reference Guide* for details.

The application mode specific names of the fields in the structures in the table above can be found in the section “Application Mode Programming Reference” on page 116.

#### APPLICATION MODE CLASS AND TYPE

The application modes are specified via a corresponding class name.

To specify the class, write the name of the class as a string, for example,

```
appl.mode.class='PiezoSolid3';
```

which specifies that the Piezo Solid application mode will be used.

When setting up an axisymmetric model, the full specification of an application mode includes a declaration of its type, as in the following example:

```
appl.mode.class = 'SmeAxialSolid';
appl.mode.type = 'axi';
```

#### DEPENDENT VARIABLES

The `dim` field in the application structure states the names of the dependent variables, and hence gives the dimension of the corresponding PDE system. If the `dim` field is missing, the software uses the standard variable names for the application mode.

For example, the default names of the dependent variables in the Piezo Axial Symmetry application mode are `uor`, `w`, and `V`. If you want to refer to the variable `uor` (representing  $u/r$ ) as  $\xi$ , you can set new names by typing

```
appl.dim = {'xi' 'w' 'V'};
```

#### SPATIAL COORDINATES

The names of the spatial coordinates are given in `fem.sdim`. However, `fem.sdim` only gives the names of the spatial coordinates of the geometry: one variable in 1D, two variables in 2D, and three variables in 3D. To specify all three spatial coordinates, use `appl.sdim`. The additional spatial coordinates are used when giving names to vector component variables.

For example, in 2D Cartesian coordinates

```
fem.sdim = {'x1' 'y1'};  
appl.sdim = {'x1' 'y1' 'z1'};  
defines the spatial coordinates  $x_1, y_1, z_1$ .
```

In cylindrical coordinates

```
fem.sdim = {'r' 'z'};  
appl.sdim = {'r' 'phi' 'z'};
```

defines the spatial coordinates  $r, \phi, z$ .

The coordinates defined in `fem.sdim` and `appl.sdim` must match.

#### APPLICATION NAME

You can use this field to give the application a descriptive name. If no name is specified, a default name is used.

```
appl.name = 'sensor';
```

#### APPLICATION MODE PROPERTIES

Some application modes define properties to, for example, specify which type of analysis to perform or which dependent variables to use. These are specified in the `appl.prop` structure. For example, the piezoelectricity application modes define the field `appl.prop.esform`, which specifies if the electrostatics formulation symmetric or unsymmetric. To set the latter option, type

```
appl.prop.esform = 'unsymmetric_es';
```

The default element type used for domains and boundaries can be specified in the `appl.prop.elemdefault` field. For example, write

```
appl.prop.elemdefault='Lag1';
```

to obtain linear Lagrange elements.

#### APPLICATION SCALAR VARIABLES

For scalar variables that are valid in the whole model, such as permittivity of vacuum and angular frequency, the values can be specified in the `appl.var` field. Note that only predefined variable names can be used in this field.

#### ASSIGNED VARIABLE NAMES

To avoid duplication of variable names, the `appl.assign` field can be used to state the relation between the assigned name of a variable that you can use in postprocessing and in the PDE and boundary coefficients on one hand, and the default name that is used in the COMSOL Multiphysics and Structural Mechanics Module algorithms on the

other. This field is thus only necessary when you solve a multiphysics problem where variable name conflicts may arise.

If you want to use the variable name `w` for the intrinsic variable `omega` in the Perpendicular currents quasi-statics mode, all you have to enter is

```
appl.assign.omega = 'w';
```

This can also be specified as a cell array,

```
appl.assign = {'omega' 'w'};
```

The odd entries in the `assign` field state the default application scalar variable names or the postprocessing variable names. These are listed in the subsection “Application Mode Variables” for each application mode in the chapter “Application Mode Variables” on page 6 of this manual. The even entries in `appl.assign` are the variable names that you want to use for the physical entities when modeling.

There is also a field `appl.assignsuffix`, which can be used to add a suffix to the name of all application mode variables. Variables appearing in `appl.assign` will take the given assigned name, while the others will get the suffix added to their names.

#### DOMAIN PROPERTIES

The application structure field `appl.equ` is used for specifying properties that will be transformed into PDE coefficients. For example the In-Plane Waves application mode defines the fields

```
appl.equ.epsilonr  
appl.equ.mur  
appl.equ.sigma  
appl.equ.n  
appl.equ.matparams  
appl.equ.magconstrel  
appl.equ.elconstrel  
appl.equ.P  
appl.equ.Dr  
appl.equ.Br  
appl.equ.M
```

Now consider an example with two materials. To use other values than the default for the physical properties, enter them as follows.

Scalars such as `sigma` are given as a cell array with the values,

```
appl.equ.sigma={'5.9e7' '0'};
```

Vectors such as `M` are given as a nested cell array with the vector components as elements in the inner cell array,

```
appl.equ.M={{'0' '0'} {'0' '500'}};
```

In the 3D case, the vector has three components,

```
appl.equ.M={{'0' '0' '0'} {'0' '500' '0'}};
```

Tensors or matrix variables are given as cell arrays with the tensor components,

```
appl.equ.mur={{'1' '2'; '3' '4'} {'5' '6' '7' '8'}};
```

This example shows two ways of writing a 2-by-2 tensor. The first tensor is written row by row with a semicolon separating the rows,

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

while the second tensor is written column by column,

$$\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}$$

There are a number of short-hand ways of writing a tensor. In 2D  $\{'1' '2'\}$  is a diagonal tensor

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$\{'1' '2' '3'\}$  is a symmetric tensor

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

and  $\{'1' '2' '3' '4'\}$  is the full tensor

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Similarly in 3D  $\{'1' '2' '3'\}$  is a diagonal tensor

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\{'1' '2' '3' '4' '5' '6'\}$  is a symmetric tensor

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

and `{'1' '2' '3' '4' '5' '6' '7' '8' '9'}` is the full tensor

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

### **BOUNDARY CONDITIONS**

The boundary conditions for the simulation is given in the `appl.bnd` field. First you have to select which type of boundary condition you want at each boundary. This is done using the field `appl.bnd.type`. Then some boundary conditions requires certain boundary variables to be set, unless the default value is sufficient. Note that not all types of boundary conditions require a boundary variable.

For example, the boundary variables defined by the In-Plane Waves application mode are

```
appl.bnd.type
appl.bnd.epsilonrbnd
appl.bnd.murbnd
appl.bnd.sigmacbnd
appl.bnd.nbnd
appl.bnd.E0
appl.bnd.Esz
appl.bnd.portnr
appl.bnd.inport
appl.bnd.Pport
appl.bnd.rectmodetype
appl.bnd.usermodeltype
appl.bnd.modespec
appl.bnd.betate
appl.bnd.betatm
appl.bnd.modenum
appl.bnd.betaport
appl.bnd.nucutoff
appl.bnd.curoffforbeta
appl.bnd.applmode
appl.bnd.wavetype
appl.bnd.srcpnt
appl.bnd.eta
appl.bnd.kdir
appl.bnd.nu0
```

```
appl.bnd.srctype  
appl.bnd.Js0  
appl.bnd.A0  
appl.bnd.matparams  
appl.bnd.H0
```

The variables are specified using the same form for scalars, vectors, and tensors as above for the subdomain variables.

#### DOMAIN GROUPS

Often you have a model with several subdomains having the same physical properties and many boundaries where you want to apply the same boundary condition. To simplify the notation, the index vectors in the fields `equ.ind` and `bnd.ind` are used. Consider this example:

```
appl.equ.sigma = {'5.9e7' '0'};  
appl.equ.ind = [1 2 1 1 1];
```

Here you have five subdomains. Subdomain 2 is nonconducting, whereas the other four subdomains have the electric conductivity  $5.9 \cdot 10^7$  S/m.

Here is another example where three different boundary conditions are applied to six boundaries:

```
appl.bnd.type = {'tH0' 'H' 'A'};  
appl.bnd.A0 = {[[] []] '100'};  
appl.bnd.H0 = {[[] {'10' '5'} []]};  
appl.bnd.ind = [1 2 2 2 3 3];
```

This specifies the magnetic field at boundaries 2, 3, and 4, the magnetic potential at boundaries 5 and 6 and electric insulation at boundary 1. Note that the perfect magnetic conductor condition does not need any boundary variable to be defined and that we have set the variables to an empty vector where the value is ignored.

There is an alternative syntax for the index vector using a cell array of numeric vectors instead of a single numeric vector as in the examples above. In this case the numeric vectors in the cell array list the domains having the same settings. For example,

```
appl.equ.ind = {[1 3 4 5] 2};
```

is equivalent to

```
appl.equ.ind = [1 2 1 1 1];
```

and

```
appl.bnd.ind = {1 [2 3 4] [5 6]};
```

is equivalent to

```
appl.bnd.ind = [1 2 2 2 3 3];
```

# Application Mode Programming Reference

This section tabulates the application-mode dependent fields of the application structure. For each application mode, the following details are listed:

- *Dependent and independent variables*—gives the variables in `appl.dim` and `appl.sdim`. In the GUI the dependent variables are given in the **Dependent variables** text field in the Model Navigator.
- *Application mode class and name*—specifies which values to use in `appl.mode` and gives the default value of `appl.name`. In the user interface, you provide `appl.name` in the **Application mode name** edit field in the Model Navigator.
- *Scalar variable*—specifies the variable in `appl.var`. The corresponding dialog box is the **Application Scalar Variables** dialog box.
- *Properties*—specifies all fields in `appl.prop`, for example which type of analysis to perform or which elements to use. In the user interface you specify the properties in the **Application Mode Properties** dialog box.
- *Application mode parameters*—specifies the parameters in `appl.equ`, `appl.bnd`, `appl.edg`, and `appl.pnt`. The dialog boxes corresponding to these fields are the **Subdomain Settings**, **Boundary Settings**, **Edge Settings**, and **Point Settings** dialog boxes.

In the tables below, words written in **Code** font refer to strings; in the corresponding structure field such a word appears in single quotes, for example '`iso`'. Furthermore, the word “expression” means that the structure field or cell array component is given either as a numeric value (a floating point value, for example, `2.0e11`) or as a string. Dimensionful default values are given in SI units.

`fem.appl` is a cell array of structures, one for each application mode. `fem.appl{i}` refers to the *i*th application mode in this cell array.

In the application mode parameters tables the field column means a field at a specific domain level given in the domain column. For example, field `alpha` and domain `equ` refers to the field `fem.appl{i}.equ.alpha`, which specifies the thermal expansion coefficient at the subdomain level. Some fields, such as loads and constraints, exist in all domains.

**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'w' 'p'}	Dependent variable names, global displacements in $x, y, z$ directions and pressure
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl{i}.mode.class	SmeSolid3	
appl{i}.name	smsld	

**SCALAR VARIABLES**

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100' 't_old_ini' '-1' 'refpntx' '0' 'refpnty' '0' 'refpntz' '0'}	Excitation frequency for frequency response analysis, initial value for previous time step used for contact with dynamic friction, and coordinates for reference point for applied moment and reaction moment computation.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5   LagU2P1   LagU3P2   LagU4P3   LagU5P4	Lag2	Default element to use. Lagrange element of order 1–5 and mixed Lagrange element of order 2–5
appl.prop.analysis	static   staticplastic   eig   eigendamped   time   freq   para   quasi   buckling	static	Analysis to be performed, static, static elasto-plastic, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient, or linear buckling analysis; see note below.
appl.prop.eigtype	lambda   freq   loadfactor	freq	Should eigenvalues, eigenfrequencies or load factors be used
appl.prop.largedef	on   off	off	Include large deformation, nonlinear geometry effects.
appl.prop.frame	name of the frame	ref	The name of the frame where the application mode lives
appl.prop.createframe	on   off	off	Controls if the application mode should create a deformed frame
appl.prop.deformframe	name of the deformed frame	deform	The name of the by the application mode created deformed frame

## APPLICATION MODE PARAMETERS

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper   visco	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, hyperelastic, or viscoelastic.
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used, 1 use mixed formulation, 0 use displacement formulation.
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy, Gyz, Gxz	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphay, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user-defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin   murnaghan	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
lMurn	expression	-3e11	equ	Murnaghan constant l
mMurn	expression	-5e11	equ	Murnaghan constant m
nMurn	expression	-6e11	equ	Murnaghan constant n
lambLame	expression	1e11	equ	Lamé constant used for Murnaghan hyperelastic materials.
muLame	expression	8e10	equ	Lamé constant used for Murnaghan hyperelastic materials.
K	expression	4e10	equ	Bulk modulus for viscoelastic materials.
G	expression	3e10	equ	Shear modulus for viscoelastic materials.

TABLE 3-I: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Gvec	cell array of expressions	2e10	equ	Shear modulus for the branches of a viscoelastic material.
tauvec	cell array of expressions	3000	equ	Relaxation time for the branches of a viscoelastic material.
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
WLFFlag	1   0	0	equ	Flag specifying whether the WLF time shift factor should be used for viscoelastic materials with thermal expansion included.
Twlf	expression	500	equ	WLF reference temperature, viscoelastic materials.
C1wlf	expression	17.44	equ	WLF constant, viscoelastic materials.
C2wlf	expression	51.6	equ	WLF constant, viscoelastic materials.
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not.
ini_strain	1   0	0	equ	Flag specifying whether initial strain should be included: 1 include strains, 0 do not.
ini_pressure	1   0	0	equ	Flag specifying whether initial pressure should be included: 1 include pressure, 0 do not.
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi, syzi, sxzi	expression	0	equ	Initial shear stresses
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi, eyzi, exzi	expression	0	equ	Initial shear strains
p_i	expression	0	equ	Initial pressure

TABLE 3-I: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined, not used for loadcond=follower_press
Fx, Fy, Fz	expression	0	all	Body load, face load, edge load, point load, $x, y, z$ direction
FxPh, FyPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation, for standard use Hx, Hy, Hz, Rx, Ry, Rz; for general use H and R
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x, y, z$ direction is constrained: 1 constrained, 0 free, used with standard notation
Rx, Ry, Rz	expression	0	all	Constraint value in $x, y, z$ direction, used with standard notation
H	cell array of expressions	{0 0 0; 0 0 0; 0 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0; 0; 0}	all	$R$ vector used for general notation constraints, $Hu = R$

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
offset	expression	0	bnd	Contact surface offset from geometric surface
pn	expression	$\min(1e-4*5^{\wedge}(\text{auglagiter}-1),0.1)E/h$	bnd	Contact normal penalty factor
pt	expression	$\min(1e-4*5^{\wedge}(\text{auglagiter}-1),0.1)E/h$	bnd	Contact tangential penalty factor
frictiontype	nofric   coulomb	nofric	bnd	Friction model
mustat	expression	0	bnd	Static coefficient of friction
cohe	expression	0	bnd	Cohesion sliding resistance
Ttmax	expression	Inf	bnd	Maximum tangential traction
dynfric	0   1	0	bnd	Should a dynamic friction model be used
mudyn	expression	0	bnd	Dynamic coefficient of friction
dcflic	expression	0	bnd	Friction decay coefficient
contacttol	auto   man	auto	bnd	Method to calculate if slave and master are in contact
mantol	expression	1e-6	bnd	Distance when slave and master are assumed to be in contact, used together with contacttol=man
searchdist	auto   man	auto	bnd	Method to calculate the distance to search for contact
mandist	expression	1e-2	bnd	Distance to search if the slave and master are in contact, used together with searchdist=man
searchmethod	fast   direct	fast	bnd	Method used when calculating if master and slave are in contact.
contact_oldi	0   1	0	bnd	If they were in contact in the previous time step
Tni	expression	1e6	bnd	Initial value for the contact pressure
Ttxi	expression	1e6	bnd	Initial value for the friction forces
xim_old	expression	1e6	bnd	The value of the mapped coordinates in the previous time step

## *Plane Stress*

---

### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'p'}	Dependent variable names, global displacements s in x, y directions and pressure
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global x, y directions

### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmePlaneStress	
appl.name		smps

### SCALAR VARIABLES

See the Solid, Stress-Strain application mode specification on page 117.

### PROPERTIES

See the Solid, Stress-Strain application mode specification on page 118.

### APPLICATION MODE PARAMETERS

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used: 1 use mixed formulation, 0 use displacement formulation.
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
thickness	expression	0.01	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphay, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4-by-4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin   murnaghan	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
lMurn	expression	-3e11	equ	Murnaghan constant l
mMurn	expression	-5e11	equ	Murnaghan constant m
nMurn	expression	-6e11	equ	Murnaghan constant n
lambLame	expression	1e11	equ	Lamé constant used for Murnaghan hyperelastic materials.
muLame	expression	8e10	equ	Lamé constant used for Murnaghan hyperelastic materials.
K	expression	4e10	equ	Bulk modulus for viscoelastic materials.
G	expression	3e10	equ	Shear modulus for viscoelastic materials.
Gvec	cell array of expressions	2e10	equ	Shear modulus for the branches of a viscoelastic material.
tauvec	cell array of expressions	3000	equ	Relaxation time for the branches of a viscoelastic material.
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
WLFFlag	1   0	0	equ	Flag specifying whether the WLF time shift factor should be used for viscoelastic materials with thermal expansion included.

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Twlf	expression	500	equ	WLF reference temperature, viscoelastic materials.
C1wlf	expression	17.44	equ	WLF constant, viscoelastic materials.
C2wlf	expression	51.6	equ	WLF constant, viscoelastic materials.
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not.
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not.
ini_pressure	1   0	0	equ	Flag specifying whether initial pressure should be included: 1 include pressure, 0 do not.
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi	expression	0	equ	Initial shear stress
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi	expression	0	equ	Initial shear strain
p_i	expression	0	equ	Initial pressure
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined, not used for loadcond=follower_press

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Fx, Fy	expression	0	all	Body load, edge load, point load, $x$ , $y$ direction
loadtype	area   volume	area	equ	Body load definition, load/volume or load/area
loadtype	area   length	length	bnd	Edge load definition, load/length or load/area
FxPh, FyPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation for standard use Hx, Hy, Rx, Ry; for general use H and R
Hx, Hy	1   0	0	all	Constraint flag controlling if $x$ , $y$ direction is constrained: 1 constrained, 0 free, used with standard notation
Rx, Ry	expression	0	all	Constraint value in $x, y$ direction, used with standard notation
H	cell array of expressions	{0 0; 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0}	all	$R$ vector used for general notation constraints, $Hu = R$
offset	expression	0	bnd	Contact surface offset from geometric surface
pn	expression	$\min(1e-4*5^{\text{auglagiter}} - 1, 0.1)E/h$	bnd	Contact normal penalty factor
pt	expression	$\min(1e-4*5^{\text{auglagiter}} - 1, 0.1)E/h$	bnd	Contact tangential penalty factor
frictiontype	nofric   coulomb	nofric	bnd	Friction model
mustat	expression	0	bnd	Static coefficient of friction
cohe	expression	0	bnd	Cohesion sliding resistance
Ttmax	expression	Inf	bnd	Maximum tangential traction
dynfric	0   1	0	bnd	Should a dynamic friction model be used
mudyn	expression	0	bnd	Dynamic coefficient of friction
dcflic	expression	0	bnd	Friction decay coefficient

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
contacttol	auto   man	auto	bnd	Method to calculate if slave and master are in contact
mantol	expression	1e-6	bnd	Distance when slave and master are assumed to be in contact, used together with contacttol=man
searchdist	auto   man	auto	bnd	Method to calculate the distance to search for contact
mandist	expression	1e-2	bnd	Distance to search if the slave and master are in contact, used together with searchdist=man
searchmethod	fast   direct	fast	bnd	Method used when calculating if master and slave are in contact
contact_oldi	0   1	0	bnd	If they were in contact in the previous time step
Tni	expression	1e6	bnd	Initial value for the contact pressure
Ttxi	expression	1e6	bnd	Initial value for the friction forces
xim_old	expression	1e6	bnd	The value of the mapped coordinates in the previous time step

## *Plane Strain*

---

### **DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'p'}	Dependent variable names, global displacements in $x, y$ directions and pressure
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y$ directions

### **APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl.mode.class	SmePlaneStrain	
appl.name	smpn	

### **SCALAR VARIABLES**

See the Solid, Stress-Strain application mode specification on page 117 for details.

### **PROPERTIES**

See the Solid, Stress-Strain application mode specification on page 118 for details.

### **APPLICATION MODE PARAMETERS**

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used: 1 use mixed formulation, 0 use displacement formulation
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
thickness	expression	1	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	equ	Type of damping; lossfactor can only be used for frequency response analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphay, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4-by-4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin  murnaghan	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
lMurn	expression	-3e11	equ	Murnaghan constant l
mMurn	expression	-5e11	equ	Murnaghan constant m
nMurn	expression	-6e11	equ	Murnaghan constant n
lambLame	expression	1e11	equ	Lamé constant used for Murnaghan hyperelastic materials.
muLame	expression	8e10	equ	Lamé constant used for Murnaghan hyperelastic materials.
K	expression	4e10	equ	Bulk modulus for viscoelastic materials.
G	expression	3e10	equ	Shear modulus for viscoelastic materials.
Gvec	cell array of expressions	2e10	equ	Shear modulus for the branches of a viscoelastic material.
tauvec	cell array of expressions	3000	equ	Relaxation time for the branches of a viscoelastic material.
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
WLFFlag	1   0	0	equ	Flag specifying whether the WLF time shift factor should be used for viscoelastic materials with thermal expansion included.
Twlf	expression	500	equ	WLF reference temperature, viscoelastic materials.
C1wlf	expression	17.44	equ	WLF constant, viscoelastic materials.
C2wlf	expression	51.6	equ	WLF constant, viscoelastic materials.
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
ini_pressure	1   0	0	equ	Flag specifying whether initial pressure should be included: 1 include pressure, 0 do not
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi	expression	0	equ	Initial shear stress
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi	expression	0	equ	Initial shear strain
p_i	expression	0	equ	Initial pressure
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fx, Fy	expression	0	all	Body load, edge load, point load, $x, y$ direction
loadtype	area   volume	volume	equ	Body load definition, load/volume or load/area
loadtype	area   length	area	bnd	Edge load definition, load/length or load/area
FxPh, FyPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Rx, Ry; for general use H and R
Hx, Hy	1   0	0	all	Constraint flag controlling if $x, y$ direction is constrained: 1 constrained, 0 free, used with standard notation
Rx, Ry	expression	0	all	Constraint value in $x, y$ direction, used with standard notation
H	cell array of expressions	{0 0;0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0}	all	$R$ vector used for general notation constraints, $Hu = R$

## Axial Symmetry, Stress-Strain

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### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'uor' 'w' 'p'}	Dependent variable names, global displacements in $r$ , $z$ directions and pressure
appl.sdim	{'r' 'phi' 'z'}	Independent variable names, space coordinates in global $r$ , $\phi$ , $z$ directions

### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmeAxialSolid	
appl.name	smaxi	

### SCALAR VARIABLES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100' 't_old_ini' '-1'}	Excitation frequency for frequency response analysis and initial value for previous time step used for contact with dynamic friction.

### PROPERTIES

All continuum application modes have the same application mode properties. See the Solid, Stress-Strain application mode specification on page 118 for details.

### APPLICATION MODE PARAMETERS

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used: 1 use mixed formulation, 0 use displacement formulation.

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density
thickness	expression	1	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
Er, Ephi, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Grz	expression	7.52e10	equ	Shear modulus for orthotropic material
nurphi, nuphiz, nurz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphar, alphaphi, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4-by-4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin   murnaghan	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
lMurn	expression	-3e11	equ	Murnaghan constant l
mMurn	expression	-5e11	equ	Murnaghan constant m
nMurn	expression	-6e11	equ	Murnaghan constant n
lambLame	expression	1e11	equ	Lamé constant used for Murnaghan hyperelastic materials.
muLame	expression	8e10	equ	Lamé constant used for Murnaghan hyperelastic materials.
K	expression	4e10	equ	Bulk modulus for viscoelastic materials.
G	expression	3e10	equ	Shear modulus for viscoelastic materials.
Gvec	cell array of expressions	2e10	equ	Shear modulus for the branches of a viscoelastic material.

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
tauvec	cell array of expressions	3000	equ	Relaxation time for the branches of a viscoelastic material.
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
WLFflag	1   0	0	equ	Flag specifying whether the WLF time shift factor should be used for viscoelastic materials with thermal expansion included.
Twlf	expression	500	equ	WLF reference temperature, viscoelastic materials.
C1wlf	expression	17.44	equ	WLF constant, viscoelastic materials.
C2wlf	expression	51.6	equ	WLF constant, viscoelastic materials.
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
ini_pressure	1   0	0	equ	Flag specifying whether initial pressure should be included: 1 include pressure, 0 do not
sri, sphii, szi	expression	0	equ	Initial normal stresses
srzi	expression	0	equ	Initial shear stress
eri, ephii, ezi	expression	0	equ	Initial normal strains
erzi	expression	0	equ	Initial shear strain
p_i	expression	0	equ	Initial pressure

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcond	free   fixed   roller (bnd only)   axisym (bnd only)   displacement   sym (bnd only)   symrphi (bnd only)   symphiz (bnd only)   antisym (bnd only)   antisymrphi (bnd only)   antisymphiz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fr, Fz	expression	0	all	Body load, edge load, point load, $r, z$ direction
loadtype	area   volume	volume	equ	Body load definition, load/volume or load/area
loadtype	area   length	area	bnd	Edge load definition, load/length or load/area
FrPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Rx, Ry; for general use H and R
Hr, Hz	1   0	0	all	Constraint flag controlling if $x, y$ direction is constrained: 1 constrained, 0 free, used with standard notation

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Rr, Rz	expression	0	all	Constraint value in $x, y$ direction, used with standard notation
H	cell array of expressions	{0 0; 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0; 0}	all	$R$ vector used for general notation constraints, $Hu = R$

### Mindlin Plate

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#### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'w' 'thx' 'thy'}	Dependent variable names, global displacements in $z$ direction and rotations about global $x, y$ -axes
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

#### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmeMindlin	
appl.name	smdrm	

#### SCALAR VARIABLES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100' 'refpntx' '0' 'refpnty' '0'}	Excitation frequency for frequency response analysis and coordinates for reference point for applied moment and reaction moment computation.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	drmplate	drmplate	Default element to be used. Mindlin plate element.
appl.prop.analysis	static   eig   eigendamped   time   freq   para   quasi	static	Analysis to be performed, static, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient.
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used

## APPLICATION MODE PARAMETERS

TABLE 3-5: APPLICATION MODE PARAMETERS FOR MINDLIN PLATE

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso	iso	equ	Material model isotropic, orthotropic, anisotropic
E	expression	2.0e11	equ	Young's modulus for isotropic material
Sf	expression	1.2	equ	Shear factor
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density
thickness	expression	0.01	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	equ	Type of damping; lossfactor can only be used for frequency response analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter

TABLE 3-5: APPLICATION MODE PARAMETERS FOR MINDLIN PLATE

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
Ex, Ey	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy, Gyz, Gxz	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy	expression	0.33	equ	Poisson's ratio for orthotropic material
alphax, alphay	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
Sfyz, Sfxz	expression	1.2	equ	Shear factors for orthotropic material
Dp	cell array of expressions	isotropic Dp matrix	equ	In-plane elasticity 3-by-3 matrix for anisotropic material; saved in symmetric format, 6 components
Ds	cell array of expressions	isotropic Ds matrix	equ	Shear elasticity 2-by-2 matrix for anisotropic material; saved in symmetric format, 3 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
dT	expression	0	equ	Temperature difference through plate
ini_load	1   0	0	equ	Flag specifying whether initial loads should be included: 1 include loads, 0 do not
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
Mxpi, Mypi	expression	0	equ	Initial plate bending moments
Mxypi	expression	0	equ	Initial plate torsional moments
Qxpi, Qypi	expression	0	equ	Initial shear forces
thxyi, thyxi	expression	0	equ	Initial curvature

TABLE 3-5: APPLICATION MODE PARAMETERS FOR MINDLIN PLATE

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
thyymthxxi	expression	0	equ	Initial warping
gyzi, gxzi	expression	0	equ	Initial average shear strain
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fz	expression	0	all	Body load, edge load, point load, $z$ direction
Mx, My	expression	0	all	Body moment, edge moment, point moment, $x, y$ direction
loadtype	area   volume	area	equ	Body load definition, load/volume or load/area
loadtype	area   length	length	bnd	Edge load definition, load/length or load/area
FzAmp	expression	1	all	Amplitude factor specifying the load's dependence on the excitation frequency $f$
FzPh	expression	0	all	Phase angle in degrees specifying the load's phase's dependence on the excitation frequency $f$
MxAmp, MyAmp	expression	1	all	Amplitude factor specifying the moment's dependence on the excitation frequency $f$
MxPh, MyPh	expression	0	all	Phase angle in degrees specifying the moment's phase's dependence on the excitation frequency $f$
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Rx, Ry; for general use H and R
constrlocaltype	free   simply   fixed   rotation   general	free	bnd	Local constraint condition on boundaries. Free, simply supported, fixed, rotation constrained, and general description

TABLE 3-5: APPLICATION MODE PARAMETERS FOR MINDLIN PLATE

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Hthx, Hthy	1   0	0	all	Constraint flag controlling if rotation about $x, y$ -axes is constrained: 1 constrained, 0 free
Hz	1   0	0	all	Constraint flag controlling if $z$ direction displacement is constrained: 1 constrained, 0 free
Rthx, Rthy	expression	0	all	Constraint value for rotation about $x, y$ -axes
Rz	expression	0	all	Constraint value in $z$ direction
Rz1	expression	0	bnd	Constraint value in $z$ direction used with simply supported in local coordinate system
Rth1	expression	0	bnd	Constraint value for tangential rotation used with rotation in local coordinate system
H	cell array of expressions	{0 0 0;0 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0;0}	all	$R$ vector used for general notation constraints, $Hu = R$
postcontr	top   bottom   mid   other	top	all	Evaluation height for stress and strain
height	expression	0	bnd	User specified evaluation height for stress and strain, used with other

## *In-Plane Euler Beam*

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### **DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'th'}	Dependent variable names, global displacements in $x, y$ directions and rotation about global $z$ -axis
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

### **APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl.mode.class	SmeInPlaneEulerBeam	
appl.name	smeulip	

### **SCALAR VARIABLES**

See the Mindlin Plate application mode specification on page 140.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	beam2d	beam2d	Default element to be used. In plane Euler beam element
appl.prop.analysis	static   eig   eigendamped   time   freq   para   quasi	static	Analysis to be performed, static, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used

## APPLICATION MODE PARAMETERS

TABLE 3-6: APPLICATION MODE PARAMETERS FOR IN-PLANE EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
E	expression	2.0e11	bnd	Young's modulus for isotropic material
alpha	expression	1.2e-5	bnd	Thermal expansion coefficient
rho	expression	7850	bnd	Density
A	expression	0.01	bnd	Cross section area
Iyy	expression	8.33e-6	bnd	Area moment of inertia
heightz	expression	0.1	bnd	Total section height
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	bnd	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	bnd	Mass damping parameter
betadK	expression	0	bnd	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
m	expression	0	pnt	Point mass
Jz	expression	0	pnt	Mass moment of inertia about z-axis

TABLE 3-6: APPLICATION MODE PARAMETERS FOR IN-PLANE EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
alphadM	expression	0	pnt	Mass damping parameter for point mass and mass moment of inertia
Tflag	1   0	0	bnd	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
Temp	expression	0	bnd	Thermal strain temperature
Tempref	expression	0	bnd	Thermal strain stress free reference temperature
dTz	expression	0	bnd	Temperature difference through beam
ini_load	1   0	0	bnd	Flag specifying whether initial loads should be included: 1 include loads, 0 do not
ini_strain	1   0	0	bnd	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
Ni	expression	0	bnd	Initial axial force
Mzi	expression	0	bnd	Initial bending moment
eni	expression	0	bnd	Initial axial strain
thsi	expression	0	bnd	Initial curvature
constrcond	free   fixed   pinned   norot   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	all	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fx, Fy	expression	0	all	Edge load, point load, $x, y$ directions
Mz	expression	0	all	Edge moment, point moment, $z$ direction

TABLE 3-6: APPLICATION MODE PARAMETERS FOR IN-PLANE EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
FxAmp, FyAmp	expression	1	all	Amplitude factor specifying the load's dependence on the excitation frequency $f$
FxPh, FyPh	expression	0	all	Phase angle in degrees specifying the load's phase's dependence on the excitation frequency $f$
MzAmp	expression	1	all	Amplitude factor specifying the moment's dependence on the excitation frequency $f$
MzPh	expression	0	all	Phase angle in degrees specifying the moment's phase's dependence on the excitation frequency $f$
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Hth, Rx, Ry, Rth; for general use H and R
Hth	1   0	0	all	Constraint flag controlling if rotation about $z$ -axis is constrained: 1 constrained, 0 free
Hx, Hy	1   0	0	all	Constraint flag controlling if $x$ , $y$ direction displacement is constrained: 1 constrained, 0 free
Rth	expression	0	all	Constraint value for rotation about $z$ -axis
Rx, Ry	expression	0	all	Constraint value in $x, y$ directions
H	cell array of expressions	{0 0 0; 0 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0;0}	all	$R$ vector used for general notation constraints, $Hu = R$

## *3D Euler Beam*

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### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'w' 'thx' 'thy' 'thz'}	Dependent variable names, global displacements in $x, y, z$ directions and rotations about global $x, y, z$ -axes
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	Sme3DEulerBeam	
appl.name		smeul3d

### SCALAR VARIABLES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100' 'refpntx' '0' 'refpnty' '0' 'refpntz' '0'}	Excitation frequency for frequency response analysis and coordinates for reference point for applied moment and reaction moment computation.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	beam3D	beam3D	Default element to be used. Euler 3D beam element.
appl.prop.analysis	static   eig   eigendamped   time   freq   para   quasi	static	Analysis to be performed, linear static, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient.
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used

## APPLICATION MODE PARAMETERS

TABLE 3-7: APPLICATION MODE PARAMETERS FOR 3D EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
E	expression	2.0e11	edg	Young's modulus for isotropic material
nu	expression	0.33	edg	Poisson's ratio
alpha	expression	1.2e-5	edg	Thermal expansion coefficient
rho	expression	7850	edg	Density
A	expression	0.01	edg	Cross section area
Iyy, Izz	expression	8.33e-6	edg	Area moment of inertia about local <i>y</i> - and <i>z</i> -axes
heighty, heightz	expression	0.1	edg	Total section height in local <i>y</i> and <i>z</i> direction
localxp, localyp	expression	1	edg	<i>x, y</i> -coordinate for point defining local <i>xy</i> -plane
localzp	expression	1	edg	<i>z</i> -coordinate for point defining local <i>xy</i> -plane
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	edg	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	edg	Mass damping parameter

TABLE 3-7: APPLICATION MODE PARAMETERS FOR 3D EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
betadK	expression	0	edg	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
m	expression	0	pnt	Point mass
Jx, Jy, Jz	expression	0	pnt	Mass moment of inertia about x, y, z-axes
alphadm	expression	0	pnt	Mass damping parameter for point mass and mass moment of inertia
masscoord	global   name of user-defined coordinate system	global	pnt	Coordinate system where mass moment of inertias are defined
Tflag	1   0	0	edg	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
Temp	expression	0	edg	Thermal strain temperature
Tempref	expression	0	edg	Thermal strain stress-free reference temperature
dTy, dTz	expression	0	edg	Temperature difference through beam in local y and z direction
ini_load	1   0	0	edg	Flag specifying whether initial loads should be included: 1 include loads, 0 do not
ini_strain	1   0	0	edg	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
Ni	expression	0	edg	Initial axial force
Mxli	expression	0	edg	Initial torsional moment
Myli, Mzli	expression	0	edg	Initial bending moment
eni	expression	0	edg	Initial axial strain
thksi	expression	0	edg	Initial torsional angle derivative
thysi, thzsi	expression	0	edg	Initial curvature

TABLE 3-7: APPLICATION MODE PARAMETERS FOR 3D EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcond	free   fixed   pinned   norot   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	all	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fx, Fy, Fz	expression	0	all	Edge load, point load, $x, y, z$ direction
Mx, My, Mz	expression	0	all	Edge moment, point moment, $x, y, z$ direction
FxAmp, FyAmp, FzAmp	expression	1	all	Amplitude factor specifying the load's dependence on the excitation frequency $f$
FxPh, FyPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phase's dependence on the excitation frequency $f$
MxAmp, MyAmp, MzAmp	expression	1	all	Amplitude factor specifying the moment's dependence on the excitation frequency $f$
MxPh, MyPh, MzPh	expression	0	all	Phase angle in degrees specifying the moment's phase's dependence on the excitation frequency $f$
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Hth, Rx, Ry, Rth; for general use H and R
Hthx, Hthy, Hthz	1   0	0	all	Constraint flag controlling if rotation about $x,y,z$ -axes is constrained: 1 constrained, 0 free
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x,y,z$ direction displacement is constrained: 1 constrained, 0 free

TABLE 3-7: APPLICATION MODE PARAMETERS FOR 3D EULER BEAM

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Rthx, Rthy, Rthz	expression	0	all	Constraint value for rotation about $x$ , $y$ , $z$ -axes
Rx, Ry, Rz	expression	0	all	Constraint value in $x$ , $y$ directions
H	cell array of expressions	{0 0 0 0 0 0;0 0 0 0 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0;0;0;0;0}	all	$R$ vector used for general notation constraints, $Hu = R$

### In-Plane Truss

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#### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v'}	Dependent variable names, global displacements in $x,y$ directions
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x,y,z$ directions

#### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmeTruss2D	
appl.name		smtr2d

#### SCALAR VARIABLES

See the Mindlin Plate application mode specification on page 140.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5	Lag1	Default element to be used
appl.prop.analysis	static   eig   eigendamped   time   freq   para   quasi   buckling	static	Analysis to be performed, static, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient, or linear buckling analysis; see note below
appl.prop.eigtype	lambda   freq   loadfactor	freq	Should eigenvalues, eigenfrequencies or load factors be used
appl.prop.largedef	on   off	off	Include large deformation, nonlinear geometry effects

## APPLICATION MODE PARAMETERS

TABLE 3-8: APPLICATION MODE PARAMETERS FOR IN-PLANE TRUSS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
E	expression	2.0e11	bnd	Young's modulus for isotropic material
alpha	expression	1.2e-5	bnd	Thermal expansion coefficient
rho	expression	7850	bnd	Density
A	expression	0.01	bnd	Cross section area
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	bnd	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	bnd	Mass damping parameter
betadK	expression	0	bnd	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
m	expression	0	pnt	Point mass

TABLE 3-8: APPLICATION MODE PARAMETERS FOR IN-PLANE TRUSS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
alphadM	expression	0	pnt	Mass damping parameter for point mass
Tflag	1   0	0	bnd	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
Temp	expression	0	bnd	Thermal strain temperature
Tempref	expression	0	bnd	Thermal strain stress free reference temperature
ini_stress	1   0	0	bnd	Flag specifying whether initial stress should be included: 1 include stresses, 0 do not
ini_strain	1   0	0	bnd	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
sni	expression	0	bnd	Initial axial stress
eni	expression	0	bnd	Initial axial strain
straight	1   0	1	bnd	Flag specifying whether constrains should be added to enforce boundary to be straight: 1 enforce boundary, 0 do not
constrcond	free   pinned   roller (bnd only)   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	bnd/edg,pnt	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fx, Fy, Fz	expression	0	all	Edge load, point load, $x, y, z$ direction

TABLE 3-8: APPLICATION MODE PARAMETERS FOR IN-PLANE TRUSS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
FxAmp, FyAmp, FzAmp	expression	1	all	Amplitude factor specifying the load's dependence on the excitation frequency $f$
FxPh, FyPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phase's dependence on the excitation frequency $f$
constrtype	standard   general	standard	all	Constraint notation: standard use Hx, Hy, Hz Rx, Ry, Rz; general use H and R
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x, y, z$ direction displacement is constrained: 1 constrained, 0 free
Rx, Ry, Rz	expression	0	all	Constraint value in $x, y, z$ directions
H	cell array of expressions	{0 0; 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0; 0}	all	$R$ vector used for general notation constraints, $Hu = R$

### 3D Truss

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#### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'w'}	Dependent variable names, global displacements in $x, y, z$ directions
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

#### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmeTruss3D	
appl.name		smtr3d

#### SCALAR VARIABLES

See the 3D Euler Beam application mode specification on page 145.

## PROPERTIES

See the In-Plane Truss application mode specification on page 154.

## APPLICATION MODE PARAMETERS

These parameters are the same as for the In-Plane Truss application mode except that there are three instead of two space variables, which results in three load components and three constraints; see page 154.

### *Shell*

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## DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'w' 'thx' 'thy' 'thz'}	Dependent variable names, global displacements in $x, y, z$ directions and rotations about global $x, y, z$ -axes
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

## APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmeShell	
appl.name	shell	

## SCALAR VARIABLES

See the 3D Euler Beam application mode specification on page 149.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	shell	shell	Default element to use. Shell element
appl.prop.analysis	static   eig   eigendamped   time   freq   para   quasi	static	Analysis to perform: static, eigenfrequency, damped eigenfrequency, time dependent, frequency response parametric, quasi-static transient
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used

## APPLICATION MODE PARAMETERS

TABLE 3-9: APPLICATION MODE PARAMETERS FOR SHELLS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
E	expression	2.0e11	bnd	Young's modulus for isotropic material
nu	expression	0.33	bnd	Poisson's ratio
alpha	expression	1.2e-5	bnd	Thermal expansion coefficient for isotropic material
rho	expression	7850	bnd	Density
Sf	expression	1.2	bnd	Shear factor
thickness	expression	0.01	bnd	Thickness of the shell
Tflag	1   0	0	bnd	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not.
Temp	expression	0	bnd	Thermal strain temperature
Tempref	expression	0	bnd	Thermal strain stress free reference temperature
dT	expression	0	bnd	Temperature difference through shell in local z direction
xlocalx, xlocaly, xlocalz	expression	t1x, t1y, t1z	bnd	x, y, z-components for projection vector defining local x-axis

TABLE 3-9: APPLICATION MODE PARAMETERS FOR SHELLS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
nsidex, nsidey, nsidez	expression	x+nx, y+ny, z+nz	bnd	$x, y, z$ -coordinates for point defining side of shell with positive $z$
dampingtype	Rayleigh   lossfactor   nodamping	nodamping	bnd	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	bnd	Mass damping parameter
betadK	expression	0	bnd	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor for loss factor damping
constrcond	free   fixed   pinned   norot   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	bnd, edg	Type of constraint condition
constrcoord	global   post (bnd only)   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   post (bnd only)   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
loadtype	area   volume	area	bnd	Body load definition, load/volume or load/area
loadtype	area   length	length	edg	Body load, edge load definition, load/length or load/area
Fx, Fy, Fz	expression	0	all	Body load, edge load, point load, $x, y, z$ directions
Mx, My, Mz	expression	0	all	Body load, edge moment, point moment, $x, y, z$ directions
FxAmp, FyAmp, FzAmp	expression	1	all	Amplitude factor specifying the load's dependence on the excitation frequency $f$

TABLE 3-9: APPLICATION MODE PARAMETERS FOR SHELLS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
FxPh, FyPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phase's dependence on the excitation frequency $f$
MxAmp, MyAmp, MzAmp	expression	1	all	Amplitude factor specifying the moment's dependence on the excitation frequency $f$
MxPh, MyPh, MzPh	expression	0	all	Phase angle in degrees specifying the moment's phase's dependence on the excitation frequency $f$
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Hth, Rx, Ry, Rth; for general use H and R
Hthx, Hthy, Hthz	1   0	0	all	Constraint flag controlling if rotation about $x, y, z$ -axes is constrained: 1 constrained, 0 free
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x, y, z$ direction displacement is constrained: 1 constrained, 0 free
Rthx, Rthy, Rthz	expression	0	all	Constraint value for rotation about $x, y, z$ -axes
Rx, Ry, Rz	expression	0	all	Constraint value in $x, y$ directions
H	cell array of expressions	{0 0 0 0 0 0;0 0 0 0 0 0}	all	$H$ matrix used for general notation constraints, $Hu = R$
R	cell array of expressions	{0;0;0;0;0;0}	all	$R$ vector used for general notation constraints, $Hu = R$
postcontr	top   bottom   mid   other	top	all	Evaluation height for stress and strain
height	expression	0	bnd	User specified evaluation height for stress and strain, used with other

### Piezoelectrical Application Modes

This section describes the application mode dependent fields of the application structure for the piezoelectrical application modes: Piezo Solid, Piezo Plane Stress, Piezo Plane Strain, and Piezo Axial Symmetry.

## DEPENDENT AND INDEPENDENT VARIABLES

The following tables describe the dependent and independent variables of each of the piezoelectrical application modes:

TABLE 3-10: DEPENDENT AND INDEPENDENT VARIABLES FOR PIEZO SOLID

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'w' 'V'}	Dependent variable names, global displacements in $x, y, z$ directions and electric potential
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

TABLE 3-11: DEPENDENT AND INDEPENDENT VARIABLES FOR PIEZO PLANE STRESS AND PIEZO PLANE STRAIN

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u' 'v' 'V'}	Dependent variable names, global displacements in $x, y$ directions and electric potential
appl.sdim	{'x' 'y' 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

TABLE 3-12: DEPENDENT AND INDEPENDENT VARIABLES FOR PIEZO AXIAL SYMMETRY

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'uor' 'w' 'V'}	Dependent variable names, global displacements in $r, z$ directions and electric potential. uor is the radial displacement divided by the radius
appl.sdim	{'r' 'phi' 'z'}	Independent variable names, space coordinates in global $r, \phi, z$ directions

### APPLICATION MODE CLASS AND NAME

The following table describes the application mode classes and names for the piezoelectrical application modes (the order of the class and name lists correspond to each other):

TABLE 3-13: APPLICATION MODE CLASSES AND NAMES FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT NAME
appl{i}.mode.class	PiezoSolid3 PiezoPlaneStress PiezoPlaneStrain PiezoAxialSymmetry	
appl{i}.name		smpz3d smpps smppn smpaxi

### SCALAR VARIABLES

The following table describes the scalar variables common to all piezoelectrical application modes:

TABLE 3-14: SCALAR VARIABLES FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	DEFAULT	DESCRIPTION
appl.var.freq	1e6	Excitation frequency for frequency response analysis
appl.var.epsilon0	8.854187817e-12	Permittivity of vacuum

### PROPERTIES

The following table describes the properties common to all piezoelectrical application modes:

TABLE 3-15: PROPERTIES FOR THE FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5	Lag2	Default element to use: Lagrange element of order 1–5
appl.prop.analysis	static   eig   eigendamped   time   freq	static	Analysis to perform: linear static, eigenfrequency, damped eigenfrequency, time dependent, and frequency response.

TABLE 3-15: PROPERTIES FOR THE FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used
appl.prop.esform	symmetric_es   unsymmetric_es   unsymmetric_ec	symmetric_es	Defines the form of the electrical part of the equation
appl.prop.largedef	on   off	off	Include large deformation, nonlinear geometry effects
appl.prop.frame	name of the frame	ref	The name of the frame where the application mode or structural degrees of freedom live
appl.prop.electrical_frame	name of the frame   defined_by_structural_frame	off	The name of the frame where the electrical degrees of freedom live
appl.prop.defines_electrical_frame	defined_by_structural_frame#	deform	Stores the string for electrical_frame property. # increments if there is a frame with the same name

#### APPLICATION MODE PARAMETERS

The following table describes the application mode parameters that appear in the piezoelectrical application modes. Unless otherwise is stated, the parameters are common to all application modes.

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
materialmodel	piezoelectric   aniso   iso	piezoelectric	equ	Defines the material model for each subdomain
constform	strain   stress	strain	equ	Form for the constitutive relation: strain-charge or stress-charge. For piezoelectric materials.
structuralon	1   0	1	equ	Defines whether structural part of the equation is active. For iso and aniso materials.
electricalon	1   0	0	equ	Defines whether electrical part of the equation is active. For iso and aniso materials.
rho	expression	7500	equ	Density

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
rhow	expression	0	equ	Space charge density
sE	cell array of expressions	Piezo material (PZT-5H)	equ	Compliance matrix (6-by-6 matrix), used for strain-charge form, saved in symmetric format, 21 components
cE	cell array of expressions	Piezo material (PZT-5H)	equ	Elasticity matrix (6-by-6 matrix), used for stress-charge form, saved in symmetric format, 21 components
d	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for strain-charge form (3-by-6 matrix)
e	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for stress-charge form (3-by-6 matrix)
epsilonnrT	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix (3-by-3 matrix), used for strain-charge form, saved in symmetric format, 6 components
epsilonnrS	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix (3-by-3 matrix), used for stress-charge form, saved in symmetric format, 6 components
sigmaT	cell array of expressions	0	equ	Electrical conductivity matrix used for strain-charge form; 3-by-3 matrix saved in symmetric format, 6 components
sigmaS	cell array of expressions	0	equ	Electrical conductivity matrix used for stress-charge form; 3-by-3 matrix saved in symmetric format, 6 components
D	cell array of expressions	Elasticity matrix of PZT-5H	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
epsilonnr	expression	1	equ	Relative permittivity for isotropic material
epsilonnrtensor	cell array of expressions	Isotropic relative permittivity 1	equ	Relative electric permittivity for anisotropic material; 3-by-3 matrix, saved in symmetric format, 6 components

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
sigma	expression	5.99e7	equ	Electrical conductivity for isotropic material
sigmatensor	cell array of expressions	Isotropic conductivity 5.99e7	equ	Electrical conductivity for anisotropic material; 3-by-3 matrix, saved in symmetric format, 6 components
dampingtype	Rayleigh   lossfactor   nodamping   equiviscous   isoloss   anisoloss	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta_s	expression	0	equ	Isotropic structural loss factor
eta_stensor	cell array of expressions	0	equ	Anisotropic structural loss factor; 6-by-6 matrix saved in symmetric format, 21 components
structural_loss	off   iso   aniso	off	equ	Defines the type of the hysteretic structural loss
coupling_loss	off   iso   aniso	off	equ	Defines the type of the hysteretic coupling loss
dielectric_loss	off   iso   aniso	off	equ	Defines the type of the hysteretic dielectric loss
eta_cE	cell array of expressions	0	equ	Anisotropic loss factor for $c_E$ ; 6-by-6 matrix saved in symmetric format, 21 components
eta_cEiso	expression	0	equ	Isotropic loss factor for $c_E$
eta_sE	cell array of expressions	0	equ	Anisotropic loss factor for $s_E$ ; 6-by-6 matrix saved in symmetric format, 21 components
eta_sEiso	expression	0	equ	Isotropic loss factor for $s_E$
eta_e	cell array of expressions	0	equ	Anisotropic loss factor for $e$ ; 3-by-6 matrix
eta_eiso	expression	0	equ	Isotropic loss factor for $e$
eta_d	cell array of expressions	0	equ	Anisotropic loss factor for $d$ ; 3-by-6 matrix
eta_diso	expression	0	equ	Isotropic loss factor for $d$

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
eta_epsT	cell array of expressions	0	equ	Anisotropic loss factor for $\epsilon_T$ ; 3-by-3 matrix saved in symmetric format, 6 components
eta_epsTiso	expression	0	equ	Isotropic loss factor for $\epsilon_T$
eta_epsS	cell array of expressions	0	equ	Anisotropic loss factor for $\epsilon_S$ ; 3-by-3 matrix saved in symmetric format, 6 components
eta_epsSiso	expression	0	equ	Isotropic loss factor for $\epsilon_S$
eta_D	cell array of expressions	0	equ	Anisotropic loss factor for the stiffness matrix $D$ ; 6-by-6 matrix saved in symmetric format, 21 components
eta_Diso	expression	0	equ	Isotropic loss factor for the stiffness matrix $D$
eta_eps	expression	0	equ	Isotropic loss factor for $\epsilon_i \epsilon_0$
eta_epstensor	cell array of expressions	0	equ	Anisotropic loss factor for $\epsilon_i \epsilon_0$ ; 3-by-3 matrix saved in symmetric format, 6 components
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties are defined
materialori	xy   yx   zx   yx   zy   xz	xz	equ	Material orientation. how the 3D material properties is oriented relative the 2D analysis plane (Piezo Plane Stress, Piezo Plane Strain, and Piezo Axial Symmetry)
thickness	expression	1	equ	Thickness of the material (Piezo Plane Stress and Piezo Plane Strain)
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not.
ini_strain	1   0	0	equ	Flag specifying whether initial strain should be included: 1 include strains, 0 do not.
ini_eldisp	1   0	0	equ	Flag specifying whether initial electric displacement should be included: 1 include pressure, 0 do not.
sxi, syi, szi	expression	0	equ	Initial normal stresses

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
sxyi, syzi, sxzi	expression	0	equ	Initial shear stresses
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi, eyzi, exzi	expression	0	equ	Initial shear strains
Di	cell array of expressions	0	equ	Initial electric displacement vector
rhos	expression	0	bnd	Surface charge density
D0	cell array of expressions	0	bnd	Electric displacement
V0	expression	0	bnd	Electric potential
J0	cell array of expressions	0	bnd	Electric current density
Jn	expression	0	bnd	Inward electric current density
electricitytype	V0   cont   D   V   r   nD0   J   nj   nJ0   dnj   fp	nD0 or cont	bnd	The type of electric boundary condition. Available conditions depend on the esform property
	ax			Additional values in Piezo Axial Symmetry
constrcond	free   fixed   roller (bnd only)   axisym (only for bnd in axisymmetry)   displacement   sym (bnd only)   antisym (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition, values common to all application modes
	symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)			Additional values for Piezo Solid
	symyz (bnd only)   symxz (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)			Additional values for Piezo Plane Stress and Piezo Plane Strain
	symrphi (bnd only)   symphiz (bnd only)   antisymrphi (bnd only)   antisymphiz (bnd only)			Type of constraint condition (additional values for Piezo Axial Symmetry)
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fx, Fy, Fz	expression	0	all	Body load, face load, edge load, point load, $x, y, z$ directions (Piezo Solid)
Fx, Fy	expression	0	all	Body load, face load, edge load, point load, $x, y$ directions (Piezo Plane Stress, Piezo Plane Strain)
Fr, Fz	expression	0	all	Body load, face load, edge load, point load, $r, z$ directions (Piezo Axial Symmetry)
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x, y, z$ direction is constrained: 1 constrained, 0 free (Piezo Solid)
Hx, Hy	1   0	0	all	Constraint flag controlling if $x, y$ direction is constrained: 1 constrained, 0 free (Piezo Plane Stress, Piezo Plane Strain)
Hr, Hz	1   0	0	all	Constraint flag controlling if $r, z$ direction is constrained: 1 constrained, 0 free (Piezo Axial Symmetry)
Rx, Ry, Rz	expression	0	all	Constraint value in $x, y, z$ direction (Piezo Solid)
Rx, Ry,	expression	0	all	Constraint value in $x, y, z$ direction (Piezo Plane Stress, Piezo Plane Strain)
Rr, Rz	expression	0	all	Constraint value in $r, z$ direction (Piezo Axial Symmetry)
HVO	1   0	0	edg   pnt	Constraint flag controlling if potential is constrained: 1 constrained, 0 free
V0	expression	0	edg   pnt	Electric potential
Q1	expression	0	edg	Line charge (Piezo Solid)
Q0	expression	0	pnt	Point charge
Q0	expression	0	bnd	Total charge on the floating potential boundary

TABLE 3-16: APPLICATION MODE PARAMETERS FOR THE PIEZOELECTRICAL APPLICATION MODES

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
I0	expression	0	bnd	Total inward current through the floating potential boundary
index	expression	0	bnd	Group index for floating potential



# 4

## Function Reference

This chapter complements the Command Reference chapter of the *COMSOL Multiphysics Reference Guide* by giving detailed reference information about the additional functions for command-line modeling that come with the Structural Mechanics Module.

# Summary of Commands

`shbar` on page 174

`shdrm` on page 176

`sheulb3d` on page 178

`sheulbps` on page 182

# Commands Grouped by Function

## *Shape Function Classes*

FUNCTION	PURPOSE
shbar	Bar element shape function object. Can be used to model bars in 1D, 2D, and 3D.
shdrm	Mindlin plate shape function object. Used to model Mindlin plates in 2D.
sheulb3d	3D Euler beam shape function object. Used to model 3D Euler beams.
sheulbps	2D in plane Euler beam shape function object. Used to model 2D Euler beams.

<b>Purpose</b>	Create an <i>n</i> D bar shape function object.
<b>Syntax</b>	<pre>obj = shbar(dispnames) obj = shbar(dispvarnames,dispdofnames) obj = shbar(dispvarnames,dispdofnames,tangdername) obj = shbar(...)</pre>
<b>Description</b>	<p><code>obj = shbar(dispnames)</code> The bar shape function object is used to implement <i>n</i>D bar elements. The cell array <code>dispnames</code> contains variable names for the displacements and the displacement degrees of freedom.</p> <p><code>obj = shbar(dispvarnames,dispdofnames)</code> The cell arrays <code>dispvarnames</code> and <code>dispdofnames</code> contain the displacement variable names and the displacement degrees of freedom names, respectively.</p> <p><code>obj = shbar(dispvarnames,dispdofnames,tangdername)</code> The variable <code>tangdername</code> additionally contains the variable name of the tangential derivative of the axial displacement.</p> <p><code>obj = shbar(...)</code> Alternate syntax based on property/values. The following property values are allowed:</p>

TABLE 4-1: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>dispnames</code>	cell array of string of length <code>sdim</code>		Default for displacement variable names and degree of freedom variable names
<code>dispvarnames</code>	cell array of string of length <code>sdim</code>	<code>dispnames</code>	Displacement variable names
<code>dispdofnames</code>	cell array of string of length <code>sdim</code>	<code>dispnames</code>	Degree of freedom variable names
<code>tangdername</code>	string	See below	Tangential derivative of the axial displacement

The property names cannot be abbreviated and must be written in lower case.

**Node points:** The end points of the bar element.

**Degrees of freedom names:** The variables in `dispdofnames`.

---

The option to manually define different degrees of freedom names and variables makes it possible to couple a bar to a solid using the degrees of freedom without having the same variables.

**Degrees of freedom:** The global displacements at both endpoints of the bar element.

**Variables:**

- The variable names in the cell array dispvarnames, signifying the displacements.
- [dispvarnames{:} 'ts'] defined on edges, meaning the tangential derivative of the axial displacement. the variable name can also be explicitly given using the property tangdername.

The global space coordinates are expressed as linear (affine) functions in the local coordinates.

**Compatibility**

The FEMLAB 2.3 syntax is obsolete but still supported:

```
obj = shbar3d(uname, vname, wname)
obj = shbar3d(uname, vname, wname, udof, vdof, wdof)
```

<b>Purpose</b>	Create a discrete Reissner-Mindlin triangular plate bending shape function object
<b>Syntax</b>	<pre>obj = shdrm(dispname,rxname,ryname) obj = shdrm(dispname,rxname,ryname,applname) obj = shdrm(...)</pre>
<b>Description</b>	<p><code>obj = shdrm(dispname,rxname,ryname,applname)</code> The DRM shape function object is used to implement discrete Reissner-Mindlin plate elements. The variable <code>dispname</code> denotes the displacement perpendicular to the plate (global <math>z</math> direction). The variables <code>rxname</code> and <code>ryname</code> denote the rotations around the global <math>x</math>-axis and <math>y</math>-axis, respectively.</p> <p><code>obj = shdrm(...)</code> Alternate syntax based on property/values. The following property values are allowed:</p>

TABLE 4-2: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>dispname</code>	string		Displacement
<code>normalnames</code>	cell array of two strings	{ <code>thnx thny</code> }	Names of the components of the vector giving the direction of the tangrotname DOF
<code>rot</code>	<code>axis</code>   <code>gradient</code>	<code>axis</code>	Rotation style
<code>rotnames</code>	cell array of two strings		Variable names of rotations
<code>shearnames</code>	cell array of two strings	{ <code>gxz gyz</code> }	Shear strain components variable names
<code>tangrotname</code>	string	<code>thn</code>	Degree of freedom name

The property names cannot be abbreviated and must be written in lower case.

In the following when the symbols are not provided as property names, let `extraname = ['_' applname]` if `applname` is given, otherwise '' , `rotnames = {rxname, ryname}`, `shearnames = ['g' indepname 'z' extraname]`, and `tangrotname = ['thn' extraname]`.

Depending on the settings of `rot`, rotations can be with respect to the coordinate axes (default) or be like gradient components.

The DRM element is a triangular element of order 1 in displacements, partly of order 2 in rotations (rotations around element 1D edges vary linearly), and partly of order 1 in shears (shear components along element 1D edges are constant).

**Node points:** Node points are second order Lagrange points. The element shape is computed from the corner points only.

**Degrees of freedom names:** `dispname` meaning the displacement perpendicular to the plane of the element. And the variables in `rotnames` are the rotations.

`tangrotnname` at each 1D edge midpoint, meaning rotation around axis perpendicular to side (with a direction convention). This direction can be evaluated using the `normalnames` components.

**Degrees of freedom:** The displacement perpendicular to the plane at each corner point. Rotation in each corner point. Rotation normal to the edge at each 1D edge midpoint.

**Variables:**

- `dispname` meaning the displacement perpendicular to the plane of the element (global  $z$  direction).
- The elements of `rotnames` are the rotations.
- `normalnames`, giving the direction of the `tangrotnname` DOF.
- `[rname indepname]`, where `rname` is a rotation, meaning the first space derivative of rotations defined for `edim==2`.
- `shearnames`, meaning shear strain components, defined for `edim==2`.

<b>Purpose</b>	Create an Euler 3D beam shape function object.
<b>Syntax</b>	<pre>obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname,     point) obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname,     point, u dof, v dof, w dof, thx dof, thy dof, thz dof) obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname) obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname,     u dof, v dof, w dof, thx dof, thy dof, thz dof) obj = sheulb3d(...)</pre>
<b>Description</b>	<p>The Euler 3D beam shape function object is used to implement Euler beam elements in 3D. The beams local coordinate system is defined by the direction of the edge=local <i>x</i>-axis and the coordinates of a point defining the local <i>xy</i>-plane.</p> <p><code>obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname, point)</code> or <code>obj = sheulb3d(uname, vname, wname, thxname, thyname, thzname, point, u dof, v dof, w dof, thx dof, thy dof, thz dof)</code>. The variables <code>uname</code>, <code>vname</code>, and <code>wname</code> denote the components of the displacement of the beam in the global coordinate system. The variables <code>thxname</code>, <code>thyname</code>, and <code>thzname</code> denote the rotation angles around the global coordinate axes. <code>point</code> is a vector with global coordinates for the point that defines the local <i>xy</i>-plane. <code>point</code> can be omitted and then defaults to [1,1,0]. The normal components of the displacements in the local <i>xy</i>- and <i>xz</i>-plane are assumed to vary as Hermitian cubic polynomials of the arclength along the beam. The displacement in the local <i>x</i> direction (edge direction) and rotation about the local <i>x</i>-axis are assumed to vary linearly along the beam.</p> <p><code>obj = sheulb3d(...)</code> Alternate syntax based on property/values. The following property values are allowed:</p>

TABLE 4-3: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>dispnames</code>	cell array of three strings		Displacement variable names and degrees of freedom names default
<code>dispvarnames</code>	cell array of three strings	<code>dispnames</code>	Displacement variable names
<code>dispdofnames</code>	cell array of three strings	<code>dispnames</code>	Displacement degrees of freedom names
<code>localdispvarnames</code>	cell array of three strings	<code>[dispvarnames[i] + '1'], i=1,...,3</code>	Displacement degrees of freedom names

TABLE 4-3: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
rotnames	cell array of three strings		Rotation variable name and degrees of freedom names default
rotvarnames	cell array of three strings	rotnames	Rotation variable names
rotdofnames	cell array of three strings	rotnames	Rotation degrees of freedom names
localrotvarnames	cell array of three strings	[rotvarnames(i) + 'l'], i=1,...,3	Local rotation variable name
tangdisptangdername	string	[dispvarnames(1)+dispvarnames(2)+dispvarnames(3) +'ts']	Name of tangential derivative of the axial displacement
tangrottangdername	string	[rotvarnames(1) + 's']	Name of tangential derivative of rotations
normrottangdernames	cell array of two strings	[rotvarnames(i) + 's'], i=2,3	Name of tangential derivative of the rotation around the local y- and z-axis
normrottangder2names	cell array of two strings	[rotvarnames(i) + 'ss'], i=2,3	Name of second tangential derivative of the rotation around the local y- and z-axis
point	cell array of three strings	{1 1 0}	

The property names cannot be abbreviated and must be written in lower case.

The arguments `uname`, `vname`, and `wname` are equivalent with the property `dispnames` if there are seven inputs, and `dispvarnames` if there are thirteen inputs. The arguments `thxname`, `thyname`, and `thzname` is equivalent with the property `rotnames` if there are seven inputs, and `rotvarnames` if there are thirteen inputs. If there are thirteen inputs, `udof`, `vdof`, and `wdof` are equivalent with the property `vardofnames`, and `thxdof`, `thydof`, and `thzdof` are equivalent with the property `rotdofnames`.

**Node points:** The end points of the beam element.

**Degrees of freedom names:** The strings `uname`, `vname`, `wname`, `thxname`, `thyname`, and `thzname` if there are six or seven inputs, or as specified by the properties `dispvarnames` and `rotvarnames`.

The strings `udof`, `vdof`, `wdof`, `thxdof`, `thydof`, and `thzdof` if there are twelve or thirteen inputs, or as specified by the properties `dispdofnames` and `rotddofnames`.

The option to manually define different degree-of-freedom names and variables makes it possible to couple a beam to a solid using the degrees of freedom without having the same variables.

**Degrees of freedom:** The global displacements and rotations at each endpoint of the beam element.

**Variables:**

- `uname` defined on edges and points, meaning the  $x$  component of the global displacement.
- `vname` defined on edges and points, meaning the  $y$  component of the global displacement.
- `wname` defined on edges and points, meaning the  $z$  component of the global displacement.
- `thxname` defined on edges and points, meaning the rotation around the global  $x$ -axis.
- `thyname` defined on edges and points, meaning the rotation around the global  $y$ -axis.
- `thzname` defined on edges and points, meaning the rotation around the global  $z$ -axis.
- `[uname '1']` defined on edges, meaning the  $x$  component of the local displacement.
- `[vname '1']` defined on edges, meaning the  $y$  component of the local displacement.
- `[wname '1']` defined on edges, meaning the  $z$  component of the local displacement.
- `[thxname '1']` defined on edges, meaning the rotation around the local  $x$ -axis.
- `[thyname '1']` defined on edges, meaning the rotation around the local  $y$ -axis.
- `[thzname '1']` defined on edges, meaning the rotation around the local  $z$ -axis.
- `[uname vname wname 'ts']` or the property `tangdisptangdername` defined on edges, meaning the tangential derivative of the axial displacement.
- `[thxname 's']` or the property `tangrottangdername` defined on edges, meaning the tangential derivative of the rotation around the local  $x$ -axis.

- [thynname 's'] or the first element of the property rottangdername defined on edges, meaning the tangential derivative of the rotation around the local  $y$ -axis.
- [thzname 's'] or the second element of the property rottangdername defined on edges, meaning the tangential derivative of the rotation around the local  $z$ -axis.
- [thynname 'ss'] or the first element of the property normrottangder2names defined on edges, meaning the second tangential derivative of the rotation around the local  $y$ -axis.
- [thzname 'ss'] or the second element of the property normrottangder2names defined on edges, meaning the second tangential derivative of the rotation around the local  $z$ -axis.

The global space coordinates are expressed as linear (affine) functions in the local coordinates.

#### Compatibility

The FEMLAB 2.3 default mechanism for handling the local coordinate system if the point vector for node 1 is close to parallel with the edge has been removed, instead an error is reported.

<b>Purpose</b>	Create a beam shape function object.
<b>Syntax</b>	<pre>obj = sheulbps(uname, vname, rotname) obj = sheulbps(uname, vname, rotname, u dof, v dof, rot dof) obj = sheulbps(...)</pre>
<b>Description</b>	<p><code>obj = sheulbps(uname, vname, rotname)</code> or <code>obj = sheulbps(uname, vname, rotname, u dof, v dof, rot dof)</code>. The beam shape function object is used to implement in-plane Euler beam elements in 2D. The variables <code>uname</code> and <code>vname</code> denote the components of the displacement of the beam in the plane. The variable <code>rotname</code> denotes the rotation angle of the beam. The normal components of the displacements are assumed to vary as Hermitian cubic polynomials in the arclength along the beam. The displacement in the local <math>x</math> direction (edge direction) is assumed to vary linearly along the beam.</p> <p><code>obj = sheulbps(...)</code> Alternate syntax based on property/values. The following property values are allowed:</p>

TABLE 4-4: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>dispnames</code>	cell array of two strings		Displacement variable names and degrees of freedom names default
<code>dispvarnames</code>	cell array of two strings	<code>dispnames</code>	Displacement variable names
<code>dispdofnames</code>	cell array of two strings	<code>dispnames</code>	Displacement degrees of freedom names
<code>rotname</code>	string		Rotation variable name and degrees of freedom name default
<code>rotvarname</code>	string	<code>rotname</code>	Rotation variable name
<code>rotnameofname</code>	string	<code>rotname</code>	Rotation degrees of freedom name
<code>disptangdername</code>	string	[ <code>dispvarnames(1)+dispvarnames(2)+'</code> <code>'ts'</code> ]	Name of tangential derivative of displacement
<code>rottangdername</code>	string	[ <code>rotvarname+'s'</code> ]	Name of tangential derivative of rotation
<code>rottangder2name</code>	string	[ <code>rotvarname+'ss'</code> ]	Name of tangential second derivative of rotation

The property names cannot be abbreviated and must be written in lower case.

The arguments `uname` and `vname` are equivalent with the property `dispnames` if there are three inputs, and `dispvarnames` if there are six inputs. The argument `rotname` is equivalent with the property `rotname` if there are three inputs, and

`rotvarnames` if there are six inputs. If there are six inputs, `udof` and `vdof` are equivalent to the property `vardofnames`, and `rotdof` is equivalent to the property `rotnameofnames`.

**Node points:** The end points of the beam.

**Degrees of freedom names:** The strings `uname`, `vname`, and `rotname` at each endpoint if 3 input arguments.

The strings `udof`, `vdof`, and `rotdof` at each endpoint if 6 input arguments.

The option to manually define different degrees of freedom names and variables makes it possible to couple a beam to a membrane using the degrees of freedom without having the same variables.

**Degrees of freedom:** The global displacement in  $x$  and  $y$  direction and the rotation angle about the global  $z$ -axis.

#### Variables:

- `uname` defined on boundaries and points, meaning the  $x$  component of the displacement.
- `vname` defined on boundaries and points, meaning the  $y$  component of the displacement.
- `rotname` defined on boundaries and points, meaning the rotation angle of the beam.
- `[uname vname 'ts']` or the property `disptangdername` defined on boundaries, meaning the tangential derivative of the axial displacement.
- `[rotname 's']` or the property `rottangdername` defined on boundaries, meaning the tangential derivative of the rotation.
- `[rotname 'ss']` or the property `rottangder2name` defined on boundaries, meaning the second tangential derivative of the rotation.

The global space coordinates are expressed as linear (affine) functions in the local coordinates.



# 5

## Fatigue Function Reference

This chapter describes the fatigue analysis functions included in the Structural Mechanics Module.

# Summary of Commands

`circumcircle` on page 188  
`elastic2plastic` on page 189  
`fatiguedamage` on page 190  
`hcfmultiax` on page 193  
`lcfmultiaxlin` on page 195  
`lcfmultiaxpla` on page 197  
`matlibfatigue` on page 199  
`rainflow` on page 201  
`sn2cycles` on page 202  
`swt2cycles` on page 204

# Commands Grouped by Function

## *Fatigue Analysis Main Functions*

FUNCTION	PURPOSE
<code>fatiguedamage</code>	High cycle fatigue analysis with proportional loading and non-constant loading using Palmgren-Miner accumulated damage rule together with rainflow count
<code>hcfmultiax</code>	High cycle fatigue analysis with non-proportional loading using critical plane method
<code>lcfmultiaxlin</code>	Low cycle fatigue analysis with non-proportional loading based on a linear elastic analysis
<code>lcfmultiaxpla</code>	Low cycle fatigue analysis with non-proportional loading based on a full elasto-plastic analysis

## *Utility Functions*

FUNCTION	PURPOSE
<code>circumcircle</code>	Calculates the minimal enclosing circles of 2D point sets. Used to find maximum shear stress in Findley's method used in <code>hcfmultiax</code> .
<code>rainflow</code>	Performs a rainflow count on a load data resulting in a count matrix. Used to get input data to Palmgren-Miner accumulated damage rule used in <code>fatiguedamage</code> .
<code>sn2cycles</code>	Compute number of cycles to fatigue, given one or two S-N curves where the stress amplitude is given as a function of number of cycles
<code>swt2cycles</code>	Compute number of cycles to fatigue given SWT parameter values and SWT model parameters
<code>elastic2plastic</code>	Compute principal stress and strain from a linear-elastic analysis using a Ramberg-Osgood elasto-plastic material law model
<code>matlibfatigue</code>	Extracts fatigue data from Material Library and creates an S-N function

<b>Purpose</b>	Calculates the minimal enclosing circles of 2D point sets.
<b>Syntax</b>	<code>[x y r] = circumcircle(coord)</code>
<b>Description</b>	<code>[x y r] = circumcircle(coord)</code> returns n-vectors of <i>x</i> -coordinates, <i>y</i> -coordinates, and radii given the coordinate matrix <code>coord</code> with dimensions 2-by-n-by-m. Each of the n cases consists of m points; <code>coord(1, :, :)</code> and <code>coord(2, :, :)</code> contain <i>x</i> - and <i>y</i> -coordinates, respectively.
<b>Examples</b>	<pre>coord = rand(2, 1, 10); [x y r] = circumcircle(coord); clf plot(coord(1, :), coord(2, :), '*') hold on v = 2*pi*0.01*(0:100); plot(x+r*cos(v), y+r*sin(v)) axis equal</pre>
<b>See Also</b>	<code>hcfmultiax</code>

**Purpose** Compute principal stress and strain from a linear-elastic analysis using a Ramberg-Osgood elasto-plastic material law model.

**Syntax** `[spric1, epric1] = elastic2plastic(spric, 'params', para)`

**Description** `[spric1, epric1] = elastic2plastic(spric, 'params', para)`

Calculates the major principal strain `epric1` and corresponding principal stress `spric1` from principal stresses `spric` from a linear-elastic analysis. Input are the principal stresses `spric` and the Ramberg-Osgood material law parameters given in the struct `para`. `spric1` and `epric1` are vectors of size `nloc`, where `nloc` are the number of locations where we like to compute the strains and stresses. `spric` are the principal stress with size 3-by-`nloc` ordered `s1, s2, s3`. The principal stresses from the linear-elastic analysis are transformed to plastic stresses and strains using an approximative method assuming a Ramberg-Osgood material law and was developed by Hoffman-Seeger.

`elastic2plastic` accepts the following property/value pairs:

TABLE 5-1: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>elplmethod</code>	string	'hoffman_seeger'	Method to use for calculating the plastic strains from linear elastic stresses
<code>params</code>	struct containing the Ramberg-Osgood parameters, <code>E</code> , <code>K</code> , <code>nu</code> , and <code>n</code> stored as double in <code>para.E</code> , <code>para.K</code> , <code>para.nu</code> , and <code>para.n</code> .		Ramberg-Osgood material law parameters

**Examples** Used in `lcfmultiaxlin`.

**See Also** `lcfmultiaxlin`

## **fatiguedamage**

---

<b>Purpose</b>	Compute fatigue usage factor for proportional loading with non-constant amplitude using Palmgren-Miner accumulation.
<b>Syntax</b>	<pre>damtot = fatiguedamage('amprange', amprange, ...) [damtot, damdistr] = fatiguedamage('amprange', amprange, ...)</pre>
<b>Description</b>	<pre>damtot = fatiguedamage('amprange', amprange, 'meanrange',...     meanrange, 'count', count, 'fatiguelim', fatiguelim, ...     'sncurve', {'SN_func'})</pre> <p>Calculates the total accumulated damage <code>damtot</code> from the loading history specified in <code>count</code>, <code>amprange</code>, and <code>meanrange</code>, and the material fatigue data specified through <code>fatiguelim</code> and the S-N function '<code>SN_func</code>'.</p> <pre>[damtot, damdistr] = fatiguedamage('amprange', amprange, ...     'meanrange', meanrange, 'count', count, 'fatiguelim', ...     fatiguelim, 'sncurve', {'sn_func'})</pre> <p>Calculates also the damage distribution matrix <code>damdistr</code> of the same size as the <code>count</code> matrix.</p>
	The minimum compulsory input to <code>fatiguedamage</code> are the following property/value pairs:

TABLE 5-2: MINIMUM COMPULSORY PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DESCRIPTION
<code>amprange</code>	double array [minstress maxstress]	The range of the stress amplitude
<code>meanrange</code>	double array [minstress maxstress]	The range of the mean stress
<code>count</code>	double array namp-by-nmean	Array with stress values counted in bins of equal size namp-by-nmean with the stress ranges specified through <code>amprange</code> and <code>meanrange</code>

TABLE 5-2: MINIMUM COMPULSORY PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DESCRIPTION
fatiguelim	double array of the same size as the number of S-N curves	Fatigue limit (below which no number of damage occurs) for the corresponding S-N curve
sncurve	cell array of strings	Names of functions giving stress amplitude as function of cycles to cracking for a specific R-value

The property names cannot be abbreviated.

In addition to the compulsory property pairs given above `fatiguedamage` accepts the following optional property/value pairs:

TABLE 5-3: OPTIONAL PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
nrepeat	double	1	Number of occurrences of the count matrix during total life
rvalue	double array of the same size as the number of S-N curves	[ -1 ]	R-values for the different S-N curves. $R = \frac{\text{mean} - \text{ampl}}{\text{mean} + \text{ampl}}$ R=-1 corresponds to alternating loading R=0 corresponds to pulse loading
method	'none'   'goodman'   'gerber'	'none'	Name of mean correction method only used if a single sn-curve is given
params	struct containing method parameters. For goodman and gerber para.ultstress		Method specific parameters. Ultimate stress for the goodman and gerber method.

The property names cannot be abbreviated.

**Examples**

See the script `frame_with_cutout_fatigue.m` for the model “Frame with Cutout” on page 245 in the *Structural Mechanics Module Model Library*.

**See Also**

`hcfmultiax`, `rainflow`, `lcfmultiaxpla`, `lcfmultiaxlin`, `sn2cycles`

**Purpose** Compute fatigue usage factor for high cycle multi axial fatigue with non-proportional loading.

**Syntax**

```
fus = hcfmultiax(stress, 'params', para)
[fus, sigmamax, deltatau] = hcfmultiax(stress, 'params', para)
[fus, sigmamax, deltatau, maxind, stresshis] =
    hcfmultiax(stress,'params', para)
```

**Description** `fus = hcfmultiax(stress, 'params', para)` calculates the fatigue usage factor `fus` for the critical plane from the stress tensor `stress` and the fatigue model parameters given in the structure `para`. `fus` is a vector of size `nloc`, where `nloc` is the number of locations for the computation of the fatigue usage factor. `stress` is the stress tensor with size 6-by-`nloc`-by-`nb1c`, ordered `xx,yy,zz,xy,yz,xz`, where `nb1c` is the number of basic load cases.

`[fus, sigmamax, deltatau] = hcfmultiax(stress, 'params', para)` also calculates the maximum normal stress `sigmamax` and the shear stress amplitude `deltatau` on the critical plane.

`[fus, sigmamax, deltatau, maxind, stresshis] = hcfmultiax(stress, 'params', para)` also calculates the index `maxind` to the location with the highest fatigue usage factor together with the stress history `stresshis` at this location.

`hcfmultiax` accepts the following property/value pairs:

TABLE 5-4: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
method	string ' <code>findley</code> '	' <code>findley</code> '	Method to use for fatigue usage factor calculation
params	struct with fields containing method parameters, for <code>findley</code> and <code>normal_stress</code> (see table below)		Method specific parameters
lccomb	matrix of size <code>nlcc</code> -by- <code>nb1c</code> where <code>nlcc</code> is the number of load case combinations	{ <code>eye(nb1c, nb1c)</code> }	Matrix containing the load case combinations of the basic load cases, used to obtain stresses from the basic load cases for the fatigue calculations. <code>lccomb(i, j) =</code> weight factor for basic load case <code>j</code> in load case combination <code>i</code> .

TABLE 5-4: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
anglestep	double	{10}	Step size in degrees used in search for critical planes
opt	{'quick'}   'full'	'quick'	Method used in calculating the shear stress range

The property names cannot be abbreviated.

TABLE 5-5: FIELDS IN THE STRUCT FOR THE PROPERTY PARAMS

FIELD	VALUE	EQUATION NAME	DESCRIPTION
f	double	$f$	Material parameter for the Findley fatigue criteria
k	double	$k$	Material parameter for the Findley fatigue criteria
fatlim	double		Fatigue stress limit used in the normal stress fatigue criteria
ultstress	double		Ultimate stress used in the normal stress fatigue criteria
yield	double		Yield stress used in the normal stress fatigue criteria

**Examples**

See the script `shaft_with_fillet.m` for the model “Shaft with Fillet” on page 234 of the *Structural Mechanics Module Model Library*.

**See Also**

`fatiguedamage`, `circumcircle`, `sn2cycles`, `rainflow`, `lcfmultiaxpla`, `lcfmultiaxlin`

**Purpose** Compute number of cycles to fatigue for low cycle multiaxial fatigue with non-proportional loading based on a linear elastic analysis.

**Syntax** `ncycle = lcfmultiaxlin(spric, 'params', para)`

**Description** `ncycle = lcfmultiaxlin(spric, 'params', para)`

Calculates the number of cycles (`ncycle`) to fatigue. Input are the principal stresses `spric` from a linear elastic analysis and the Ramberg-Osgood and fatigue model parameters given in the struct `para`. `ncycle` is a vector of size `nloc`, where `nloc` are the number of locations where we like to compute the number of cycles to fatigue. `spric` are the principal stress with size 3-by-`nloc`, ordered `s1, s2, s3`. The principal stresses from the linear-elastic analysis are transformed to plastic stresses and strains using an approximative method assuming a Ramberg-Osgood material law and was developed by Hoffman-Seeger.

`lcfmultiaxlin` accepts the following property/value pairs:

TABLE 5-6: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>method</code>	string	'swt'	Method to use for calculating the number of cycles to fatigue
<code>elplmethod</code>	string	'hoffman_seeger'	Method to use for calculating the plastic strains from linear elastic stresses
<code>params</code>	struct with fields containing method parameters, for swt and Ramberg-Osgood material law parameters (see table below)		Method specific parameters

The property names cannot be abbreviated.

TABLE 5-7: FIELDS IN THE STRUCT FOR THE PROPERTY PARAMS

FIELD	VALUE	EQUATION NAME	DESCRIPTION
<code>epsf</code>	double	$\epsilon_f'$	Fatigue ductility coefficient
<code>c</code>	double	$c$	Fatigue ductility exponent

TABLE 5-7: FIELDS IN THE STRUCT FOR THE PROPERTY PARAMS

FIELD	VALUE	EQUATION NAME	DESCRIPTION
sigmaf	double	$\sigma_f'$	Fatigue strength coefficient
b	double	$b$	Fatigue strength exponent
E	double	$E$	Young's modulus
K	double	$K$	Ramberg-Osgood material law parameter
n	double	$n$	Ramberg-Osgood material law parameter

**Examples**

See the script `cylinder_hole_linear_fatigue.m` for the model “Cylinder with Hole” on page 261 of the *Structural Mechanics Module Model Library*.

**See Also**

`hcfmultiax`, `fatiguedamage`, `swt2cycles`, `lcfmultiaxpla`

**Purpose** Compute number of cycles to fatigue for low cycle multiaxial fatigue with non-proportional loading from an elasto-plastic analysis.

**Syntax**

```
ncycle = lcfmultiaxpla(stress, strain, 'params', para)
[ncycle, sigmamax, maxdeltaeps, swt] = lcfmultiaxpla(stress, ...
    strain, 'params', para)
```

**Description**

```
ncycle = lcfmultiaxpla(stress, strain, 'params', para)
```

Calculates the number of cycles (`ncycle`) to fatigue for the critical plane. Input are the stress tensor `stress` and the strain tensor `strain` from an elasto-plastic analysis and the fatigue model parameters given in the struct `para`. `ncycle` is a vector of size `nloc`, where `nloc` is the number of locations where you want to compute the number of cycles to fatigue. `stress` and `strain` are the stress and strain tensors with size 6-by-`nloc`-by-`np`, ordered *xx,yy,zz,xy,yz,xz*, where `np` is the number of saved load steps in the elasto-plastic analysis.

```
[ncycle, sigmamax, maxdeltaeps, swt] = lcfmultiaxpla(stress, ...
    strain, 'params', para)
```

Calculates also the maximum normal stress `sigmamax`, the maximum strain amplitude or strain amplitude maximizing the stress times strain product `maxdeltaeps` depending of the selected critical plane method, and the SWT (Smith-Watson-Topper) parameter.

`lcfmultiaxpla` accepts the following property/value pairs:

TABLE 5-8: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
<code>method</code>	string	'swt'	Method to use for calculating the number of cycles to fatigue
<code>critplane</code>	'strain'   'damage'	'strain'	Option determining what method to use when calculating the critical plane to determine the damage parameter: <code>strain</code> for plane of maximum strain range; <code>damage</code> for plane with maximum damage parameter

TABLE 5-8: VALID PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
params	struct with fields containing method parameters, for swt (see table below)		Method specific parameters
anglestep	double	{10}	Step size in search for critical planes, degrees

The property names cannot be abbreviated.

TABLE 5-9: FIELDS IN THE STRUCT FOR THE PROPERTY PARAMS

FIELD	VALUE	EQUATION NAME	DESCRIPTION
epsf	double	$\varepsilon_f^{'}$	Fatigue ductility coefficient
c	double	$c$	Fatigue ductility exponent
sigmaf	double	$\sigma_f^{'}$	Fatigue strength coefficient
b	double	$b$	Fatigue strength exponent
E	double	$E$	Youngs modulus

**Examples**

See script `cylinder_hole_plastic_fatigue.m` for the model “Cylinder with Hole” on page 261 of the *Structural Mechanics Module Model Library*.

**See Also**

`hcfmultiax`, `fatiguedamage`, `swt2cycles`, `lcfmultiaxlin`

---

<b>Purpose</b>	Extracts fatigue data from the Material Library and returns it in form of an S-N function to be used in <code>fatiguedamage</code> for instance.
<b>Syntax</b>	<pre>found = matlibfatigue('material', matname,'phase', phase, 'ori',     ori, 'funcname', funcname, 'rvalue', r)</pre>
<b>Description</b>	<pre>matlibfatigue('material', matname, 'phase', phase...     'ori', ori, 'function', funcname, 'rvalue', r)</pre> <p>Locates S-N curves in Material Library for the material <code>matname</code> with the phase <code>phase</code> and orientation <code>ori</code>. Returns true if an S-N curve was found and false if it couldn't be found. The function itself is written to the file <code>funcname</code>. The S-N curve is transformed (In Material Library the S-N curves are stored as maximum stress as a function of number of cycles to fatigue.) to return stress amplitude as a function of number of cycles to fatigue. To do this an R value need to specified. The S-N values outside the definition interval are extrapolated to have the same value as the value at the end of the interval.</p>

---

**Note:** You need COMSOL Multiphysics Material Library in order to extract fatigue data using this function. Smoothing is not supported through the script interface.

The name of the material, the phase/condition, and the orientation/condition need to be specified exactly as they are spelled out in the COMSOL Multiphysics Material Library. The easiest way to find the names is to use COMSOL Multiphysics and open the **Materials/Coefficients Library** dialog box from the options menu.

The minimum compulsory input to `matlibfatigue` are the following property/value pairs:

TABLE 5-10: MINIMUM COMPULSORY PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DESCRIPTION
<code>material</code>	string	Name of the material
<code>funcname</code>	string	Name of file/function where the S-N function will be defined
<code>phase</code>	string	Phase/condition for the material
<code>ori</code>	string	Orientation/condition for the material with the specified phase/condition

The property names cannot be abbreviated.

In addition to the compulsory property pairs given above `matlibfatigue` accepts the following optional property/value pair:

TABLE 5-II: OPTIONAL PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
rvalue	double	-1	R-value for the S-N-curve $R = \frac{\text{mean} - \text{ampl}}{\text{mean} + \text{ampl}}$ R=-1 corresponds to alternating loading R=0 corresponds to pulse loading

The property names cannot be abbreviated.

**Examples**

See the script `extractSNCurvesFromMatlib.m` used in the model “Frame with Cutout” on page 245 of the *Structural Mechanics Module Model Library*.

**See Also**

`fatiguedamage`

**Purpose** Rainflow counting of load data

**Syntax** [rangerange meanrange count] = rainflow(load, nrange, nmean)

**Description** [rangerange meanrange count] = rainflow(load, nrange, nmean) performs rainflow counting of the loadvector load. Ranges rangerange and meanrange for ranges and mean values of load cycles are returned, as well as a count matrix with discretized occurrence counts for (range, mean) pairs. count has size nrange-by-nmean.

**Examples**

```
t = 1:1e3;
r = rand(2, length(t));
load = t.^0.25.* (1+1*r(1, :))+(1+t.^0.2.* (1+3*r(2, :))).*...
mod(t, 2);
nrange = 20;
nmean = 15;
[rr mr c] = rainflow(load, nrange, nmean);
r = rr(1)+sort([(0:(nrange-1))+0.01 (1:nrange)-0.01])*...
diff(rr)/nrange;
m = mr(1)+sort([(0:(nmean-1))+0.01 (1:nmean)-0.01])*...
diff(mr)/nmean;
[R M] = meshgrid(m, r);
surf(R, M, c(floor(0.5*(2:(2*nrange+1))),
floor(0.5*(2:(2*nmean+1)))));
title('Cycle count as a function of (range, mean)')
```

**See Also** fatiguedamage

**Purpose** Compute number of cycles to fatigue, given one or two SN-curves where the stress amplitude is given as a function of the number of cycles.

**Syntax**

```
n = sn2cycles('amp', amp, 'mean', mean, ...
    'fatiguelim', fatiguelim, 'sncurve', {'sn_func'})
```

**Description**

```
n = sn2cycles('amp', amp, 'mean', mean, ...
    'fatiguelim', fatiguelim, 'sncurve', {'SN_func'})
```

Calculates the number of cycles to fatigue from the stress amplitude `amp`, the mean stress `mean`, and the material fatigue data specified through `fatiguelim` and the SN-function '`SN_func`'.

The minimum compulsory input to `sn2cycles` are the following property/value pairs:

TABLE 5-12: MINIMUM COMPULSORY PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DESCRIPTION
<code>amp</code>	double	Stress amplitude
<code>mean</code>	double	Mean stress
<code>fatiguelim</code>	double array of the same size as the number of S-N curves	Fatigue limit (below which no damage occurs) for the corresponding S-N curve
<code>sncurve</code>	cell array of strings	Names of functions giving stress amplitude as function of cycles to cracking for a specific r-value. If two or more curves are specified they must be given in ascending r-value order

In addition to the compulsory property pairs given above `sn2cycles` accepts the following optional property/value pairs:

TABLE 5-13: OPTIONAL PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DEFAULT	DESCRIPTION
rvalue	double array of the same size as the number of S-N curves	[ -1 ]	R-values for the different S-N curves. $R = \frac{\text{mean} - \text{ampl}}{\text{mean} + \text{ampl}}$ R=-1 corresponding to alternating loading R=0 corresponding to pulse loading
method	'none'   'goodman'   'gerber'	'none'	Name of mean correction method only used if a single S-N curve is given
params	struct containing method parameters. For goodman and gerber para.ultstress		Method specific parameters. Ultimate stress for the goodman and gerber method

The property names cannot be abbreviated.

**Examples**

Used in `fatiguedamage.m`.

**See Also**

`fatiguedamage`

**Purpose** Compute number of cycles to fatigue given SWT parameter values and SWT model parameters.

**Syntax**

```
ncycles = swt2cycles(swtvalues, 'params', para)
```

**Description**

```
ncycles = swt2cycles(swtvalues, 'PARAMS', PARA)
```

Calculates the number of cycles (`ncycles`) to fatigue. Inputs are the SWT parameter values `swtvalues`, a vector of size `nloc` and the SWT fatigue model parameters given in the struct `para`. `ncycles` has the same size (`nloc`) as `swtvalues`, that is, the number of locations where you want to compute the number of cycles to fatigue.

`swt2cycles` accepts the following property/value pairs:

TABLE 5-14: PROPERTY/VALUE PAIRS

PROPERTY	VALUE	DESCRIPTION
params	struct with fields containing swt parameters (see table below)	SWT model parameters

The property names cannot be abbreviated.

TABLE 5-15: FIELDS IN THE STRUCT FOR THE PROPERTY PARAMS

FIELD	VALUE	EQUATION NAME	DESCRIPTION
epsf	double	$\varepsilon_f'$	Fatigue ductility coefficient
c	double	$c$	Fatigue ductility exponent
sigmaf	double	$\sigma_f'$	Fatigue strength coefficient
b	double	$b$	Fatigue strength exponent
E	double	$E$	Young's modulus

**Examples** Used in `lcfmultiaxlin.m` and `lcfmultiaxlin.m`.

**See Also** `lcfmultiaxlin`, `sn2cycles`





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