

# MEMS MODULE

REFERENCE  
GUIDE

**VERSION 3.4**

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# Introduction

The MEMS Module 3.4 is an optional package that extends the COMSOL Multiphysics modeling environment with customized user interfaces and functionality optimized for MEMS modeling.

The documentation set for the MEMS Module consists of the *MEMS Module User's Guide*, the *MEMS Module Model Library*, and this *MEMS Module Reference Guide*. All books are available in PDF and HTML versions from the COMSOL Help Desk. This book contains reference information about the application modes in the MEMS Module.

We hope the MEMS Module becomes a valuable tool in your modeling work, and we are convinced that the effort you put into understanding COMSOL Multiphysics will be repaid several times over. If you have any feedback on the models in this set, please let us know. Likewise, if you have any ideas for additional models that we could add to the library, we welcome your suggestions. Finally, if in your work you have developed a model you think would be a good candidate for inclusion in this model set, please let us hear about it. In any case, feel free to contact us at [info@comsol.com](mailto:info@comsol.com).

## *Typographical Conventions*

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All COMSOL manuals use a set of consistent typographical conventions that should make it easy for you to follow the discussion, realize what you can expect to see on the screen, and know which data you must enter into various data-entry fields. In particular, you should be aware of these conventions:

- A **boldface** font of the shown size and style indicates that the given word(s) appear exactly that way on the COMSOL graphical user interface (for toolbar buttons in the corresponding tooltip). For instance, we often refer to the **Model Navigator**, which is the window that appears when you start a new modeling session in COMSOL; the corresponding window on the screen has the title **Model Navigator**. As another example, the instructions might say to click the **Multiphysics** button, and the boldface font indicates that you can expect to see a button with that exact label on the COMSOL user interface.
- The names of other items on the graphical user interface that do not have direct labels contain a leading uppercase letter. For instance, we often refer to the Draw toolbar; this vertical bar containing many icons appears on the left side of the user interface during geometry modeling. However, nowhere on the screen will you see the term “Draw” referring to this toolbar (if it were on the screen, we would print it in this manual as the **Draw** menu).
- The symbol > indicates a menu item or an item in a folder in the **Model Navigator**. For example, **Physics>Equation System>Subdomain Settings** is equivalent to: On the **Physics** menu, point to **Equation System** and then click **Subdomain Settings**. **COMSOL Multiphysics>Heat Transfer>Conduction** means: Open the **COMSOL Multiphysics** folder, open the **Heat Transfer** folder, and select **Conduction**.
- A `Code` (monospace) font indicates keyboard entries in the user interface. You might see an instruction such as “Type 1.25 in the **Current density** edit field.” The monospace font also indicates COMSOL Script codes.
- An *italic* font indicates the introduction of important terminology. Expect to find an explanation in the same paragraph or in the Glossary. The names of books in the COMSOL documentation set also appear using an italic font.

# Application Modes Variables

This chapter provides listings of the application mode variables that you have access to in the MEMS Module's application modes.

# Variables in the Application Modes

A large number of variables are available for use in expressions and postprocessing. This chapter lists the variables defined in each application mode. In addition to the variables listed herein, you have always access to variable related to the geometry and the mesh, for example.

The application mode variable tables are organized as follows:

- The **Name** column lists the names of the variables that you can use in the equations or for postprocessing. Almost all variables, such as stresses and strains, are also available as the amplitude and phase of those variables by appending `_amp` or `_ph` to the variable name. Exceptions are variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`. A single index  $i$  on the displacement,  $u_i$ , means that  $u_i$  runs over the available global displacements, for example  $(u, v, w)$  in 3D. A single index on other names, for example  $s_i$ , means that  $i$  runs over the global space variables  $(x, y, z)$ . A double index  $s_{ij}$  means that  $ij$  runs over the combination of the space variables  $(xy, yz, xz)$ . Exceptions to these conventions are noted in the tables. For example,  $s_i$  means the principle stresses when  $i$  runs over  $(1, 2, 3)$ . For elasto-plastic materials the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined: the normally defined variable and the Gauss-point evaluated variable. Notationally, the latter are distinguished by an added suffix `Gp` to the variable name, for example, `sxGp` instead of `sx`. It is only possible to use the Gauss-point evaluated variables for postprocessing.
- The **Symbol** column lists the symbol notation for each variable.
- In the **Analysis** column you can see the availability of variables for the different analysis types. The following abbreviations are used:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Parametric	P
Time dependent	T
Eigenfrequency	E

- The **Domain** column lists whether variables are available on subdomains (S), boundaries (B), edges (E), points (P), or all domains (All).



- In the **Description** column you can find a short description for each variable.
- Where applicable, the **Expression** column lists the expression used for determining each variable.

# Structural Mechanics Application Modes

## *Solid, Stress-Strain*

A large number of variables are available for use in expressions and postprocessing. In addition to the variables in Table 2-1, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

Table 2-1 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ).

For elasto-plastic materials the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined: the normally defined variable and the Gauss-point evaluated variable. Notationally, the latter are distinguished by an added suffix `Gp` to the variable name, for example, `sxGp` instead of `sx`. It is only possible to use the Gauss-point evaluated variables for postprocessing.

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{itph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{itph} + 180^\circ, 360^\circ)$
$p$	$p$	All	All	Pressure	$p$
$p\_amp$	$p_{amp}$	F	All	Pressure amplitude	$ p $
$p\_ph$	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
$disp$	$disp$	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
$e_i, e_{ij}$	$\varepsilon_i, \varepsilon_{ij}$	All	S	Strain, global coord. system	Engineering or Green strain depending if small or large deformation.
$ep_i, ep_{ij}$	$\varepsilon_{pi}, \varepsilon_{pij}$	S T	S	Plastic strain, global coord. system	
$epe$	$\varepsilon_{pe}$	S T	S	Effective plastic strain	
$e_{il}, e_{ijl}$	$\varepsilon_{il}, \varepsilon_{ijl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon T_{\text{coord}}$
$e_{i\_t}, e_{ij\_t}$	$\varepsilon_{it}, \varepsilon_{ijt}$	F T	S	Velocity strain, global coord system	Engineering or Green strain time derivative depending if small or large deformation
$e_{i\_tl}, e_{ij\_tl}$	$\varepsilon_{itl}, \varepsilon_{ijt l}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
$s_i, s_{ij}$	$\sigma_i, \tau_{ij}$	All	S	Cauchy stress, global coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$s_{il}, s_{ijl}$	$\sigma_i, \tau_{ij}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{i\_t}, s_{ij\_t}$	$\sigma_{it}, \tau_{ijt}$	F T	S	Time derivative of Cauchy stress, global coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$sil\_t, sijl\_t$	$\sigma_{ilt}, \tau_{ijlt}$	F T	S	Time derivative of Cauchy stress, user-defined local coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$S_i, S_{ij}$	$S_i, S_{ij}$	All	S	Second Piola Kirchhoff stress, global coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$S_{il}, S_{ijl}$	$S_{il}, S_{ijl}$	All	S	Second Piola Kirchhoff stress, user-defined local coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$S_{i\_t}, S_{ij\_t}$	$S_{it}, S_{ijt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$S_{il\_t}$	$S_{ilt}, S_{ijlt}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined local coord. system	Defined differently depending of coordinate system, material model, and if mixed or displacement formulation, and if loss factor damping is used
$P_i, P_{ij}$	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
$e_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$e_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
evol	$\varepsilon_{vol}$	All	All	volumetric strain	Defined differently for small and large deformations
$F_{ij}$	$F_{ij},$ $i,j=1,2,3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$c_{ij}$	$c_{ij},$ $i,j=1,2,3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
invFij	invF <sub>ij</sub> , $i,j=1,2,3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
detF	detF	All	All	Determinant of deformation gradient	detF
J	J	All	All	Volume ratio	detF
Je1	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
I1	$I_1$	All	All	First strain invariant	trace( $C^2$ ) = $C_{11}^2 + C_{22}^2 + C_{33}^2$
I2	$I_2$	All	All	Second strain invariant	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
I3	$I_3$	All	All	Third strain invariant	$J_{el}^2$
II1	$\bar{I}_1$	All	All	First modified strain invariant	$I_1 J_{el}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	All	Second modified strain invariant	$I_2 J_{el}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending of material model and mixed or displacement formulation
Ent	$S_{elast}$	All	All	Entropy per unit volume	Defined only for small deformations and either no damping or loss factor damping. See definition in theory section
Qdamp	$Q_d$	F	All	Heat associated with mechanical losses in material	Defined only for loss factor damping $0.5\omega\eta \text{Real}(\varepsilon \cdot \text{Conj}(D\varepsilon))$

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in x <sub>i</sub> -direction	Defined differently depending of large or small deformation
F <sub>ig</sub>	F <sub>ig</sub>	All	All	Body load, face load, edge load, point load, in global x <sub>i</sub> -direction	Defined differently depending of force definition
F <sub>tij</sub>	F <sub>tij</sub>	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i T x_j$
wcn_cpi	w <sub>cn</sub>	S P	B	Contact help variable for contact pair <i>i</i>	$\text{nojac}(T_{np}) - T_n$
wctxj_cpi	w <sub>ctj</sub>	P	B	Contact help variable for contact pair <i>i</i>	See definition in theory section
slipxj_cpi	slip <sub>xj</sub>	P	B	Slip vector x <sub>j</sub> dir. reference frame, contact pair <i>i</i>	$\text{map}(x_j) + x_j^m \text{old}$
slip_cpi	slip	P	B	Slip vector magnitude reference frame, contact pair <i>i</i>	$\text{sqrt} \sum_j (\text{slip}_{xj}^2)$
slipdxrj_cpi	slip <sub>xrj</sub>	P	B	Slip vector x <sub>rj</sub> dir. deformed frame, contact pair <i>i</i>	$\sum_k \text{map}(F_{tij}) \text{slip}_{xj}$
slipd_cpi	slipd	P	B	Slip vector magnitude deformed frame, contact pair <i>i</i>	$\text{sqrt} \left( \sum_j (\text{slipd}_{xrj}^2) \right)$
Tnp_cpi	T <sub>np</sub>	S P	B	Penalized contact pressure, contact pair <i>i</i>	See definition in theory section
Ttpxj_cpi	T <sub>tpj</sub>	P	B	Penalized friction traction x <sub>j</sub> dir., contact pair <i>i</i>	See definition in theory section
Tttrialxj_cpi	T <sub>ttrialj</sub>	P	B	Trial friction force x <sub>j</sub> dir., contact pair <i>i</i>	See definition in theory section

TABLE 2-1: SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$vslip_{x_rj\_cpi}$	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair $i$	$\frac{slipd_{x_rj}}{t - t_{old}}$
$vslip_{cpi}$	$v_s$	P	B	Slip velocity magnitude, contact pair $i$	$\sqrt{\sum_j vslip_{x_rj}^2}$
$mu_{cpi}$	$\mu$	S P	B	Frictional coefficient, contact pair $i$	See definition in theory section
$Ttcrit_{cpi}$	$\mu$	P	B	Maximum friction traction, contact pair $i$	See definition in theory section
$gap_{cpi}$	$g$	S P	B	Gap distance including offsets, contact pair $i$	$Geomgap_{cpi} - offset_{cpi} - map(offset_{cpi})$
$contact_{cpi}$	contact	S P	B	In contact variable, contact pair $i$	Defined differently depending on the pair setting
$friction_{cpi}$	friction	S P	B	Enabling friction variable, contact pair $i$	$contact_{cpi\_old}$

### Plane Stress

A large number of variables are available for use in expressions and for postprocessing. In addition to the variables listed in Table 2-2, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added

Gp to the variable name. Example, sxGp instead of sx. The Gauss point evaluated variables can only be used for postprocessing.

Table 2-2 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v$ ).

TABLE 2-2: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
$p$	$p$	All	All	Pressure	$p$
$p\_amp$	$p_{amp}$	F	All	Pressure amplitude	$ p $
$p\_ph$	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
$disp$	$disp$	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
$e_i, e_z, e_{xy}$	$\varepsilon_i, \varepsilon_z, \varepsilon_{xy}$	All	S	Strain global system	Engineering or Green strain depending if small or large deformation. $\varepsilon_z$ defined differently if loss factor damping is used.
$ep_i, ep_z, ep_{xy}$	$\varepsilon_{pi}, \varepsilon_{pz}, \varepsilon_{pxy}$	S T	S	Plastic strain global system	
$epe$	$\varepsilon_{pe}$	S T	S	Effective plastic strain	



TABLE 2-2: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$e_{il}, e_{xyl}$	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$e_{i\_t}, e_{z\_t}, e_{xy\_t}$	$\epsilon_{it}, \epsilon_{zt}, \epsilon_{xyt}$	F T	S	Velocity strain, global coord. system	Defined differently depending of small or large deformation and analysis type
$e_{il\_t}, e_{xyl\_t}$	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
$s_i, s_{xy}$	$\sigma_i, \tau_{xy}$	All	S	Cauchy stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il}, s_{xyl}$	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending of material model, and if loss factor damping is used
$s_{i\_t}, s_{xy\_t}$	$\sigma_{it}, \tau_{xyt}$	F T	S	Time derivative of Cauchy stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il\_t}, s_{xyl\_t}$	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_i, S_{xy}$	$S_i, S_{xy}$	All	S	Second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_{il}, S_{xyl}$	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, and if loss factor damping is used
$S_{i\_t}, S_{xy\_t}$	$S_{it}, S_{xyt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used

TABLE 2-2: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$S_{il\_t}, S_{xy1\_t}$	$S_{it}, S_{xy}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$P_i, P_{ij}$	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
$s_i$	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
$e_i$	$\epsilon_i$	All	S	Principal strains, $i=1,2,3$	
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i, j=1,2,3$	
$e_{ixj}$	$\epsilon_{ixj}$	All	S	Principal strain directions, $i, j=1,2,3$	
evol	$\epsilon_{vol}$	All	All	volumetric strain	Defined differently depending of small or large displacement
$F_{ij}$	$F_{ij}, i,j=1,2,3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$c_{ij}$	$c_{ij}, i,j=1,2,3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
detF	$\det F$	All	All	Determinant of deformation gradient	$\det F$
invFij	$invF_{ij}, i,j=1,2,3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
J	$J$	All	All	Volume ratio	$\det F$
Je1	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
I1	$I_1$	All	All	First strain invariant	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
I2	$I_2$	All	All	Second strain invariant	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$

TABLE 2-2: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
I3	$I_3$	All	All	Third strain invariant	$J_{el}^2$
II1	$\bar{I}_1$	All	All	First modified strain invariant	$I_1 J_{el}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	All	Second modified strain invariant	$I_2 J_{el}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ,  \sigma_1 - \sigma_3 ))$
mises	$\sigma_{mises}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending of material model and if mixed or displacement formulation
Tai	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	Defined differently depending of small or large deformation
Fig	$F_{ig}$	All	S	Point, Edge, Body load, in global $x_i$ direction	Defined differently depending of force definition
Ftij	$F_{tij}$	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i T x_j$
wcn_cpi	$w_{cn}$	S P	B	Contact help variable for contact pair $i$	$nojac(T_{np}) - T_n$
wctxj_cpi	$w_{ctj}$	P	B	Contact help variable for contact pair $i$	See definition in theory section
slipxj_cpi	$slip_{xj}$	P	B	Slip vector $x_j$ dir. reference frame, contact pair $i$	$map(x_j) + x_{jold}^m$
slip_cpi	$slip$	P	B	Slip vector magnitude reference frame, contact pair $i$	$\text{sqrt}\left(\sum_j \text{slip}_{xj}^2\right)$

TABLE 2-2: PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
slipdxrj_cpi	slip <sub>xrj</sub>	P	B	Slip vector xrj dir. deformed frame, contact pair <i>i</i>	$\sum_j \text{map}(F_{tij}) \text{slip}_{xj}$
slipd_cpi	slipd	P	B	Slip vector magnitude deformed frame, contact pair <i>i</i>	$\text{sqrt}\left(\sum_j \text{slipd}_{xrj}^2\right)$
Tnp_cpi	$T_{np}$	S P	B	Penalized contact pressure, contact pair <i>i</i>	See definition in theory section
Ttpxj_cpi	$T_{tpj}$	P	B	Penalized friction force $x_j$ dir., contact pair <i>i</i>	See definition in theory section
Tttrialxj_cpi	$T_{ttrialj}$	P	B	Trial friction force $x_j$ dir., contact pair <i>i</i>	See definition in theory section
vslipxrj_cpi	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair <i>i</i>	$\frac{\text{slipd}_{xrj}}{t - t_{\text{old}}}$
vslip_cpi	$v_s$	P	B	Slip velocity, contact pair <i>i</i>	$\text{sqrt}\left(\sum_j (v\text{slip}_{xrj})^2\right)$
mu_cpi	$\mu$	S P	B	Frictional coefficient, contact pair <i>i</i>	See definition in theory section
Ttcrit_cpi	$\mu$	P	B	Maximum frictional traction, contact pair <i>i</i>	See definition in theory section
gap_cpi	$g$	S P	B	Gap distance including offsets, contact pair <i>i</i>	$\text{Geomgap}_{cpi} - \text{offset}_{cpi} - \text{map}(\text{offset}_{cpi})$
contact_cpi	contact	S P	B	In contact variable, contact pair <i>i</i>	Defined differently depending on the pair setting
friction_cpi	friction	S P	B	Enabling friction variable, contact pair <i>i</i>	contact_cpi_old

## Plane Strain

A large number of variables are available for use in expressions and for postprocessing. In addition to the variables in Table 2-3, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added `Gp` to the variable name. Example, `sxGp` instead of `sx`. It is only possible to use the Gauss point evaluated variables for postprocessing.

Table 2-3 uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x$  and  $y$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v$ ).

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u\_amp, v\_amp$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_i\_ph$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{itph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
p	$p$	All	All	Pressure	$p$
p_amp	$p_{amp}$	F	All	Pressure amplitude	$ p $
p_ph	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
$e_i, e_{xy}$	$\epsilon_i, \epsilon_{xy}$	All	S	Strain, global coord. system	Engineering or Green strain depending if small or large deformation
$ep_i, e_{pxy}$	$\epsilon_{pi}, \epsilon_{pxy}$	S T	S	Plastic strain, global coord. system	
epe	$\epsilon_{pe}$	S T	S	Effective plastic strain, global coord. system	
$e_{il}, e_{xyl}$	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
$e_{i\_t}, e_{xy\_t}$	$\epsilon_{it}, \epsilon_{xyt}$	F T	S	Velocity strain, global coord. system	Defined differently depending of small or large deformation and analysis type
$e_{il\_t}, e_{xyl\_t}$	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
$s_i, s_z, s_{xy}$	$\sigma_i, \sigma_z, \tau_{xy}$	All	S	Cauchy stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il}, s_{xyl}$	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{i\_t}, s_{z\_t}, s_{xy\_t}$	$\sigma_{it}, \sigma_z, \tau_{xyt}$	F T	S	Time derivative of Cauchy stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$s_{il\_t}, s_{xyl\_t}$	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_i, S_z, S_{xy}$	$S_i, S_z, S_{xy}$	All	S	Second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_{il}, S_{xyl}$	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_{i\_t}, S_{z\_t}, S_{xy\_t}$	$S_{it}, S_{zt}, S_{xyt}$	T	S	Time derivative of second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$S_{il\_t}, S_{xyl\_t}$	$S_{ilt}, S_{xylt}$	T	S	Time derivative of second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, mixed or displacement formulation, coordinate system, and if small or large deformation, and if loss factor damping is used
$P_i, P_z, P_{xy}$	$P_i, P_z, P_{xy}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
$s_i$	$\sigma_i$	All	S	Principal stresses, $i = 1, 2, 3$	
$e_i$	$\varepsilon_i$	All	S	Principal strains, $i = 1, 2, 3$	

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
$e_{ixj}$	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
evol	$\varepsilon_{vol}$	All	All	volumetric strain	Defined differently for small and large displacement
$F_{ij}$	$F_{ij}$ , $i,j=1,2,3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \mathbf{X}}$
$c_{ij}$	$c_{ij}$ , $i,j=1,2,3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
detF	$\det F$	All	All	Determinant of deformation gradient	$\det F$
invF $_{ij}$	inv $F_{ij}$ , $i,j=1,2,3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
J	$J$	All	All	Volume ratio	$\det F$
Je1	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
I1	$I_1$	All	All	First strain invariant	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
I2	$I_2$	All	All	Second strain invariant	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
I3	$I_3$	All	All	Third strain invariant	$J_{el}^2$
II1	$\bar{I}_1$	All	All	First modified strain invariant	$I_1 J_{el}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	All	Second modified strain invariant	$I_2 J_{el}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$



TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ,  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending of material model and if mixed or displacement formulation
Ta <sub>i</sub>	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ -direction	Defined differently depending of the force definition
Fig	$F_{ig}$	All	S	Point, Edge, Body load in global $x_i$ -direction	Defined differently depending on the force definition
Ft <sub>ij</sub>	$F_{tij}$	All	B	Deformation gradient projected on the tangent plane	$\delta_{ij} + u_i T x_j$
wcn_cp <sub>i</sub>	$w_{cn}$	SP	B	Contact help variable for contact pair $i$	$\text{nojac}(T_{np}) - T_n$
wctxj_cp <sub>i</sub>	$w_{ctxj}$	P	B	Contact help variable for contact pair $i$	See definition in theory section
slipxj_cp <sub>i</sub>	$\text{slip}_{xj}$	P	B	Slip vector $x_j$ dir. reference frame, contact pair $i$	$\text{map}(x_j) + x_{j\text{old}}^m$
slip_cp <sub>i</sub>	slip	P	B	Slip vector magnitude reference frame, contact pair $i$	$\text{sqrt}\left(\sum_j \text{slip}_{xj}^2\right)$
slipdxrj_cp <sub>i</sub>	$\text{slip}_{xrj}$	P	B	Slip vector $x_{rj}$ dir. deformed frame, contact pair $i$	$\sum_j \text{map}(F_{tij}) \text{slip}_{xj}$

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
slipd_cpi	slipd	P	B	Slip vector magnitude deformed frame, contact pair $i$	$\text{sqrt}\left(\sum_j (\text{slipd}_{x_{rj}}^2)\right)$
Tnp_cpi	$T_{np}$	SP	B	Penalized contact pressure, contact pair $i$	See definition in theory section
Ttpxj_cpi	$T_{tpj}$	P	B	Penalized friction force $x_j$ dir., contact pair $i$	See definition in theory section
Tttrialxj_cpi	$T_{ttrialj}$	P	B	Trial friction force $x_j$ dir., contact pair $i$	See definition in theory section
vslipxrj_cpi	$v_{sxj}$	P	B	Slip velocity vector $x_j$ dir., contact pair $i$	$\frac{\text{slipd}_{x_{rj}}}{t - t_{\text{old}}}$
vslip_cpi	$v_s$	P	B	Slip velocity, contact pair $i$	$\text{sqrt}\left(\sum_j (\text{vslip}_{x_{rj}}^2)\right)$
mu_cpi	$\mu$	SP	B	Frictional coefficient, contact pair $i$	See definition in theory section
Ttcrit_cpi	$\mu$	P	B	Maximum friction traction, contact pair $i$	See definition in theory section
gap_cpi	$g$	SP	B	Gap distance including offsets, contact pair $i$	$\text{Geomgap}_{\text{cpi}} - \text{offset}_{\text{cpi}} - \text{map}(\text{offset}_{\text{cpi}})$

TABLE 2-3: PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
contact_cpi	contact	SP	B	In contact variable, contact pair $i$	Defined differently depending on the pair setting
friction_cpi	friction	SP	B	Enabling friction variable, contact pair $i$	contact_cpi_old

### *Axial Symmetry, Stress-Strain*

A large number of variables are available for use in expressions and postprocessing. In addition to the variables in Table 2-4, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` is the amplitude of the normal stress in the  $x$  direction
- `ex_ph` is the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

For elasto-plastic material the plastic strain, effective strain, effective stress, principal stress, and all stress components have two different variables defined. The normal defined variable and the Gauss point evaluated variable. The different being an added `Gp` to the variable name. Example, `sxGp` instead of `sx`. It is only possible to use the gauss point evaluated variables for postprocessing.

TABLE 2-4: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uor	uor	All	All	$r$ displacement divided by $r$	uor
uaxi	uaxi	All	All	$r$ displacement	uor · $r$
w	$w$	All	All	$z$ displacement	$w$
uort	uor <sub><math>t</math></sub>	T	All	$r$ velocity divided by $r$	uor <sub><math>t</math></sub>

TABLE 2-4: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
uaxi_t	$uaxi_t$	T	All	$r$ velocity	$uor_t \cdot r$
w_t	$w_t$	T	All	$z$ velocity	$w_t$
uaxi_amp	$uaxi_{amp}$	F	All	$r$ displacement amplitude	$ uaxi $
w_amp	$w_{amp}$	F	All	$z$ displacement amplitude	$ w $
uaxi_ph	$uaxi_{ph}$	F	All	$r$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(uaxi), 2\pi)$
w_ph	$w_{ph}$	F	All	$z$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(w), 2\pi)$
uaxi_t	$uaxi_t$	F	All	$r$ velocity	$j\omega uaxi$
w_t	$w_t$	F	All	$z$ velocity	$j\omega w$
uaxi_t_amp	$uaxi_{tamp}$	F	All	$r$ velocity amplitude	$\omega uaxi_{amp}$
w_t_amp	$w_{tamp}$	F	All	$z$ velocity amplitude	$\omega w_{amp}$
uaxi_t_ph	$uaxi_{tph}$	F	All	$r$ velocity phase	$\text{mod}(uaxi_{ph} + 90^\circ, 360^\circ)$
w_t_ph	$w_{tph}$	F	All	$z$ velocity phase	$\text{mod}(w_{ph} + 90^\circ, 360^\circ)$
uaxi_tt	$uaxi_{tt}$	F	All	$r$ acceleration	$-\omega^2 uaxi$
w_tt	$w_{tt}$	F	All	$z$ acceleration	$-\omega^2 w$
uaxi_tt_amp	$uaxi_{ttamp}$	F	All	$r$ acceleration amplitude	$\omega^2 uaxi_{amp}$
w_tt_amp	$w_{ttamp}$	F	All	$z$ acceleration amplitude	$\omega^2 w_{amp}$
uaxi_tt_ph	$uaxi_{ttph}$	F	All	$r$ acceleration phase	$\text{mod}(uaxi_{ph} + 180^\circ, 360^\circ)$
w_tt_ph	$w_{ttph}$	F	All	$z$ acceleration phase	$\text{mod}(w_{ph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{uaxi^2 + w^2}$
p	$p$	All	All	Pressure	$p$
p_amp	$p_{amp}$	F	All	Pressure amplitude	$ p $

TABLE 2-4: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
p_ph	$p_{ph}$	F	All	Pressure phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(p), 2\pi)$
er, ez, ephi, erz	$\epsilon_r, \epsilon_z, \epsilon_\phi,$ $\epsilon_{rz}$	All	S	Strain, global coord. system	Engineering or Green strain depending if small or large deformation
epr, epz, epphi, eprz	$\epsilon_{pr}, \epsilon_{pz},$ $\epsilon_{p\phi}, \epsilon_{prz}$	S T	S	Plastic strain, global coord. system	
epe	$\epsilon_{pe}$	S T	S	Plastic strain, global coord. system	
eil, exyl	$\epsilon_{il}, \epsilon_{xyl}$	All	S	Strains, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
er_t, ez_t, ephi_t, erz_t	$\epsilon_{rt}, \epsilon_{zt},$ $\epsilon_{\phi t}, \epsilon_{rzt}$	F T	S	Velocity strain, global coord. system	Defined differently depending of small or large displacement
eil_t, exyl_t	$\epsilon_{ilt}, \epsilon_{xylt}$	F T	S	Velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
sr, sphi, sz, srz	$\sigma_r, \sigma_\phi,$ $\sigma_z, \tau_{rz}$	All	S	Cauchy stress, global coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
sil, sxyl	$\sigma_{il}, \tau_{xyl}$	All	S	Cauchy stress, user-defined coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
sr_t, sphi_t, sz_t, srz_t	$\sigma_{rt}, \sigma_{\phi t},$ $\sigma_{zt}, \sigma_{rzt}$	F T	S	Time derivative of Cauchy stress, global coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
sil_t, sxyl_t	$\sigma_{ilt}, \tau_{xylt}$	F T	S	Time derivative of Cauchy stress, user-defined coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used

TABLE 2-4: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Sr, Sphi, Sz, Srz	$S_r, S_\phi,$ $S_z, S_{rz}$	All	S	Second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
Sil, Sxyl	$S_{il}, S_{xyl}$	All	S	Second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
Sr_t, Sphi_t, Sz_t, Srz_t	$S_{rt}, S_{\phi t},$ $S_{zt}, S_{rzt}$	T	S	Time der. of second Piola Kirchhoff stress, global coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
Sil_t, Sxyl_t	$S_{ilt}, S_{xylt}$	T	S	Time der. of second Piola Kirchhoff stress, user-defined coord. system	Defined differently depending of material model, coordinate system, mixed or displacement formulation, and small or large displacement, and if loss factor damping is used
Pi, Pij	$P_i, P_{ij}$	All	S	First Piola Kirchhoff stress, global coord. system	Only defined for hyperelastic material. Defined differently if loss factor damping is used
si	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	
ei	$\varepsilon_i$	All	S	Principal strains, $i=1,2,3$	
sixj	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	
eixj	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	
evol	$\varepsilon_{vol}$	All	All	volumetric strain	Defined differently for small and large displacement
Fij	$F_{ij},$ $i, j=1,2,3$	All	All	Deformation gradient	$\frac{\partial \mathbf{x}}{\partial \bar{\mathbf{X}}}$

TABLE 2-4: AXIAL SYMMETRY, STRESS-STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$c_{ij}$	$c_{ij}$ , $i, j = 1, 2, 3$	All	All	Right Cauchy-Green symmetric tensor all components are defined	$F^T F$
detF	$\det F$	All	All	Determinant of deformation gradient	$\det F$
invF $_{ij}$	invF $_{ij}$ , $i, j = 1, 2, 3$	All	All	Inverse of deformation gradient	$F^{-1}$ (calculated symbolically from $F_{ij}$ )
J	$J$	All	All	Volume ratio	$\det F$
Je1	$J_{el}$	All	All	Elastic volume ratio	Defined differently if thermal loads or not
I1	$I_1$	All	All	First strain invariant	$\text{trace}(C^2) = C_{11}^2 + C_{22}^2 + C_{33}^2$
I2	$I_2$	All	All	Second strain invariant	$\frac{1}{2}(I_1^2 - \text{trace}(C^2))$
I3	$I_3$	All	All	Third strain invariant	$J_{el}^2$
II1	$\bar{I}_1$	All	All	First modified strain invariant	$I_1 J_{el}^{-\frac{2}{3}} = I_1 I_3^{-\frac{1}{3}}$
II2	$\bar{I}_2$	All	All	Second modified strain invariant	$I_2 J_{el}^{-\frac{4}{3}} = I_1 I_3^{-\frac{2}{3}}$
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
Ws	$W_s$	All	S	Strain energy density	Defined differently depending on material model and if mixed or displacement formulation
Tar, Taz	$Ta_r, Ta_z$	All	B	Surface traction (force/area) in $r$ and $z$ directions	Defined differently depending on small or large deformation
Frg, Fzg	$F_{rg}, F_{zg}$	All	All	Body, edge, point load in global $r$ and $z$ directions	Defined differently depending on force definition

# Piezoelectric Application Modes

## *Piezo Solid*

A large number of variables are available for use in expressions and for postprocessing purposes. In addition to the variables listed below, almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` represents the amplitude of the normal stress in the  $x$  direction
- `ex_ph` represents the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Time dependent	T

## VARIABLES

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
V	V	All	All	Electric potential	V
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$



TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
V_amp	$V_{amp}$	F	All	Electric potential amplitude	$ V $
V_ph	$V_{ph}$	F	All	Electric potential phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(V), 2\pi)$
u <sub>i</sub> _t	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
u <sub>i</sub> _t_amp	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
u <sub>i</sub> _t_ph	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{itph} + 90^\circ, 360^\circ)$
u <sub>i</sub> _tt	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
u <sub>i</sub> _tt_amp	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
u <sub>i</sub> _tt_ph	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{ittph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
e <sub>i</sub>	$\varepsilon_i$	All	S	$\varepsilon_i$ normal strain global coord. system	$\frac{\partial u_i}{\partial x_i}$
e <sub>ij</sub>	$\varepsilon_{ij}$	All	S	$\varepsilon_{ij}$ shear strain global coord. system	$\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$
E <sub>i</sub>	$E_i$	All	S	Electric field	$-\left( \frac{\partial V}{\partial x_i} \right)$
normE	$E_i$	All	S	Electric field	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
e <sub>i1</sub>	$\varepsilon_{i1}$	All	S	$\varepsilon_{i1}$ normal strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon T_{\text{coord}}$
e <sub>ij1</sub>	$\varepsilon_{ij1}$	All	S	$\varepsilon_{ij1}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon T_{\text{coord}}$

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$E_{il}$	$E_{il}$	All	S	Electric field, user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}$
$V_{il}$	$V_{il}$	All	S	Electric potential gradient, user-defined coord. system	$T_{\text{coord}}^T \nabla V$
$e_{i\_t}$	$\varepsilon_{it}$	T	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{it}}{\partial x_i}$
$e_{i\_t}$	$\varepsilon_{it}$	F	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{i\_j\omega}}{\partial x_i}$
$e_{ij\_t}$	$\varepsilon_{ijt}$	T	S	$\varepsilon_{ijt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u_{it}}{\partial x_j} + \frac{\partial u_{jt}}{\partial x_i} \right)$
$e_{ij\_t}$	$\varepsilon_{ijt}$	F	S	$\varepsilon_{ijt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) j\omega$
$e_{il\_t}$	$\varepsilon_{ilt}$	F T	S	$\varepsilon_{ilt}$ normal velocity strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
$e_{ijl\_t}$	$\varepsilon_{ijlt}$	F T	S	$\varepsilon_{ijlt}$ shear velocity strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
cE	$c_E$	All	S	Stiffness matrix components	$s_E^{-1}$ , if material is specified on strain-charge form, calculated by a special inverting-matrices element.

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
e	$e$	All	S	Piezoelectric coupling matrix, if material is specified on strain-charge form	$ds_E^{-1}$
epsilonT	$\epsilon_T$	All	S	Electric permittivity with stress field constant	$\epsilon_0 \epsilon_{rT}$
epsilonS	$\epsilon_S$	All	S	Electric permittivity with strain field constant	If material defined on stress-charge from $\epsilon_0 \epsilon_{rS}$ If material defined on strain-charge from $\epsilon_0 \epsilon_{rT} - d \cdot s_E^{-1} \cdot d^t$
D	$D$	All	S	Stiffness matrix components	For isotropic and anisotropic material
epsilon	$\epsilon_e$	All	S	Electric permittivity matrix components	$\epsilon_0 \epsilon_r$ , for isotropic and anisotropic material
sigma	$\sigma_e$	freq	S	Electric conductivity matrix components	For isotropic and anisotropic material
si	$\sigma_i$	All	S	$\sigma_i$ normal stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon - e^t \mathbf{E}$ or $D \epsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \epsilon - e^t \mathbf{E}$ or $(1 + j\eta)D \epsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$D_i$	$D_i$	All	S	Electric displacement, $x_i$ component	If material defined in global coord. sys. $e\varepsilon + \varepsilon_S \mathbf{E}$ or $\varepsilon_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{D}_1$
$J_i$	$J_i$	T F	S	Total current density, $x_i$ component	$J_{d,i} + J_{p,i}$ or $J_{d,i}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
$J_{d,i}$	$J_{d,i}$	T	S	Displacement current density, $x_i$ component	$\frac{\partial D_i}{\partial t}$
$J_{d,i}$	$J_{d,i}$	F	S	Displacement current density, $x_i$ component	$j\omega D_i$
$J_{p,i}$	$J_{p,i}$	T F	S	Potential current density, $x_i$ component	$\sigma_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
$s_{ij}$	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^t \mathbf{E}$ or $D\varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}$ or $(1 + j\eta)D\varepsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$
$s_{i1}$	$\sigma_i$	All	S	$\sigma_i$ normal stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D\varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D\varepsilon_1$
$D_{i1}$	$D_{i1}$	All	S	Electric displacement, $x_i$ component, local coord. sys.	$e\varepsilon_1 + \varepsilon_S \mathbf{E}_1$ or $\varepsilon_e \mathbf{E}_1$

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
J <sub>i1</sub>	$J_{i1}$	T F	S	Total current density, $x_i$ component, local coord. sys.	$J_{d,i1} + J_{p,i1}$ or $J_{d,i1}$
Jd <sub>i1</sub>	$J_{d,i1}$	T	S	Displacement current density, $x_i$ component, local coord. sys.	$\frac{\partial D_{i1}}{\partial t}$
Jd <sub>i1</sub>	$J_{d,i1}$	F	S	Displacement current density, $x_i$ component, local coord. sys.	$j\omega D_{i1}$
Jp <sub>i1</sub>	$J_{p,i1}$	F	S	Potential current density, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_1$
si <sub>j1</sub>	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$
si <sub>t</sub>	$\sigma_{it}$	F T	S	$\sigma_{it}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$
si <sub>j_t</sub>	$\tau_{ijt}$	T	S	$\tau_{ijt}$ time derivative of shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
si <sub>l</sub> _t	$\sigma_{ilt}$	F T	S	$\sigma_{ilt}$ time derivative of normal stress, user-defined local coord. system	$c_E \varepsilon_{lt}$ or $D \varepsilon_{lt}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$
si <sub>j</sub> l_t	$\tau_{ijlt}$	F T	S	$\tau_{ijlt}$ time derivative of shear stress, user-defined local coord. system	$c_E \varepsilon_{lt}$ or $D \varepsilon_{lt}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$
si	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	Defined by elpric element
ei	$\varepsilon_i$	All	S	Principal strains, $i=1,2,3$	Defined by elpric element
si <sub>xj</sub>	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	Defined by elpric element
ei <sub>xj</sub>	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	Defined by elpric element
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
normD	$ \mathbf{D} $	All	S	Electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Ws	$\bar{W}_s$	All	S	Strain energy density	<p>If material properties defined in global coord. sys.</p> $0.5 \sigma \cdot \varepsilon$ $\frac{\sigma \cdot \varepsilon}{2}, \frac{1}{2} \text{real}(\sigma \cdot \text{conj}(\varepsilon))$ in frequency response analyses <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\sigma_1 \cdot \varepsilon_1}{2}, \frac{1}{2} \text{real}(\sigma_1 \cdot \text{conj}(\varepsilon_1))$ in freq. resp.
We	$\bar{W}_e$	All	S	Electric energy density	<p>If material properties defined in global coord. sys.</p> $\mathbf{E} \cdot \mathbf{D} / 2, \text{real}(\text{conj}(\mathbf{E}) \cdot \mathbf{D}) / 2$ in freq. resp. <p>If material properties defined in local user-defined coord. sys.</p> $\mathbf{E}_1 \cdot \mathbf{D}_1 / 2, \text{real}(\text{conj}(\mathbf{E}_1) \cdot \mathbf{D}_1) / 2$ in freq. resp.
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in x <sub>i</sub> direction	$\begin{bmatrix} \text{Ta}_x \\ \text{Ta}_y \\ \text{Ta}_z \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$
nD	nD	All	B	Surface charge density	$\mathbf{n}_{\text{up}} \cdot (\mathbf{D}_{\text{down}} - \mathbf{D}_{\text{up}})$
nJ	nJ	F T	B	Current density outflow	$\mathbf{n} \cdot \mathbf{J}$
nJs	nJs	F	B	Source current density	<p>Only for unsymmetric electric currents.</p> $\mathbf{n}_{\text{up}} \cdot (\mathbf{J}_{\text{down}} - \mathbf{J}_{\text{up}})$ or, <p>with weak constraints, the Lagrange multiplier for V.</p>

TABLE 2-5: PIEZO SOLID APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<i>F<sub>ig</sub></i>	<i>F<sub>ig</sub></i>	All	All	Body load, face load, edge load, point load, in global $x_i$ direction	<p>If global coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$ <p>If other coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$
smon	smon	All	S	Structural equation available	1 or 0
eson	eson	All	S	Electrical equation available	1 or 0

### *Piezo Plane Stress*

A large number of variables are available for use in expressions and for postprocessing purposes. In addition to the variables listed below, almost all application mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append *\_amp* or *\_ph* to the variable name. For example:

- *sx\_amp* represents the amplitude of the normal stress in the  $x$  direction.
- *ex\_ph* represents the phase of the normal strain in the  $x$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as *mises*, *disp*, *Tresca*, or *s1*.



The table uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column employs the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Time dependent	T

## VARIABLES

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
V	$V$	All	All	Electric potential	$V$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
V_amp	$V_{amp}$	F	All	Electric potential amplitude	$ V $
V_ph	$V_{ph}$	F	All	Electric potential phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(V), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
e <sub>i</sub>	$\epsilon_i$	All	S	$\epsilon_i$ normal strain, global coord. system	$\frac{\partial u_i}{\partial x_i}$
e <sub>z</sub>	$\epsilon_z$	All	S	$\epsilon_z$ normal strain, out of the <i>xy</i> -plane	$\left( \frac{\sum_j e_{j3} E_j - \sum_{k=1,2,4} (c_E)_{3k} \epsilon_k}{(c_E)_{33}} \right)$ or $- \frac{\sum_{k=1,2,4} (D)_{3k} \epsilon_k}{(D)_{33}}$ <p>With loss factor damping in frequency response analysis</p> $\left( \frac{\sum_j e_{j3} E_j - \sum_{k=1,2,4} (1+j\eta)(c_E)_{3k} \epsilon_k}{(1+j\eta)(c_E)_{33}} \right)$ or $- \frac{\sum_{k=1,2,4} (1+j\eta)(c_E)_{3k} \epsilon_k}{(1+j\eta)(c_E)_{33}}$
e <sub>xy</sub>	$\epsilon_{xy}$	All	S	$\epsilon_{xy}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$
E <sub>i</sub>	$E_i$	All	S	Electric field	$-\left( \frac{\partial V}{\partial x_i} \right)$
normE	$E_i$	All	S	Electric field	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
e <sub>i1</sub>	$\epsilon_{i1}$	All	S	$\epsilon_{i1}$ normal strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
e <sub>ij1</sub>	$\epsilon_{ij1}$	All	S	$\epsilon_{ij1}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
E <sub>il</sub>	$E_{il}$	All	S	Electric field, user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}$
V <sub>il</sub>	$V_{il}$	All	S	Electric potential gradient, user-defined coord. system	$T_{\text{coord}}^T \nabla V$
e <sub>i_t</sub>	$\varepsilon_{it}$	T	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{it}}{\partial x_i}$
ez <sub>t</sub>	$\varepsilon_z$	F T	S	$\varepsilon_z$ normal velocity strain out of the xy-plane	$\frac{\left( - \sum_{k=1,2,4} (M)_{3k} \varepsilon_{kt} \right)}{(M)_{33}}$ ( $M$ is $c_E$ or $D$ )
e <sub>i_t</sub>	$\varepsilon_{it}$	F	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{ij}}{\partial x_i} j \omega$
exy <sub>t</sub>	$\varepsilon_{xyt}$	T	S	$\varepsilon_{xyt}$ shear velocity strain global coord. system	$\frac{1}{2} \left( \frac{\partial u_t}{\partial y} + \frac{\partial v_t}{\partial x} \right)$
exy <sub>t</sub>	$\varepsilon_{xyt}$	F	S	$\varepsilon_{xyt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) j \omega$
e <sub>il_t</sub>	$\varepsilon_{ilt}$	F T	S	$\varepsilon_{ilt}$ normal velocity strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
exyl <sub>t</sub>	$\varepsilon_{xylt}$	F T	S	$\varepsilon_{xylt}$ shear velocity strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon_t T_{\text{coord}}$
cE	$c_E$	All	S	Stiffness matrix components	$s_E^{-1}$ , if material is specified on strain-charge form, calculated by a special inverting-matrices element.

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
e	$e$	All	S	Piezoelectric coupling matrix, if material is specified on strain-charge form	$ds_E^{-1}$
epsilonT	$\epsilon_T$	All	S	Electric permittivity with stress field constant	$\epsilon_0 \epsilon_{rT}$
epsilonS	$\epsilon_S$	All	S	Electric permittivity with strain field constant	If material defined on stress-charge from $\epsilon_0 \epsilon_{rS}$ If material defined on strain-charge from $\epsilon_0 \epsilon_{rT} - d \cdot s_E^{-1} \cdot d^t$
D	$D$	All	S	Stiffness matrix components	For isotropic and anisotropic material
epsilon	$\epsilon_e$	All	S	Electric permittivity matrix components	$\epsilon_0 \epsilon_r$ , for isotropic and anisotropic material
sigma	$\sigma_e$	freq	S	Electric conductivity matrix components	For isotropic and anisotropic material
si	$\sigma_i$	All	S	$\sigma_i$ normal stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon - e^t \mathbf{E}$ or $D \epsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \epsilon - e^t \mathbf{E}$ or $(1 + j\eta)D \epsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$D_i$	$D_i$	All	S	Electric displacement, $x_i$ component	If material defined in global coord. sys. $e\epsilon + \epsilon_S \mathbf{E}$ or $\epsilon_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{D}_1$
$J_i$	$J_i$	T F	S	Total current density, $x_i$ component	$J_{d,i} + J_{p,i}$ or $J_{d,i}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
$J_{d,i}$	$J_{d,i}$	T	S	Displacement current density, $x_i$ component	$\frac{\partial D_i}{\partial t}$
$J_{d,i}$	$J_{d,i}$	F	S	Displacement current density, $x_i$ component	$j\omega D_i$
$J_{p,i}$	$J_{p,i}$	T F	S	Potential current density, $x_i$ component	$\sigma_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
$\tau_{ij}$	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, global coord. system	If material defined in global coord. sys. $c_E \epsilon - e^t \mathbf{E}$ or $D\epsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \epsilon - e^t \mathbf{E}$ or $(1 + j\eta)D\epsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$
$\sigma_i$	$\sigma_i$	All	S	$\sigma_i$ normal stress, user-defined local coord. system	$c_E \epsilon_1 - e^t \mathbf{E}_1$ or $D\epsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \epsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D\epsilon_1$
$D_{i1}$	$D_{i1}$	All	S	Electric displacement, $x_i$ component, local coord. sys.	$e\epsilon_1 + \epsilon_S \mathbf{E}_1$ or $\epsilon_e \mathbf{E}_1$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
J <sub>i1</sub>	$J_{i1}$	T F	S	Total current density, $x_i$ component, local coord. sys.	$J_{d,i1} + J_{p,i1}$ or $J_{d,i1}$
Jd <sub>i1</sub>	$J_{d,i1}$	T	S	Displacement current density, $x_i$ component, local coord. sys.	$\frac{\partial D_{i1}}{\partial t}$
Jd <sub>i1</sub>	$J_{d,i1}$	F	S	Displacement current density, $x_i$ component, local coord. sys.	$j\omega D_{i1}$
Jp <sub>i1</sub>	$J_{p,i1}$	F	S	Potential current density, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_1$
si <sub>j1</sub>	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$
si <sub>t</sub>	$\sigma_{it}$	F T	S	$\sigma_{it}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$
si <sub>j_t</sub>	$\tau_{ijt}$	T	S	$\tau_{ijt}$ time derivative of shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
si _t	$\sigma_{i t}$	F T	S	$\sigma_{i t}$ time derivative of normal stress, user-defined local coord. system	$c_E \varepsilon_{ t}$ or $D \varepsilon_{ t}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$
si _t	$\tau_{i t}$	F T	S	$\tau_{i t}$ time derivative of shear stress, user-defined local coord. system	$c_E \varepsilon_{ t}$ or $D \varepsilon_{ t}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$
si	$\sigma_i$	All	S	Principal stresses, $i = 1, 2, 3$	Defined by elpric element
ei	$\varepsilon_i$	All	S	Principal strains, $i = 1, 2, 3$	Defined by elpric element
si_xj	$\sigma_{ixj}$	All	S	Principal stress directions, $i, j = 1, 2, 3$	Defined by elpric element
ei_xj	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i, j = 1, 2, 3$	Defined by elpric element
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
normD	normD	All	S	Electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$

TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Ws	$W_s$	All	S	Strain energy density	<p>If material properties defined in global coord. sys.</p> $\frac{\sigma \cdot \varepsilon}{2} \text{th}, \frac{1}{2} \text{real}(\sigma \cdot \text{conj}(\varepsilon)) \text{th}$ <p>in frequency resp.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\sigma_1 \cdot \varepsilon_1}{2} \text{th}, \frac{1}{2} \text{real}(\sigma_1 \cdot \text{conj}(\varepsilon_1)) \text{th}$ <p>in freq. resp.</p>
We	$W_e$	All	S	Electric energy density	<p>If material properties defined in global coord. sys.</p> $\frac{\mathbf{E} \cdot \mathbf{D}}{2} \text{th}, \frac{1}{2} \text{real}(\text{conj}(\mathbf{E}) \cdot \mathbf{D}) \text{th}$ <p>in frequency resp.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\mathbf{E}_1 \cdot \mathbf{D}_1}{2} \text{th}, \frac{1}{2} \text{real}(\text{conj}(\mathbf{E}_1) \cdot \mathbf{D}_1) \text{th}$ <p>in frequency response analyses.</p>
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in x <sub>i</sub> direction	$\begin{bmatrix} T a_x \\ T a_y \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$
nD	nD	All	B	Surface charge density	$\mathbf{n}_{\text{up}} \cdot (\mathbf{D}_{\text{down}} - \mathbf{D}_{\text{up}})$
nJ	nJ	F T	B	Current density outflow	$\mathbf{n} \cdot \mathbf{J}$
nJs	nJs	F	B	Source current density	<p>Only for unsymmetric electric currents.</p> $\mathbf{n}_{\text{up}} \cdot (\mathbf{J}_{\text{down}} - \mathbf{J}_{\text{up}}) \text{ or,}$ <p>with weak constraints, the Lagrange multiplier for V.</p>



TABLE 2-6: PIEZO PLANE STRESS APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Fig	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	<p>If global coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_z \end{bmatrix}$ <p>If other coordinate system</p> $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$
smon	smon	All	S	Structural equation available	1 or 0
eson	eson	All	S	Electrical equation available	1 or 0

### *Piezo Plane Strain*

A large number of variables are available for use in expressions and for postprocessing purposes. In addition to the variables listed below, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sx_amp` represents the amplitude of the normal stress in the  $x$  direction
- `ex_ph` represents the phase of the normal strain in the  $x$  direction.

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`.

The table uses a convention where indices  $i, j, \dots$  (or  $i, j, \dots$ ) run over the geometry's Cartesian coordinate axes,  $x, y$ , and  $z$ . In particular,  $u_i$  ( $u_i$ ) refers to the global displacements ( $u, v, w$ ). The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Time dependent	T

## VARIABLES

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$u_i$	$u_i$	All	All	$x_i$ displacement	$u_i$
V	$V$	All	All	Electric potential	$V$
$u_{it}$	$u_{it}$	T	All	$x_i$ velocity	$u_{it}$
$u_{i\_amp}$	$u_{iamp}$	F	All	$x_i$ displacement amplitude	$ u_i $
$u_{i\_ph}$	$u_{iph}$	F	All	$x_i$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(u_i), 2\pi)$
V_amp	$V_{amp}$	F	All	Electric potential amplitude	$ V $
V_ph	$V_{ph}$	F	All	Electric potential phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(V), 2\pi)$
$u_{i\_t}$	$u_{it}$	F	All	$x_i$ velocity	$j\omega u_i$
$u_{i\_t\_amp}$	$u_{itamp}$	F	All	$x_i$ velocity amplitude	$\omega u_{iamp}$
$u_{i\_t\_ph}$	$u_{itph}$	F	All	$x_i$ velocity phase	$\text{mod}(u_{iph} + 90^\circ, 360^\circ)$
$u_{i\_tt}$	$u_{itt}$	F	All	$x_i$ acceleration	$-\omega^2 u_i$
$u_{i\_tt\_amp}$	$u_{ittamp}$	F	All	$x_i$ acceleration amplitude	$\omega^2 u_{iamp}$
$u_{i\_tt\_ph}$	$u_{ittph}$	F	All	$x_i$ acceleration phase	$\text{mod}(u_{iph} + 180^\circ, 360^\circ)$

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
e <sub>i</sub>	$\varepsilon_i$	All	S	$\varepsilon_i$ normal strain, global coord. system	$\frac{\partial u_i}{\partial x_i}$
exy	$\varepsilon_{xy}$	All	S	$\varepsilon_{xy}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$
E <sub>i</sub>	$E_i$	All	S	Electric field	$-\left( \frac{\partial V}{\partial x_i} \right)$
normE	$E_i$	All	S	Electric field	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
e <sub>il</sub>	$\varepsilon_{il}$	All	S	$\varepsilon_{il}$ normal strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon T_{\text{coord}}$
e <sub>ijl</sub>	$\varepsilon_{ijl}$	All	S	$\varepsilon_{ijl}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \varepsilon T_{\text{coord}}$
E <sub>il</sub>	$E_{il}$	All	S	Electric field, user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}$
V <sub>il</sub>	$V_{il}$	All	S	Electric potential gradient, user-defined coord. system	$T_{\text{coord}}^T \nabla V$
e <sub>i_t</sub>	$\varepsilon_{it}$	T	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{it}}{\partial x_i}$
e <sub>i_t</sub>	$\varepsilon_{it}$	F	S	$\varepsilon_{it}$ normal velocity strain, global system	$\frac{\partial u_{i_j \omega}}{\partial x_i}$
exy <sub>t</sub>	$\varepsilon_{xyt}$	T	S	$\varepsilon_{xyt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u_t}{\partial y} + \frac{\partial v_t}{\partial x} \right)$

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
exy_t	$\epsilon_{xyt}$	F	S	$\epsilon_{xyt}$ shear velocity strain, global coord. system	$\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) j\omega$
eil_t	$\epsilon_{ilt}$	F T	S	$\epsilon_{ilt}$ normal velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
exyl_t	$\epsilon_{xylt}$	F T	S	$\epsilon_{xylt}$ shear velocity strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
cE	$c_E$	All	S	Stiffness matrix components	$s_E^{-1}$ , if material is specified on strain-charge form, calculated by a special inverting-matrices element.
e	$e$	All	S	Piezoelectric coupling matrix if material is specified on strain-charge form	$d s_E^{-1}$
epsilonT	$\epsilon_T$	All	S	Electric permittivity with stress field constant	$\epsilon_0 \epsilon_r T$
epsilonS	$\epsilon_S$	All	S	Electric permittivity with strain field constant	If material defined on stress-charge from $\epsilon_0 \epsilon_r S$ If material defined on strain-charge from $\epsilon_0 \epsilon_r T - d \cdot s_E^{-1} \cdot d^t$
D	$D$	All	S	Stiffness matrix components	For isotropic and anisotropic material
epsilon	$\epsilon_e$	All	S	Electric permittivity matrix components	$\epsilon_0 \epsilon_r$ , for isotropic and anisotropic material

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sigma	$\sigma_e$	freq	S	Electric conductivity matrix components	For isotropic and anisotropic material
s <i>i</i>	$\sigma_i$	All	S	$\sigma_i$ normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^t \mathbf{E}$ or $D \varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}$ or $(1 + j\eta)D \varepsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$
D <i>i</i>	$D_i$	All	S	Electric displacement, $x_i$ component	If material defined in global coord. sys. $e \varepsilon + \varepsilon_S \mathbf{E}$ or $\varepsilon_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{D}_1$
J <i>i</i>	$J_i$	T F	S	Total current density, $x_i$ component	$J_{d,i} + J_{p,i}$ or $J_{d,i}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
Jd <i>i</i>	$J_{d,i}$	T	S	Displacement current density, $x_i$ component	$\frac{\partial D_i}{\partial t}$
Jd <i>i</i>	$J_{d,i}$	F	S	Displacement current density, $x_i$ component	$j\omega D_i$
Jp <i>i</i>	$J_{p,i}$	T F	S	Potential current density, $x_i$ component	$\sigma_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{ij}$	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^t \mathbf{E}$ or $D \varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}$ or $(1 + j\eta)D \varepsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$
$s_{il}$	$\sigma_i$	All	S	$\sigma_i$ normal stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$
$D_{il}$	$D_{i1}$	All	S	Electric displacement, $x_i$ component, local coord. sys.	$e \varepsilon_1 + \varepsilon_S \mathbf{E}_1$ or $\varepsilon_e \mathbf{E}_1$
$J_{il}$	$J_{i1}$	T F	S	Total current density, $x_i$ component, local coord. sys.	$J_{d,i1} + J_{p,i1}$ or $J_{d,i1}$
$J_{dil}$	$J_{d,i1}$	T	S	Displacement current density, $x_i$ component, local coord. sys.	$\frac{\partial D_{i1}}{\partial t}$
$J_{dil}$	$J_{d,i1}$	F	S	Displacement current density, $x_i$ component, local coord. sys.	$j\omega D_{i1}$
$J_{pil}$	$J_{p,i1}$	F	S	Potential current density, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_1$
$s_{ij1}$	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
si_t	$\sigma_{it}$	F T	S	$\sigma_{it}$ time derivative of normal stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \varepsilon_t</math> or <math>D \varepsilon_t</math>                      With loss factor damping in frequency response analysis  <math>(1 + j\eta)j\omega c_E \varepsilon_t</math> or <math>(1 + j\eta)j\omega D \varepsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T</math></p>
sj_t	$\tau_{ijt}$	T	S	$\tau_{ijt}$ time derivative of shear stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \varepsilon_t</math> or <math>D \varepsilon_t</math>                      With loss factor damping in frequency response analysis  <math>(1 + j\eta)j\omega c_E \varepsilon</math> or <math>(1 + j\eta)j\omega D \varepsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T</math></p>
sil_t	$\sigma_{ilt}$	F T	S	$\sigma_{ilt}$ time derivative of normal stress, user-defined local coord. system	<p><math>c_E \varepsilon_{1t}</math> or <math>D \varepsilon_{1t}</math>                      With loss factor damping in frequency response analysis  <math>(1 + j\eta)j\omega c_E \varepsilon_1</math> or <math>(1 + j\eta)j\omega D \varepsilon_1</math></p>
sjl_t	$\tau_{ijlt}$	F T	S	$\tau_{ijlt}$ time derivative of shear stress, user-defined local coord. system	<p><math>c_E \varepsilon_{1t}</math> or <math>D \varepsilon_{1t}</math>                      With loss factor damping in frequency response analysis  <math>(1 + j\eta)j\omega c_E \varepsilon_1</math> or <math>(1 + j\eta)j\omega D \varepsilon_1</math></p>

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sz	$\sigma_z$	All	S	$\sigma_z$ normal stress	<p>If material defined in global coord. sys.</p> $\sum_k (c_E)_{3k} \varepsilon_k - \sum_j e_{j3} E_j, \text{ or } \sum_k (D)_{3k} \varepsilon_k$ <p>With loss factor damping in frequency response analysis</p> $\sum_k (1 + j\eta)(c_E)_{3k} \varepsilon_k - \sum_j e_{j3} E_j, \text{ or}$ $\sum_k (1 + j\eta)(D)_{3k} \varepsilon_k$ <p>If material defined in user-def. coord. sys.</p> $\sum_k (c_E)_{3k} (\varepsilon_1)_k - \sum_j e_{j3} (E_1)_j, \text{ or}$ $\sum_k (D)_{3k} (\varepsilon_1)_k$
sz_t	$\sigma_{zt}$	All	S	$\sigma_{zt}$ time derivative of normal stress	<p>If material defined in global coord. sys.</p> $\sum_k (D)_{3k} (\varepsilon_t)_k \quad (M \text{ is } c_E \text{ or } D)$ <p>With loss factor damping in frequency response analysis</p> $\sum_k (1 + j\eta)(M)_{3k} j\omega \varepsilon_k \quad (M \text{ is } c_E \text{ or } D)$ <p>If material defined in user-def. coord. sys.</p> $\sum_k (M)_{3k} (\varepsilon_{1t})_k \quad (M \text{ is } c_E \text{ or } D)$
si	$\sigma_i$	All	S	Principal stresses, $i=1,2,3$	Defined by elpric element
ei	$\varepsilon_i$	All	S	Principal strains, $i=1,2,3$	Defined by elpric element



TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
$s_{ixj}$	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j=1,2,3$	Defined by elpric element
$e_{ixj}$	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j=1,2,3$	Defined by elpric element
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
normD	normD	All	S	Electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$
Ws	$W_s$	All	S	Strain energy density	<p>If material properties defined in global coord. sys.</p> $\frac{\sigma \cdot \varepsilon}{2} \text{th}, \frac{1}{2} \text{real}(\sigma \cdot \text{conj}(\varepsilon)) \text{th}$ <p>in frequency response analyses.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\sigma_1 \cdot \varepsilon_1}{2} \text{th}, \frac{1}{2} \text{real}(\sigma_1 \cdot \text{conj}(\varepsilon_1)) \text{th}$ <p>in frequency response analyses.</p>
We	$W_e$	All	S	Electric energy density	<p>If material properties defined in global coord. sys.</p> $\frac{\mathbf{E} \cdot \mathbf{D}}{2} \text{th}, \frac{1}{2} \text{real}(\text{conj}(\mathbf{E}) \cdot \mathbf{D}) \text{th}$ <p>in frequency response analyses.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\mathbf{E}_1 \cdot \mathbf{D}_1}{2} \text{th}, \frac{1}{2} \text{real}(\text{conj}(\mathbf{E}_1) \cdot \mathbf{D}_1) \text{th}$ <p>in frequency response analyses.</p>

TABLE 2-7: PIEZO PLANE STRAIN APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Tai	$Ta_i$	All	B	Surface traction (force/area) in $x_i$ direction	$\begin{bmatrix} T a_x \\ T a_y \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix}$
nD	nD	All	B	Surface charge density	$\mathbf{n}_{up} \cdot (\mathbf{D}_{down} - \mathbf{D}_{up})$
nJ	nJ	F T	B	Current density outflow	$\mathbf{n} \cdot \mathbf{J}$
nJs	nJs	F	B	Source current density	Only for unsymmetric electric currents. $\mathbf{n}_{up} \cdot (\mathbf{J}_{down} - \mathbf{J}_{up})$ or, with weak constraints, the Lagrange multiplier for V.
Fig	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{yg} \end{bmatrix} = T_{coord} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$
smon	smon	All	S	Structural equation available	1 or 0
eson	eson	All	S	Electrical equation available	1 or 0

### *Piezo Axial Symmetry*

A large number of variables are available for use in expressions and for postprocessing purposes. In addition to the variables listed below, almost all application-mode parameters are available as variables. Some variables change their availability with the type of analysis, as noted in the Analysis column. For frequency-response analysis a number of additional variables are available. Furthermore, the amplitudes and phases

of variables such as strains and stresses are available; to access them, append `_amp` or `_ph` to the variable name. For example:

- `sr_amp` represents amplitude of the normal stress in the  $r$  direction.
- `ephi_ph` represents the phase of the normal strain in the  $\phi$  direction

The exception to this scheme consists of variables defined using a nonlinear operator such as `mises`, `disp`, `Tresca`, or `s1`. The Analysis column uses the following abbreviations:

ANALYSIS	ABBREVIATION
Static	S
Frequency response	F
Time dependent	T

## VARIABLES

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
<code>uor</code>	<code>uor</code>	All	All	$r$ displacement divided by $r$	<code>uor</code>
<code>uaxi</code>	<code>uaxi</code>	All	All	$r$ displacement	<code>uor</code> · $r$
<code>w</code>	$w$	All	All	$z$ displacement	<code>w</code>
<code>V</code>	$V$	All	All	Electric potential	<code>V</code>
<code>uort</code>	<code>uor<sub>t</sub></code>	T	All	$r$ velocity divided by $r$	<code>uor<sub>t</sub></code>
<code>uaxi_t</code>	<code>uaxi<sub>t</sub></code>	T	All	$r$ velocity	<code>uor<sub>t</sub></code> · $r$
<code>w_t</code>	<code>w<sub>t</sub></code>	T	All	$z$ velocity	<code>w<sub>t</sub></code>
<code>uaxi_amp</code>	<code>uaxi<sub>amp</sub></code>	F	All	$r$ displacement amplitude	<code> uaxi </code>
<code>w_amp</code>	<code>w<sub>amp</sub></code>	F	All	$z$ displacement amplitude	<code> w </code>
<code>uaxi_ph</code>	<code>uaxi<sub>ph</sub></code>	F	All	$r$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(\text{uaxi}), 2\pi)$
<code>w_ph</code>	<code>w<sub>ph</sub></code>	F	All	$z$ displacement phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(w), 2\pi)$

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
V_amp	$V_{amp}$	F	All	Electric potential amplitude	$ V $
V_ph	$V_{ph}$	F	All	Electric potential phase	$\frac{180^\circ}{\pi} \text{mod}(\text{angle}(V), 2\pi)$
uaxi_t	$uaxi_t$	F	All	$r$ velocity	$j\omega uaxi$
w_t	$w_t$	F	All	$z$ velocity	$j\omega w$
uaxi_t_amp	$uaxi_{tamp}$	F	All	$r$ velocity amplitude	$\omega uaxi_{amp}$
w_t_amp	$w_{tamp}$	F	All	$z$ velocity amplitude	$\omega w_{amp}$
uaxi_t_ph	$uaxi_{tph}$	F	All	$r$ velocity phase	$\text{mod}(uaxi_{ph} + 90^\circ, 360^\circ)$
w_t_ph	$w_{tph}$	F	All	$z$ velocity phase	$\text{mod}(w_{ph} + 90^\circ, 360^\circ)$
uaxi_tt	$uaxi_{tt}$	F	All	$r$ acceleration	$-\omega^2 uaxi$
w_tt	$w_{tt}$	F	All	$z$ acceleration	$-\omega^2 w$
uaxi_tt_amp	$uaxi_{ttamp}$	F	All	$r$ acceleration amplitude	$\omega^2 uaxi_{amp}$
w_tt_amp	$w_{ttamp}$	F	All	$z$ acceleration amplitude	$\omega^2 w_{amp}$
uaxi_tt_ph	$uaxi_{ttph}$	F	All	$r$ acceleration phase	$\text{mod}(uaxi_{ph} + 180^\circ, 360^\circ)$
w_tt_ph	$w_{ttph}$	F	All	$z$ acceleration phase	$\text{mod}(w_{ph} + 180^\circ, 360^\circ)$
disp	disp	All	All	Total displacement	$\sqrt{uaxi^2 + w^2}$
er	$\epsilon_r$	All	S	$\epsilon_r$ normal strain, global system	$uor + \frac{\partial}{\partial r}(uor) \cdot r$
ez	$\epsilon_z$	All	S	$\epsilon_z$ normal strain, global system	$\frac{\partial w}{\partial z}$
ephi	$\epsilon_\phi$	All	S	$\epsilon_\phi$ normal strain	uor
erz	$\epsilon_{rz}$	All	S	$\epsilon_{rz}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z}(uor) \cdot r + \frac{\partial w}{\partial r} \right)$

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
exl, eyl	$\epsilon_{xl}, \epsilon_{yl}$	All	S	$\epsilon_{xl}, \epsilon_{yl}$ normal strains, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
exyl	$\epsilon_{xyl}$	All	S	$\epsilon_{xy}$ shear strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon T_{\text{coord}}$
er_t	$\epsilon_{rt}$	T	S	$\epsilon_{rt}$ velocity normal strain, global system	$uor_t + \frac{\partial}{\partial r}(uor_t) \cdot r$
er_t	$\epsilon_{rt}$	F	S	$\epsilon_{rt}$ velocity normal strain, global system	$j\omega \left( uor + \frac{\partial}{\partial r} uor \cdot r \right)$
ez_t	$\epsilon_{zt}$	T	S	$\epsilon_{zt}$ velocity normal strain, global system	$\frac{\partial w_t}{\partial z}$
ez_t	$\epsilon_{zt}$	F	S	$\epsilon_{zt}$ velocity normal strain, global system	$j\omega \left( \frac{\partial w}{\partial z} \right)$
ephi_t	$\epsilon_{\phi t}$	T	S	$\epsilon_{\phi t}$ velocity normal strain	$uor_t$
ephi_t	$\epsilon_{\phi t}$	F	S	$\epsilon_{\phi t}$ velocity normal strain	$j\omega uor$
erz_t	$\epsilon_{rzt}$	T	S	$\epsilon_{rzt}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z} (uor_t) \cdot r + \frac{\partial w_t}{\partial r} \right)$
erz_t	$\epsilon_{rzt}$	F	S	$\epsilon_{rzt}$ shear strain, global coord. system	$\frac{1}{2} \left( \frac{\partial}{\partial z} (uor) \cdot r + \frac{\partial w}{\partial r} \right) j\omega$
exl_t, eyl_t	$\epsilon_{xlt}, \epsilon_{ylt}$	F T	S	$\epsilon_{xlt}, \epsilon_{ylt}$ velocity normal strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$
exyl_t	$\epsilon_{xylt}$	F T	S	$\epsilon_{xylt}$ velocity shear strain, user-defined coord. system	$T_{\text{coord}}^T \epsilon_t T_{\text{coord}}$

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
disp	disp	All	All	Total displacement	$\sqrt{\sum_i (\text{real}(u_i))^2}$
E <sub>i</sub>	$E_i$	All	S	Electric field	$-\left(\frac{\partial V}{\partial x_i}\right)$
normE	$ \mathbf{E} $	All	S	Electric field	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
E <sub>i1</sub>	$E_{i1}$	All	S	Electric field, user-defined coord. system	$T_{\text{coord}}^T \mathbf{E}$
V <sub>i1</sub>	$V_{i1}$	All	S	Electric potential gradient, user-defined coord. system	$T_{\text{coord}}^T \nabla V$
cE	$c_E$	All	S	Stiffness matrix components	$s_E^{-1}$ , if material is specified on strain-charge form, calculated by a special inverting-matrices element.
e	$e$	All	S	Piezoelectric coupling matrix if material is specified on strain-charge form	$ds_E^{-1}$
epsilonT	$\epsilon_T$	All	S	Electric permittivity with stress field constant	$\epsilon_0 \epsilon_{rT}$
epsilonS	$\epsilon_S$	All	S	Electric permittivity with strain field constant	If material defined on stress-charge from $\epsilon_0 \epsilon_{rS}$ If material defined on strain-charge from $\epsilon_0 \epsilon_{rT} - d \cdot s_E^{-1} \cdot d^t$
D	$D$	All	S	Stiffness matrix components	For isotropic and anisotropic material

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
epsilon	$\epsilon_e$	All	S	Electric permittivity matrix components	$\epsilon_0 \epsilon_r$ , for isotropic and anisotropic material
sigma	$\sigma_e$	freq	S	Electric conductivity matrix components	For isotropic and anisotropic material
sr, sz	$\sigma_r, \sigma_z$	All	S	$\sigma_{r,z}$ normal stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \epsilon - e^t \mathbf{E}</math> or <math>D \epsilon</math></p> <p>With loss factor damping in frequency response analysis  <math>(1 + j\eta) c_E \epsilon - e^t \mathbf{E}</math> or <math>(1 + j\eta) D \epsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \sigma_1 T_{\text{coord}}^T</math></p>
sphi	$\sigma_\phi$	All	S	$\sigma_\phi$ normal stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \epsilon - e^t \mathbf{E}</math> or <math>D \epsilon</math></p> <p>With loss factor damping in frequency response analysis  <math>(1 + j\eta) c_E \epsilon - e^t \mathbf{E}</math> or <math>(1 + j\eta) D \epsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>c_E \epsilon_1 - e^t \mathbf{E}_1</math></p>

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
s r z	$\tau_{rz}$	All	S	$\tau_{rz}$ shear stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \varepsilon - e^t \mathbf{E}</math> or <math>D \varepsilon</math></p> <p>With loss factor damping in frequency response analysis  <math>(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}</math> or <math>(1 + j\eta)D \varepsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T</math></p>
s i	$\sigma_i$	All	S	$\sigma_i$ normal stress, global coord. system	<p>If material defined in global coord. sys.  <math>c_E \varepsilon - e^t \mathbf{E}</math> or <math>D \varepsilon</math></p> <p>With loss factor damping in frequency response analysis  <math>(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}</math> or <math>(1 + j\eta)D \varepsilon</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \sigma_1 T_{\text{coord}}^T</math></p>
D i	$D_i$	All	S	Electric displacement, $x_i$ component	<p>If material defined in global coord. sys.  <math>e \varepsilon + \varepsilon_S \mathbf{E}</math> or <math>\varepsilon_e \mathbf{E}</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \mathbf{D}_1</math></p>
J i	$J_i$	T F	S	Total current density, $x_i$ component	<p><math>J_{d,i} + J_{p,i}</math> or <math>J_{d,i}</math></p> <p>If material defined in user-def. coord. sys.  <math>T_{\text{coord}} \mathbf{J}_1</math></p>
J d i	$J_{d,i}$	T	S	Displacement current density, $x_i$ component	$\frac{\partial D_i}{\partial t}$
J d i	$J_{d,i}$	F	S	Displacement current density, $x_i$ component	$j\omega D_i$



TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
J <i>p</i> <i>i</i>	$J_{p,i}$	T F	S	Potential current density, $x_i$ component	$\sigma_e \mathbf{E}$ If material defined in user-def. coord. sys. $T_{\text{coord}} \mathbf{J}_1$
s <i>i</i> <i>j</i>	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon - e^t \mathbf{E}$ or $D \varepsilon$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon - e^t \mathbf{E}$ or $(1 + j\eta)D \varepsilon$ If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_1 T_{\text{coord}}^T$
s <i>i</i> <i>l</i>	$\sigma_i$	All	S	$\sigma_i$ normal stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$
D <i>i</i> <i>l</i>	$D_{i1}$	All	S	Electric displacement, $x_i$ component, local coord. sys.	$e \varepsilon_1 + \varepsilon_S \mathbf{E}_1$ or $\varepsilon_e \mathbf{E}_1$
J <i>i</i> <i>l</i>	$J_{i1}$	T F	S	Total current density, $x_i$ component, local coord. sys.	$J_{d,i1} + J_{p,i1}$ or $J_{d,i1}$
J <i>d</i> <i>i</i> <i>l</i>	$J_{d,i1}$	T	S	Displacement current density, $x_i$ component, local coord. sys.	$\frac{\partial D_{i1}}{\partial t}$
J <i>d</i> <i>i</i> <i>l</i>	$J_{d,i1}$	F	S	Displacement current density, $x_i$ component, local coord. sys.	$j\omega D_{i1}$

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
Jp <sub>il</sub>	$J_{p,il}$	F	S	Potential current density, $x_i$ component, local coord. sys.	$\sigma_e \mathbf{E}_1$
si <sub>j</sub> l	$\tau_{ij}$	All	S	$\tau_{ij}$ shear stress, user-defined local coord. system	$c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $D \varepsilon_1$ With loss factor damping in frequency response analysis $(1 + j\eta)c_E \varepsilon_1 - e^t \mathbf{E}_1$ or $(1 + j\eta)D \varepsilon_1$
si <sub>t</sub>	$\sigma_{it}$	F T	S	$\sigma_{it}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon_t$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$
si <sub>j</sub> t	$\tau_{ijt}$	T	S	$\tau_{ijt}$ time derivative of shear stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon_t$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$
si <sub>l</sub> t	$\sigma_{ilt}$	F T	S	$\sigma_{ilt}$ time derivative of normal stress, user-defined local coord. system	$c_E \varepsilon_{lt}$ or $D \varepsilon_{lt}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
sijl_t	$\tau_{ijl_t}$	F T	S	$\tau_{ijl_t}$ time derivative of shear stress, user-defined local coord. system	$c_E \varepsilon_{1t}$ or $D \varepsilon_{1t}$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_1$ or $(1 + j\eta)j\omega D \varepsilon_1$
sr_t, sz_t	$\sigma_{rt}, \sigma_{zt}$	F T	S	$\sigma_{rt}, \sigma_{zt}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $T_{\text{coord}} \sigma_{1t} T_{\text{coord}}^T$
sphi_t	$\sigma_{\phi t}$	F T	S	$\sigma_{\phi t}$ time derivative of normal stress, global coord. system	If material defined in global coord. sys. $c_E \varepsilon_t$ or $D \varepsilon_t$ With loss factor damping in frequency response analysis $(1 + j\eta)j\omega c_E \varepsilon_t$ or $(1 + j\eta)j\omega D \varepsilon$  If material defined in user-def. coord. sys. $c_E \varepsilon_{1t}$
si	$\sigma_i$	All	S	Principal stresses, $i = 1,2,3$	Defined by elpric element
ei	$\varepsilon_i$	All	S	Principal strains, $i = 1,2,3$	Defined by elpric element
sixj	$\sigma_{ixj}$	All	S	Principal stress directions, $i,j = 1,2,3$	Defined by elpric element
eixj	$\varepsilon_{ixj}$	All	S	Principal strain directions, $i,j = 1,2,3$	Defined by elpric element

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
tresca	$\sigma_{tresca}$	All	S	Tresca stress	$\max(\max( \sigma_1 - \sigma_2 ,  \sigma_2 - \sigma_3 ),  \sigma_1 - \sigma_3 )$
mises	$\sigma_{mises}$	All	S	von Mises stress	
normD	normD	All	S	Electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$
Ws	$W_s$	All	S	Strain energy density	<p>If material properties defined in global coord. sys.</p> $\frac{\sigma \cdot \varepsilon}{2}, \frac{1}{2} \text{real}(\sigma \cdot \text{conj}(\varepsilon))$ <p>in frequency response analyses.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\frac{\sigma_1 \cdot \varepsilon_1}{2},$ $\frac{\text{real}(\sigma_1 \cdot \text{conj}(\varepsilon_1))}{2} + \frac{\text{real}(\sigma_\phi \cdot \text{conj}(\varepsilon_\phi))}{2}$ <p>in freq. resp.</p>
We	$W_e$	All	S	Electric energy density	<p>If material properties defined in global coord. sys.</p> $\mathbf{E} \cdot \mathbf{D} / 2, \text{real}(\text{conj}(\mathbf{E}) \cdot \mathbf{D}) / 2$ <p>in freq. resp.</p> <p>If material properties defined in local user-defined coord. sys.</p> $\mathbf{E}_1 \cdot \mathbf{D}_1 / 2, \text{real}(\text{conj}(\mathbf{E}_1) \cdot \mathbf{D}_1) / 2$ <p>in freq. resp.</p>
Ta <sub>i</sub>	Ta <sub>i</sub>	All	B	Surface traction (force/area) in x <sub>i</sub> direction	$\begin{bmatrix} \text{Ta}_r \\ \text{Ta}_z \end{bmatrix} = \begin{bmatrix} \sigma_r & \tau_{rz} \\ \tau_{rz} & \sigma_z \end{bmatrix} \begin{bmatrix} n_r \\ n_z \end{bmatrix}$
nD	nD	All	B	Surface charge density	$\mathbf{n}_{up} \cdot (\mathbf{D}_{down} - \mathbf{D}_{up})$
smon	smon	All	S	Structural equation available	1 or 0

TABLE 2-8: PIEZO AXIAL SYMMETRY APPLICATION MODE VARIABLES

NAME	SYMBOL	ANALYSIS	DOMAIN	DESCRIPTION	EXPRESSION
eson	eson	All	S	Electrical equation available	1 or 0
nJ	nJ	F T	B	Current density outflow	$\mathbf{n} \cdot \mathbf{J}^d$
nJs	nJs	F	B	Source current density	Only for unsymmetric electric currents. $\mathbf{n}_{\text{up}} \cdot (\mathbf{J}_{\text{down}} - \mathbf{J}_{\text{up}})$ or, with weak constraints, the Lagrange multiplier for V.
Fig	$F_{ig}$	All	All	Body load, edge load, point load, in global $x_i$ direction	If global coordinate system $\begin{bmatrix} F_{rg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} F_r \\ F_z \end{bmatrix}$ If other coordinate system $\begin{bmatrix} F_{xg} \\ F_{zg} \end{bmatrix} = T_{\text{coord}} \begin{bmatrix} F_r \\ F_z \end{bmatrix}$

# Film Damping Application Modes

A number of variables and physical quantities are available for postprocessing and for use in equations and boundary conditions. This chapter describes these expressions for the Film Damping application modes in 2D, 2D axisymmetric, and 3D.

The **Type** column in the following table indicates where you can use a given variable (**b** = boundary, **p/e** = point/edge):

TABLE 2-9: VARIABLES AND QUANTITIES

NAME	SYMBOL	UNIT	DESCRIPTION	TYPE	EXPRESSION/DEFAULT
pf	$p_F$	Pa	Dependent variable, film pressure variation	b	0
$\frac{dx_i}{dy} \frac{dx}{dz}$	$\mathbf{d}, dx_i$	m	Components of the boundary-deformation vector	b	0
$\frac{d0x_i}{d0y} \frac{d0x}{d0z}$	$\mathbf{d}_0, d_{0,xi}$	m	Components of the boundary-deformation vector of the channel base	b	0
h0	$h_0$	m	Initial film thickness	b	$10^{-5}$
pA	$p_A$	Pa	Initial pressure	b	100
plambda0	$p_{\lambda, 0}$	Pa	Reference pressure for $\lambda_0$	b	101325
lambda0	$\lambda_0$	m	Mean free path of gas molecules at $p_{\lambda, 0}$	b	$10^{-7}$
eta	$\eta$	Pa·s	Dynamic viscosity	b	$2 \cdot 10^{-5}$
alphav	$\alpha_v$	l	Tangential momentum accommodation coefficient	b	l
Dh	$D_h$	l	Relative diffusivity	b	l
Ch	$C_h$	l	Relative compressibility	b	l
Yh	$Y_h$	m/s/Pa	Perforation admittance	b	0
dph	$\Delta p_h$	Pa	Perforation pressure difference	b	pf
dL	$\Delta L$	m	Border elongation	p/e	$0.7 \cdot 10^{-5}$
dLr	$\Delta L_r$	l	Relative border elongation	p/e	0.7

TABLE 2-9: VARIABLES AND QUANTITIES

NAME	SYMBOL	UNIT	DESCRIPTION	TYPE	EXPRESSION/DEFAULT
pf0	$p_{F,0}$	Pa	Film pressure variation	p/e	0
Kn	Kn	l	Knudsen number	b	$\lambda/h$ (time dependent) $\lambda/h_0$ (frequency response)
D	$D$	l	Inverse scaled Knudsen number	b	$\frac{\sqrt{\pi}}{2Kn}$
Ks	$K_s$	l	Scaled Knudsen number	b	$\sigma_p Kn$
sigmap	$\sigma_p$	l	Slip coefficient	b	Equation 6-5 in “Rarefaction and Slip Effects” on page 197 in the MEMS Module User’s Guide
dn	$d_n$	m	Normal boundary deformation	b	$-\mathbf{n} \cdot \mathbf{d}$
d0n	$d_{0,n}$	m	Normal boundary deformation of the channel base	b	$\mathbf{n} \cdot \mathbf{d}_0$
d_txi	$\mathbf{d}_t, d_{t,xi}$	m	$x_i$ components of the tangential boundary deformation	b	$\mathbf{d} - \mathbf{n}d_n$
d0_txi	$\mathbf{d}_{0,t}, d_{0,t,xi}$	m	$x_i$ components of the tangential boundary deformation of the channel base	b	$\mathbf{d}_0 - \mathbf{n}d_{0,n}$
u_txi	$\mathbf{u}_t, u_{t,xi}$	m/s	$x_i$ components of the tangential boundary velocity		$d\mathbf{d}_t / dt$ (time dependent) $j\omega\mathbf{d}_t$ (frequency response)
u0_txi	$\mathbf{u}_{0,t}, u_{0,t,xi}$	m/s	$x_i$ components of the tangential boundary velocity of the channel base		$d\mathbf{d}_{0,t} / dt$ (time dependent) $j\omega\mathbf{d}_{0,t}$ (frequency response)
dh	$\Delta h$	m	Film thickness variation	b	$-d_n - d_{0,n}$
dht	$\frac{d(\Delta h)}{dt}$	m/s	Film thickness variation rate	b	diff(dh,t) (time dependent) $j\omega dh$ (frequency response)
h	$h$	m	Film thickness	b	$h_0 + \Delta h$

TABLE 2-9: VARIABLES AND QUANTITIES

NAME	SYMBOL	UNIT	DESCRIPTION	TYPE	EXPRESSION/DEFAULT
lambda	$\lambda$	m	Gas molecules mean free path	b	$\lambda_0 p_{\lambda,0} / (p_a + p_f)$ (time dependent) $\lambda_0 p_{\lambda,0} / p_a$ (frequency response)
Qch	$Q_{ch}$		Relative flow rate function	b	Different expressions according to equations Equation 6-6 – Equation 6-9 in “Rarefaction and Slip Effects” on page 197 in the MEMS Module User’s Guide
pf_amp	$p_{F,amp}$		Pressure variation amplitude, in Freq	b	abs(pf)
pf_ph	$p_{F,ph}$		Pressure variation phase, in Freq	b	$180/\pi * \text{mod}(\text{angle}(pf), 2 * \pi)$
F_xi	$\mathbf{F}, F_{xi}$		$x_i$ components of the film load	b	$\mathbf{F}_n$ for squeezed film $\mathbf{F}_n + \mathbf{F}_t$ otherwise
F_nxi	$\mathbf{F}_n, F_{n,xi}$		$x_i$ components of the normal film load	b	$-\mathbf{n} p_f$
F_txi	$\mathbf{F}_t, F_{t,xi}$		$x_i$ components of the tangential film load	b	$-\eta \frac{\mathbf{u}_t - \mathbf{u}_{0,t}}{h} - \frac{h}{2} \nabla_t p_f$ for No slip Relative flow rate function, $-\eta \frac{\mathbf{u}_t - \mathbf{u}_{0,t}}{h} - \frac{h}{2} \nabla_t p_f$ otherwise



# Electrostatic Fields

A number of variables and physical quantities are available for postprocessing and for use in equations and boundary conditions. They are all given in the following tables.

## *Conductive Media DC Application Mode*

The fundamental fields that can be derived from the electric potential are available for postprocessing and for use in equations and boundary conditions.

### **APPLICATION MODE SUBDOMAIN VARIABLES**

The subdomain variables for Conductive Media DC are given the table below.

TABLE 2-10: APPLICATION MODE SUBDOMAIN VARIABLES, CONDUCTIVE MEDIA DC

NAME	DESCRIPTION	EXPRESSION
V	electric potential	$V$
sigma	electric conductivity	$\sigma$
sigma <sup>i,j</sup>	electric conductivity, $x_i x_j$ component	$\sigma_{ij}$
Q <sub>j</sub>	current source	$Q_j$
d	thickness	$d$
Je <sup>i</sup>	external current density, $x_i$ component	$J_i^e$
normJe	external current density, norm	$\sqrt{\mathbf{J}^e \cdot \mathbf{J}^e}$
Ji <sup>i</sup>	potential current density, $x_i$ component	$\sigma_{ij} E_j$
normJi	potential current density, norm	$\sqrt{\mathbf{J}^i \cdot \mathbf{J}^i}$
J <sup>i</sup>	total current density, $x_i$ component	$J_i^e + J_i^i$
normJ	total current density, norm	$\sqrt{\mathbf{J} \cdot \mathbf{J}}$
E <sup>i</sup>	electric field, $x_i$ component	$\frac{\partial V}{\partial x_i}$
normE	electric field, norm	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
Q	resistive heating	$\mathbf{J} \cdot \mathbf{E}$

## APPLICATION BOUNDARY VARIABLES

The boundary variables for Conductive Media DC are given in the table below.

TABLE 2-11: APPLICATION MODE BOUNDARY VARIABLES, CONDUCTIVE MEDIA DC

NAME	DESCRIPTION	EXPRESSION
tE <sub>i</sub>	tangential electric field, $x_i$ component	$-\mathbf{t}_i \cdot \nabla_t V$
normtE	tangential electric field, norm	$\sqrt{\mathbf{t} \cdot \mathbf{E} \cdot \mathbf{t}}$
nj	current density outflow	$\mathbf{n} \cdot \mathbf{J}$
njs	source current density	$\mathbf{n}_{\text{up}} \cdot (\mathbf{J}_{\text{down}} - \mathbf{J}_{\text{up}})$
js <sub>i</sub>	surface current density, $x_i$ component	$d\sigma t E_i$
normjs	surface current density, norm	$\sqrt{\mathbf{J}_s \cdot \mathbf{J}_s}$
sigmabnd	electric conductivity on boundary	$\sigma_{\text{bnd}}$
Qs	surface resistive heating	$\mathbf{J}_s \cdot \mathbf{t} \mathbf{E}$
Qjl	line current source	$Q_{jl}$
Qj0	point current source	$Q_{j0}$

## APPLICATION POINT VARIABLES

The point variable for the Conductive Media DC application mode appears in the following table.

TABLE 2-12: APPLICATION MODE POINT VARIABLES, CONDUCTIVE MEDIA DC

NAME	TYPE	DESCRIPTION	EXPRESSION
Qj0	P	Point current source	$Q_{j0}$

## *The Electrostatics Application Mode*

The fundamental fields that can be derived from the electric potential are available for postprocessing and for use in equations and boundary conditions.

## APPLICATION MODE SCALAR VARIABLES

The application-specific scalar variable in this mode is given in the following table.

TABLE 2-13: APPLICATION MODE SCALAR VARIABLES, ELECTROSTATICS,

NAME	DESCRIPTION	EXPRESSION
epsilon0	permittivity of vacuum	$\epsilon_0$

## APPLICATION MODE SUBDOMAIN VARIABLES

The subdomain variables for Electrostatics are given the table below.

TABLE 2-14: APPLICATION MODE SUBDOMAIN VARIABLES, ELECTROSTATICS,

NAME	CONSTITUTIVE RELATION	DESCRIPTION	EXPRESSION
V		electric potential	$V$
epsilon <sub>r</sub>	epsr, Dr	relative permittivity	$\epsilon_r$
epsilon <sub>r</sub>	P	relative permittivity	1
epsilon <sub>r<sub>ij</sub></sub>	epsr, Dr	relative permittivity, $x_i x_j$ component	$\epsilon_{rij}$
epsilon <sub>r<sub>ij</sub></sub>	P	relative permittivity, $x_i x_j$ component	1
epsilon		permittivity	$\epsilon_0 \epsilon_r$
epsilon <sub>ij</sub>		permittivity, $x_i x_j$ component	$\epsilon_0 \epsilon_{rij}$
P <sub>i</sub>	P	electric polarization, $x_i$ component	$P_i$
P <sub>i</sub>	epsr, Dr	electric polarization, $x_i$ component	$D_i - \epsilon_0 E_i$
normP		electric polarization, norm	$\sqrt{\mathbf{P} \cdot \mathbf{P}}$
Dr <sub>i</sub>	epsr	remanent displacement, $x_i$ component	0
Dr <sub>i</sub>	P	remanent displacement, $x_i$ component	$P_i$
Dr <sub>i</sub>	Dr	remanent displacement, $x_i$ component	$D_{ri}$
normDr		remanent displacement, norm	$\sqrt{\mathbf{D}_r \cdot \mathbf{D}_r}$
rho		space charge density	$\rho$
E <sub>i</sub>		electric field, $x_i$ component	$-\frac{\partial V}{\partial x_i}$
normE		electric field, norm	$\sqrt{\mathbf{E} \cdot \mathbf{E}}$
D <sub>i</sub>	epsr	electric displacement, $x_i$ component	$\epsilon_0 \epsilon_{rij} E_j$
D <sub>i</sub>	P	electric displacement, $x_i$ component	$\epsilon_0 E_i + P_i$
D <sub>i</sub>	Dr	electric displacement, $x_i$ component	$\epsilon_0 \epsilon_{rij} E_j + D_{ri}$
normD		electric displacement, norm	$\sqrt{\mathbf{D} \cdot \mathbf{D}}$
We		electric energy density	$\frac{\mathbf{E} \cdot \mathbf{D}}{2}$

### APPLICATION BOUNDARY VARIABLES

The boundary variables for Electrostatics are given in the table below.

TABLE 2-15: APPLICATION MODE BOUNDARY VARIABLES, ELECTROSTATICS.

NAME	DESCRIPTION	EXPRESSION
nD	surface charge density	$\mathbf{n}_{\text{up}} \cdot (\mathbf{D}_{\text{down}} - \mathbf{D}_{\text{up}})$
epsilon_bnd	relative permittivity on boundary	$\epsilon_{\text{bnd}}$
unT <sub>i</sub>	Maxwell surface stress tensor, $x_i$ component, up side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{up}} \cdot \mathbf{D}_{\text{up}})n_{i\text{down}}$ $+ (\mathbf{n}_{\text{down}} \cdot \mathbf{D}_{\text{up}})E_{i\text{up}}$
dnT <sub>i</sub>	Maxwell surface stress tensor, $x_i$ component, down side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{down}} \cdot \mathbf{D}_{\text{down}})n_{i\text{up}}$ $+ (\mathbf{n}_{\text{up}} \cdot \mathbf{D}_{\text{down}})E_{i\text{down}}$
unTE <sub>i</sub>	electric Maxwell surface stress tensor, $x_i$ component, up side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{up}} \cdot \mathbf{D}_{\text{up}})n_{i\text{down}}$ $+ (\mathbf{n}_{\text{down}} \cdot \mathbf{D}_{\text{up}})E_{i\text{up}}$
dnTE <sub>i</sub>	electric Maxwell surface stress tensor, $x_i$ component, down side of boundary	$-\frac{1}{2}(\mathbf{E}_{\text{down}} \cdot \mathbf{D}_{\text{down}})n_{i\text{up}}$ $+ (\mathbf{n}_{\text{up}} \cdot \mathbf{D}_{\text{down}})E_{i\text{down}}$

### APPLICATION EDGE VARIABLES

The edge variable for Electrostatics appears in the following table:

TABLE 2-16: APPLICATION MODE EDGE VARIABLES, ELECTROSTATICS.

NAME	DESCRIPTION	EXPRESSION
Ql	line charge density	$Q_1$

### APPLICATION POINT VARIABLES

The point variable for Electrostatics appears in the following table:

TABLE 2-17: APPLICATION MODE POINT VARIABLES, ELECTROSTATICS.

NAME	DESCRIPTION	EXPRESSION
Q0	charge	$Q_0$



# Microfluidics Application Modes

## *Application Modes for Laminar Flow*

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A number of variables and physical quantities are available for postprocessing and for use in equations and boundary conditions. The **Type** column in the following table indicates where you can use a given variable:

- P = points
- B = boundaries
- S = subdomains
- V = vector expressions

Many of the boundary variables belong to certain boundary condition and thus are available only on certain boundaries. The table points out these cases with the following code: lf—laminar inflow/outflow boundary condition; eo—electroosmotic velocity boundary condition; and sv—slip velocity boundary condition.

Most labels for postprocessing variables are followed by the code given to the respective application mode. For example, in any of the microfluidics application modes the modulus of the velocity, denoted  $U$ , becomes `U_mmg1f`. Even so, the name `mmg1f` can change (for example, to `mmg1f2`) if the model includes multiple microfluidics application modes or if you have renamed the application mode. Always check the label of the appropriate variables before using them in equation-based modeling.

The following table reviews the application modes variables:

TABLE 2-18: APPLICATION MODES FOR LAMINAR FLOW—AVAILABLE APPLICATION MODE VARIABLES

LABEL	TYPE	DESCRIPTION	EXPRESSION
<code>u, v, w</code>	B/S/V	Dependent variable, $x_i$ velocity ( $u_i$ )	<code>u, v, w</code>
<code>p</code>	P/B/S	Dependent variable, pressure	<code>p</code>
<code>rho</code>	S	Density	$\rho$
<code>eta</code>	S	Dynamic viscosity	$\eta$
<code>kappadv</code>	S	Dilatational viscosity	$\kappa$
<code>thickness</code>	S	Thickness of the channel	Thickness (2D)

TABLE 2-18: APPLICATION MODES FOR LAMINAR FLOW—AVAILABLE APPLICATION MODE VARIABLES

LABEL	TYPE	DESCRIPTION	EXPRESSION
epsilon <sub>r</sub>	S	Relative permittivity of the fluid	$\epsilon_r$
meanfrp	S	Mean free path of the gas molecules	$\lambda$
F <sub>xi</sub>	S	Volume force	$\mathbf{F}$
u0, v0, w0	B	Components of the fluid's boundary velocity	$\mathbf{u}_0$
p0	B/P	Pressure	$p_0$
f0	B	Normal stress	$f_0$
F <sub>bdxi</sub>	B/V	Boundary stress	$\mathbf{F}_{\text{bnd}}$
U0in	B	Normal inflow velocity	$U_0$
U0out	B	Normal outflow velocity	$V_0$
uvw <sub>-</sub>	B	Tangential wall velocity	$U_w$
uw, vw, ww	B	Wall velocity components	$u_w, v_w, w_w$
U	B/S	Velocity field	$ \mathbf{u} $
V	S	Vorticity (in 2D, 2D-axi)	$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ (2D), $\frac{\partial u}{\partial z} - \frac{\partial v}{\partial r}$ (2D-axi)
V <sub>xi</sub>	S	Vorticity (in 3D)	$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ ( $u, v, w$ components)
divU	S	Divergence of velocity field	$\nabla \cdot \mathbf{u}$
K <sub>xi</sub>	B	Viscous force per area	$\sum_j n_j \left[ \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - (2\eta/3 - \kappa)(\nabla \cdot \mathbf{u})\delta_{ij} \right]$ (Non-Isothermal Flow and Non-Isothermal Stokes Flow)
K <sub>xi</sub>	B	Viscous force per area	$\sum_j n_j \left[ \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$ (Incompressible Navier-Stokes and Stokes Flow)

TABLE 2-18: APPLICATION MODES FOR LAMINAR FLOW—AVAILABLE APPLICATION MODE VARIABLES

LABEL	TYPE	DESCRIPTION	EXPRESSION
T_xi	B	Total force per area	$\sum_j n_j \left[ -p \delta_{ij} + \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - (2\eta/3 - \kappa)(\nabla \cdot \mathbf{u}) \delta_{ij} \right]$ (Non-Isothermal Flow and Non-Isothermal Stokes Flow)
T_xi	B	Total force per area	$\sum_j n_j \left[ -p \delta_{ij} + \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$ (Incompressible Navier-Stokes and Stokes Flow)
cellRe	S	Cell Reynolds number	$\frac{\rho  \mathbf{u}  h}{\eta}$
res_xi	S	Equation residual	$\rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mathbf{F} - \nabla \cdot [\eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - (2\eta/3 - \kappa)(\nabla \cdot \mathbf{u})\mathbf{I}]$
res_sc_xi	S	Shock-capturing residual	$\rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mathbf{F}$
beta_xi	S	Convective field	$\rho u_i \text{ (2D, 3D), } r \rho u_i \text{ (2D-axi)}$
Dm	S	Mean-diffusion coefficient	$\eta \text{ (2D, 3D), } r \eta \text{ (2D-axi)}$
da	S	Total time-scale factor	$\rho \text{ (2D, 3D), } r \rho \text{ (2D-axi)}$
taum	S	GLS time scale	$\min \left( \frac{\Delta t}{\rho}, \frac{0.5h}{\max \left( \rho  \mathbf{u} , \frac{6\eta}{h} \right)} \right)$
tauc	S	GLS time scale	$0.5  \mathbf{u}  h \min \left( 1, \frac{\rho  \mathbf{u}  h}{\eta} \right)$
Pinl	P	Dependent variable, pressure at the end of entrance or exit channel	$p_{\text{entr}} \text{ or } p_{\text{exit}} \text{ (If)}$
Pinl	B	Pressure at the end of entrance or exit channel	$p_{\text{entr}} \text{ or } p_{\text{exit}}, \text{ from weak form point equation (If)}$
U0	B	Average velocity through laminar inflow inlet	$U_0 \text{ (If)}$
V0	B	Volume flow through the laminar inflow inlet	$V_0 \text{ (If)}$



TABLE 2-18: APPLICATION MODES FOR LAMINAR FLOW—AVAILABLE APPLICATION MODE VARIABLES

LABEL	TYPE	DESCRIPTION	EXPRESSION
Lentr	B	Length of the entrance channel	$L_{entr}$ (lf)
Lexit	B	Length of the exit channel	$L_{exit}$ (lf)
p0_entr	B	Pressure at the end of the entrance channel	$p_{0,entr}$ (lf)
p0_exit	B	Pressure at the end of the exit channel	$p_{0,exit}$ (lf)
lm <sub>xi</sub>	B/V	Lagrange multiplier corresponding to viscous boundary force	From weak form boundary equation (eo,sv)
N <sub>xi</sub>	B/V	Boundary normal vector component	$nx_{i-}$ (eo,sv)
mueo	B	Electroosmotic mobility	$\mu_{eo}$
zeta	B	Zeta potential of the boundary	$\zeta$
E <sub>xi</sub>	B/V	Electric field in the fluid	$Ex_i$ (eo)
ET <sub>xi</sub>	B/V	Tangential component of the electric field	$Ex_i - Nx_i \sum_j Ex_j Nx_j$ (eo)
uiw0	B/V	Wall velocity	$u_{i \text{ wall}}$ (sv)
Ls	B	Slip length	$L_s$ (sv)
alphav	B	Momentum accommodation coefficient	$\alpha_v$ (sv)
T	B	Temperature of the fluid	$T$ (sv)
sigmat	B	Thermal slip coefficient	$\sigma_T$ (sv)
TT <sub>xi</sub>	B/V	Tangential component of the temperature gradient	$Tx_i - Nx_i \sum_j Nx_j Tx_j$ (sv)
uiwT	B/V	Tangential wall velocity	$u_{i, \text{ wall}} - Nx_i \sum_j Nx_j u_{j, \text{ wall}}$ (sv)

## *The Level Set Two Phase Flow Application Mode*

The Level Set Two Phase Flow application mode contains all the variables for the Application Modes for Laminar Flow (Table 2-18 on page 74). In addition it has the following variables:

TABLE 2-19: LEVEL SET TWO-PHASE FLOW—AVAILABLE APPLICATION MODE VARIABLES

<b>LABEL</b>	<b>TYPE</b>	<b>DESCRIPTION</b>	<b>EXPRESSION</b>
phi	S	Dependent variable, the level set variable	$\phi$
delta_	S	Dirac delta function	$6 \nabla\phi  \phi(1-\phi) $
gamma_	S	Reinitialization parameter	$\gamma$
u_	S	Fluid velocity	<b>u</b>
gradphi_	S	Gradient of phi	$\nabla\phi$
norm_	S	interface normal	$\mathbf{n}_{\text{interface}} = \frac{\nabla\phi}{ \nabla\phi }$
kappa_	S	Mean curvature	$(-\nabla \cdot \mathbf{n}_{\text{interface}})(\phi > 0)(\phi < 0.9)$
Vf1_	S	Volume fraction of fluid 1	$1 - \phi$
Vf2_	S	Volume fraction of fluid 2	$\phi$
hmaxi_	B	Maximum mesh size in subdomain i	
hmax_	B/P	Maximum mesh size	

# Application Mode Programming Reference

This chapter provides details about the fields in the application mode structure for the structural and piezoelectric application modes.

# Structural Mechanics Application Modes

This reference chapter tabulates the application mode dependent fields of the application structure. For each application mode these are the following sections:

- *Dependent and independent variables*, which gives the variables in `appl.dim` and `appl.sdim`. In the GUI the dependent variables are given in the **Dependent variables** text field in the Model Navigator.
- *Application mode class and name*, which specifies which values to use in `appl.mode` and gives the default value of `appl.name`. In the user interface, you provide `appl.name` in the **Application mode name** edit field in the Model Navigator.
- *Scalar variable*, which specifies the variable in `appl.var`. The corresponding dialog box is the **Application Scalar Variables** dialog box.
- *Properties*, which specifies all fields in `appl.prop`, for example which type of analysis to perform or which elements to use. In the user interface you specify the properties in the **Application Mode Properties** dialog box.
- *Application mode parameters*, which specifies the parameters in `appl.equ`, `appl.bnd`, `appl.edg`, and `appl.pnt`. The dialog boxes corresponding to these fields are the **Subdomain Settings**, **Boundary Settings**, **Edge Settings**, and **Point Settings** dialog boxes.

In the tables below, words written in code format means that the structure field is given as a string ('iso'); the word “expression” means that the structure field or cell array component is given either as a numeric value (a floating point value, 2.0E11) or as a string.

`fem.appl` is a cell array of structures, one for each application mode. `fem.appl{i}` refers to the application mode in question.

In the application mode parameters tables the field column means a field on a specific domain level given in the domain column. Example: field `alpha`, domain `equ`, refers to the field `fem.appl{i}.equ.alpha`, thermal expansion coefficient on subdomain level. Some fields exist in all domains, such as loads and constraints.

**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u','v','w','p'}	Dependent variable names, global displacements in $x, y, z$ directions and pressure
appl.sdim	{'x','y','z'}	Independent variable names, space coordinates in global $x, y, z$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl{i}.mode.class	SmeSolid3	
appl{i}.name		smsld

**SCALAR VARIABLE**

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100' 't_old_ini' '-1'}	Excitation frequency for frequency response analysis and initial value for previous time step used for contact with dynamic friction.

## PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5   LagU2P1   LagU3P2   LagU4P3   LagU5P4	Lag2	Default element to use. Lagrange element of order 1–5 and mixed Lagrange element of order 2–5
appl.prop.analysis	static   staticplastic   eig   time   freq   para   quasi   buckling	static	Analysis to be performed, static, static elasto-plastic, eigenfrequency, time dependent, frequency response parametric, quasi-static transient, or linear buckling analysis; see note below.
appl.prop.eigtype	lambda   freq   loadfactor	freq	Should eigenvalues, eigenfrequencies or load factors be used
appl.prop.largedef	on   off	off	Include large deformation, nonlinear geometry effects.
appl.prop.frame	name of the frame	ref	The name of the frame where the application mode lives
appl.prop.createframe	on   off	off	Controls if the application mode should create a deformed frame
appl.prop.deformframe	name of the deformed frame	deform	The name of the by the application mode created deformed frame

## APPLICATION MODE PARAMETERS

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used, 1 use mixed formulation, 0 use displacement formulation.
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy, Gyz, Gxz	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphay, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
dampingtype	Rayleigh   lossfactor   nodamping	Rayleigh	equ	Type of damping; lossfactor can only be used for frequency response analysis
alphadM	expression	1	equ	Mass damping parameter
betadK	expression	0.001	equ	Stiffness damping parameter
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user-defined
Sys	expression	2.0e8	equ	Yield stress level

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)^{\epsilon_{pe}}}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included, 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included, 1 include stresses, 0 do not.
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included, 1 include strains, 0 do not.
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi, syzi, sxzi	expression	0	equ	Initial shear stresses
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi, eyzi, exzi	expression	0	equ	Initial shear strains



TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition.
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined, not used for loadcond=follower_press
Fx, Fy, Fz	expression	0	all	Body load, face load, edge load, point load, $x, y, z$ direction
FxPh, FyPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation, for standard use Hx, Hy, Hz, Rx, Ry, Rz; for general use H and R
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if $x, y, z$ direction is constrained. 1 constrained, 0 free, used with standard notation
Rx, Ry, Rz	expression	0	all	Constraint value in $x, y, z$ direction, used with standard notation
H	cell array of expressions	{0 0 0;0 0 0;0 0 0}	all	$H$ matrix used for general notation constraints, $Hu=R$
R	cell array of expressions	{0;0;0}	all	$R$ vector used for general notation constraints, $Hu=R$

TABLE 3-1: APPLICATION MODE PARAMETERS FOR SOLID, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
offset	expression	0	bnd	Contact surface offset from geometric surface
pn	expression	$\min(1e-4*5^{(a \text{ ug}lagiter-1)}, 0.1)E/h$	bnd	Contact normal penalty factor
pt	expression	$\min(1e-4*5^{(a \text{ ug}lagiter-1)}, 0.1)E/h$	bnd	Contact tangential penalty factor
frictiontype	nofric   coulomb	nofric	bnd	Friction model
mustat	expression	0	bnd	Static coefficient of friction
cohe	expression	0	bnd	Cohesion sliding resistance
Ttmax	expression	Inf	bnd	Maximum tangential traction
dynfric	0   1	0	bnd	Should a dynamic friction model be used
mudyn	expression	0	bnd	Dynamic coefficient of friction
dcfric	expression	0	bnd	Friction decay coefficient
contacttol	auto   man	auto	bnd	Method to calculate if slave and master are incontact
mantol	expression	1e-6	bnd	Distance when slave and master are assumed to be in contact, used together with contacttol=man
searchdist	auto   man	auto	bnd	Method to calculate the distance to search for contact
mandist	expression	1e-2	bnd	Distance to search if the slave and master are in contact, used together with searchdist=man
searchmethod	fast   direct	fast	bnd	Method used when calculating if master and slave are in contact.
contact_oldi	0   1	0	bnd	If they where in contact in the previous time step
Tni	expression	1e6	bnd	Initial value for the contact pressure
Ttxi	expression	1e6	bnd	Initial value for the friction forces
xim_old	expression	1e6	bnd	The value of the mapped coordinates in the previous time step

**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u', 'v', 'p'}	Dependent variable names, global displacements $s$ in $x, y$ directions and pressure
appl.sdim	{'x', 'y', 'z'}	Independent variable names, space coordinates in global $x, y$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl.mode.class	SmePlaneStress	
appl.name		smps

**SCALAR VARIABLE**

See the solid, stress-strain application mode specification on page 81.

**PROPERTIES**

See the solid, stress-strain application mode specification on page 82.

**APPLICATION MODE PARAMETERS**

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used, 1 use mixed formulation, 0 use displacement formulation.
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
thickness	expression	0.01	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	Rayleigh	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	1	equ	Mass damping parameter
betadK	expression	0.001	equ	Stiffness damping parameter
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphas, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4-by-4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
hypertype	neo_hookean   mooney_rivlin	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included, 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included, 1 include stresses, 0 do not.
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included, 1 include strains, 0 do not.
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi	expression	0	equ	Initial shear stress
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi	expression	0	equ	Initial shear strain
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition.
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined, not used for loadcond=follower_press
Fx, Fy	expression	0	all	Body load, edge load, point load, x, y direction
loadtype	area   volume	area	equ	Body load definition, load/volume or load/area
loadtype	area   length	length	bnd	Edge load definition, load/length or load/area
FxPh, FyPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation for standard use Hx, Hy, Rx, Ry; for general use H and R
Hx, Hy	1   0	0	all	Constraint flag controlling if x,y direction is constrained. 1 constrained, 0 free, used with standard notation
Rx, Ry	expression	0	all	Constraint value in x, y direction, used with standard notation
H	cell array of expressions	{0 0; 0 0}	all	$H$ matrix used for general notation constraints, $Hu=R$
R	cell array of expressions	{0;0}	all	$R$ vector used for general notation constraints, $Hu=R$
offset	expression	0	bnd	Contact surface offset from geometric surface
pn	expression	$\min(1e-4*5^{(a-uglagiter-1)}, 0.1)E/h$	bnd	Contact normal penalty factor
pt	expression	$\min(1e-4*5^{(a-uglagiter-1)}, 0.1)E/h$	bnd	Contact tangential penalty factor
frictiontype	nofric   coulomb	nofric	bnd	Friction model
mustat	expression	0	bnd	Static coefficient of friction

TABLE 3-2: APPLICATION MODE PARAMETERS FOR PLANE STRESS

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
cohe	expression	0	bnd	Cohesion sliding resistance
Ttmax	expression	Inf	bnd	Maximum tangential traction
dynfric	0   1	0	bnd	Should a dynamic friction model be used
mudyn	expression	0	bnd	Dynamic coefficient of friction
dcfric	expression	0	bnd	Friction decay coefficient
contacttol	auto   man	auto	bnd	Method to calculate if slave and master are incontact
mantol	expression	1e-6	bnd	Distance when slave and master are assumed to be in contact, used together with contacttol=man
searchdist	auto   man	auto	bnd	Method to calculate the distance to search for contact
mandist	expression	1e-2	bnd	Distance to search if the slave and master are in contact, used together with searchdist=man
searchmethod	fast   direct	fast	bnd	Method used when calculating if master and slave are in contact
contact_oldi	0   1	0	bnd	If they where in contact in the previous time step
Tni	expression	1e6	bnd	Initial value for the contact pressure
Ttxi	expression	1e6	bnd	Initial value for the friction forces
xim_old	expression	1e6	bnd	The value of the mapped coordinates in the previous time step

## Plane Strain

### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{ 'u', 'v', 'p' }	Dependent variable names, global displacements in $x,y$ directions and pressure
appl.sdim	{ 'x', 'y', 'z' }	Independent variable names, space coordinates in global $x,y$ directions

### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl.mode.class	SmePlaneStrain	
appl.name		smpn

### SCALAR VARIABLE

See the solid, stress-strain application mode specification on page 81 for details.

### PROPERTIES

See the solid, stress-strain application mode specification on page 82 for details.

### APPLICATION MODE PARAMETERS

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used: 1 use mixed formulation, 0 use displacement formulation
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density



TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
thickness	expression	1	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	Rayleigh	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	1	equ	Mass damping parameter
betadK	expression	0.001	equ	Stiffness damping parameter
Ex, Ey, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Gxy	expression	7.52e10	equ	Shear modulus for orthotropic material
nuxy, nuyz, nuxz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphax, alphas, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4x4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not.
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
sxi, syi, szi	expression	0	equ	Initial normal stresses
sxyi	expression	0	equ	Initial shear stress
exi, eyi, ezi	expression	0	equ	Initial normal strains
exyi	expression	0	equ	Initial shear strain
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition

TABLE 3-3: APPLICATION MODE PARAMETERS FOR PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where load are defined
Fx, Fy	expression	0	all	Body load, edge load, point load, $x, y$ direction
loadtype	area   volume	volume	equ	Body load definition, load/volume or load/area
loadtype	area   length	area	bnd	Edge load definition, load/length or load/area
FxPh, FyPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Rx, Ry; for general use H and R
Hx, Hy	1   0	0	all	Constraint flag controlling if $x, y$ direction is constrained: 1 constrained, 0 free, used with standard notation
Rx, Ry	expression	0	all	Constraint value in $x, y$ direction, used with standard notation
H	cell array of expressions	{0 0; 0 0}	all	$H$ matrix used for general notation constraints, $Hu=R$
R	cell array of expressions	{0; 0}	all	$R$ vector used for general notation constraints, $Hu=R$

**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'uor','w','p'}	Dependent variable names, global displacements in $r, z$ directions and pressure
appl.sdim	{'r','phi','z'}	Independent variable names, space coordinates in global $r, \phi, z$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl.mode.class	SmeAxialSolid	
appl.name		smaxi

**SCALAR VARIABLE**

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.var	cell array with variable name and value	{'freq' '100'}	Excitation frequency for frequency response analysis

**PROPERTIES**

All continuum application modes have the same application mode properties. See the solid, stress-strain application mode specification on page 82 for details.

**APPLICATION MODE PARAMETERS**

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
matmodel	iso   ortho   aniso   plastic   hyper	iso	equ	Material model isotropic, orthotropic, anisotropic, elasto-plastic, or hyperelastic
mixedform	1   0	0	equ	Flag specifying whether mixed or displacement formulation should be used, 1 use mixed formulation, 0 use displacement formulation.
E	expression	2.0e11	equ	Young's modulus for isotropic material

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
nu	expression	0.33	equ	Poisson's ratio for isotropic material
alpha	expression	1.2e-5	equ	Thermal expansion coefficient for isotropic material
rho	expression	7850	equ	Density
thickness	expression	1	equ	Thickness of the plate
dampingtype	Rayleigh   lossfactor   nodamping	Rayleigh	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	1	equ	Mass damping parameter
betadK	expression	0.001	equ	Stiffness damping parameter
Er, Ephi, Ez	expression	2.0e11	equ	Young's modulus for orthotropic material
Grz	expression	7.52e10	equ	Shear modulus for orthotropic material
nurphi, nuphiz, nurz	expression	0.33	equ	Poisson's ratios for orthotropic material
alphan, alphaphi, alphaz	expression	1.2e-5	equ	Thermal expansion coefficients for orthotropic material
D	cell array of expressions	isotropic D matrix	equ	Elasticity 4x4 matrix for anisotropic material, saved in symmetric format, 10 components
alphavector	cell array of expressions	isotropic expansion	equ	Thermal expansion coefficient vector for anisotropic material
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties and initial stress and strain are defined
hardeningmodel	iso   kin   ideal	iso	equ	Hardening model isotropic, kinematic or ideal-plastic
yieldtype	mises   userdef	mises	equ	Yield function, mises or user defined
Sys	expression	2.0e8	equ	Yield stress level
Syfunc	expression	mises	equ	User-defined yield function

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
isodata	tangent   userdef	tangent	equ	Isotropic hardening specification, tangent data or user-defined function
Syhard	expression	$\frac{2 \cdot 10^{10}}{\left(1 - \frac{2 \cdot 10^{10}}{2 \cdot 10^{11}}\right)} \epsilon_{pe}$	equ	User-defined hardening function
ETiso	expression	2.0e10	equ	Tangent modulus for isotropic hardening
ETkin	expression	2.0e10	equ	Tangent modulus for kinematic hardening
hypertype	neo_hookean   mooney_rivlin	neo_hookean	equ	Hyperelastic model
mu	expression	8e5	equ	Neo-Hookean hyperelastic material parameters, initial shear modulus
C10, C01	expression	2e5	equ	Mooney-Rivlin hyperelastic material parameters
kappa	expression	1e10	equ	Hyperelastic material parameters, initial bulk modulus
Tflag	1   0	0	equ	Flag specifying whether thermal expansion should be included: 1 include thermal expansion, 0 do not
Temp	expression	0	equ	Thermal strain temperature
Tempref	expression	0	equ	Thermal strain stress free reference temperature
ini_stress	1   0	0	equ	Flag specifying whether initial stresses should be included: 1 include stresses, 0 do not
ini_strain	1   0	0	equ	Flag specifying whether initial strains should be included: 1 include strains, 0 do not
sri, sphii, szl	expression	0	equ	Initial normal stresses
srzi	expression	0	equ	Initial shear stress
eri, ephii, ezi	expression	0	equ	Initial normal strains
erzi	expression	0	equ	Initial shear strain

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symrphi (bnd only)   symphiz (bnd only)   antisym (bnd only)   antisymrphi (bnd only)   antisymphiz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcond	distr_force   follower_press	distr_force	bnd	Type of load
P	expression	0	bnd	Follower pressure, only used for loadcond=follower_press
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fr, Fz	expression	0	all	Body load, edge load, point load, $r$ , $z$ direction
loadtype	area   volume	volume	equ	Body load definition, load/volume or load/area
loadtype	area   length	area	bnd	Edge load definition, load/length or load/area
FrPh, FzPh	expression	0	all	Phase angle in degrees specifying the load's phases
constrtype	standard   general	standard	all	Constraint notation: for standard use Hx, Hy, Rx, Ry; for general use H and R
Hr, Hz	1   0	0	all	Constraint flag controlling if $x$ , $y$ direction is constrained: 1 constrained, 0 free, used with standard notation

TABLE 3-4: APPLICATION MODE PARAMETERS FOR AXIAL SYMMETRY, STRESS-STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
Rr, Rz	expression	0	all	Constraint value in $x, y$ direction, used with standard notation
H	cell array of expressions	{0 0;0 0}	all	$H$ matrix used for general notation constraints, $Hu=R$
R	cell array of expressions	{0;0}	all	$R$ vector used for general notation constraints, $Hu=R$



**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u', 'v', 'w', 'V'}	Dependent variable names, global displacements in $x, y, z$ directions and electric potential
appl.sdim	{'x', 'y', 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl{i}.mode.class	PiezoSolid3	
appl{i}.name		smpz3d

**SCALAR VARIABLE**

FIELD	DEFAULT	DESCRIPTION
appl.var.freq	1e6	Excitation frequency for frequency response analysis
appl.var.epsilon0	8.854187817e-12	Permittivity of vacuum

**PROPERTIES**

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5	Lag2	Default element to use: Lagrange element of order 1–5
appl.prop.analysis	static   eig   time   freq	static	Analysis to perform: linear static, eigenfrequency, time-dependent, and frequency response.

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used
appl.prop.esform	symmetric_es   unsymmetric_es   unsymmetric_ec	unsymmetric_ec	Defines the form of the electrostatic part of the equation

### APPLICATION MODE PARAMETERS

TABLE 3-5: APPLICATION MODE PARAMETERS FOR PIEZO SOLID

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
materialmodel	piezoelectric   aniso   iso	piezoelectric	equ	Defines the material model for each subdomain
constform	strain   stress	strain	equ	Form for the constitutive relation, strain-charge, stress-charge, for piezoelectric material
structuralon	1   0	1	equ	Defines wheter structural part of the equation is active. For iso and aniso materials.
electricalon	1   0	0	equ	Defines wheter electrical part of the equation is active. For iso and aniso materials.
rho	expression	7850	equ	Density
rhov	expression	0	equ	Space charge density
sE	cell array of expressions	Piezo material (PZT-5H)	equ	Compliance matrix 6-by-6 matrix, used for strain-charge form, saved in symmetric format, 21 components
cE	cell array of expressions	Piezo material (PZT-5H)	equ	Elasticity matrix 6-by-6 matrix, used for stress-charge form, saved in symmetric format, 21 components
d	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for strain-charge form 3-by-6 matrix
e	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for stress-charge form 3-by-6 matrix
epsiloniT	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for strain-charge form, saved in symmetric format, 6 components

TABLE 3-5: APPLICATION MODE PARAMETERS FOR PIEZO SOLID

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
epsilon <sub>rS</sub>	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for stress-charge form, saved in symmetric format, 6 components
D	cell array of expressions	Elasticity matrix of PZT-5H	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
epsilon <sub>r</sub>	expression	1	equ	Relative permittivity for isotropic material
epsilon <sub>r</sub> tensor	cell array of expressions	Isotropic relative permittivity 1	equ	Relative electric permittivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
sigma	expression	5.99e7	equ	Electrical conductivity for isotropic material
sigma <sub>tensor</sub>	cell array of expressions	Isotropic conductivity 5.99e7	equ	Electrical conductivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
dampingtype	Rayleigh   lossfactor   nodamping   equiviscous	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphadM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta	expression	0	equ	Loss factor can only be used for frequency response damping
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties are defined
rhos	expression	0	bnd	Surface charge density
D0	cell array of expressions	0	bnd	Electric displacement
V0	expression	0	bnd	Electric potential
J0	cell array of expressions	0	bnd	Electric current density
Jn	expression	0	bnd	Inward electric current density

TABLE 3-5: APPLICATION MODE PARAMETERS FOR PIEZO SOLID

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
electricitytype	V0   cont   D   V   r   nD0   J   nJ   nJ0   dnJ   fp	V0 or cont	bnd	The type of electric boundary condition. Available conditions depend on the esform property
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symxy (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymxy (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fx, Fy, Fz	expression	0	all	Body load, face load, edge load, point load, x, y, z directions
Hx, Hy, Hz	1   0	0	all	Constraint flag controlling if x,y,z direction is constrained: 1 constrained, 0 free
Rx, Ry, Rz	expression	0	all	Constraint value in x, y, z direction
HV0	1   0	0	edg   pnt	Constraint flag controlling if potential is constrained: 1 constrained, 0 free
V0	expression	0	edg   pnt	Electric potential
Q1	expression	0	edg	Line charge
Q0	expression	0	pnt	Point charge
Q0	expression	0	bnd	Total charge on the floating potential boundary
I0	expression	0	bnd	Total inward current through the floating potential boundary
index	expression	0	bnd	Grouping index for floating potential

## *Piezo Plane Stress and Piezo Plane Strain*

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### DEPENDENT AND INDEPENDENT VARIABLES

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'u', 'v', 'V'}	Dependent variable names, global displacements in $x, y$ directions and electric potential
appl.sdim	{'x', 'y', 'z'}	Independent variable names, space coordinates in global $x, y, z$ directions

### APPLICATION MODE CLASS AND NAME

FIELD	VALUE	DEFAULT
appl{i}.mode.class	PiezoPlaneStress PiezoPlaneStrain	
appl{i}.name		smpps, smppn

### SCALAR VARIABLE

FIELD	DEFAULT	DESCRIPTION
appl.var.freq	1e6	Excitation frequency for frequency response analysis
appl.var.epsilon0	8.854187817e-12	Permittivity of vacuum

### PROPERTIES

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5	Lag2	Default element to use: Lagrange element of order 1–5
appl.prop.analysis	static   eig   time   freq	static	Analysis to perform: linear static, eigenfrequency, time-dependent, and frequency response.

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used
appl.prop.esform	symmetric_es   unsymmetric_es   unsymmetric_ec	unsymmetric_ec	Defines the form of the electrostatic part of the equation

### APPLICATION MODE PARAMETERS

TABLE 3-6: APPLICATION MODE PARAMETERS FOR PIEZO PLANE STRESS AND PIEZO PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
materialmodel	piezoelectric   aniso   iso	piezoelectric	equ	Defines the material model for each subdomain
constform	strain   stress	strain	equ	Form for the constitutive relation, strain-charge, stress-charge, for piezoelectric material
structuralon	1   0	1	equ	Defines wheter structural part of the equation is active. For iso and aniso materials.
electricalon	1   0	0	equ	Defines wheter electrical part of the equation is active. For iso and aniso materials.
rho	expression	7850	equ	Density
rhov	expression	0	equ	Space charge density
sE	cell array of expressions	Piezo material (PZT-5H)	equ	Compliance matrix 6-by-6 matrix, used for strain-charge form, saved in symmetric format, 21 components
cE	cell array of expressions	Piezo material (PZT-5H)	equ	Elasticity matrix 6-by-6 matrix, used for stress-charge form, saved in symmetric format, 21 components
d	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for strain-charge form 3-by-6 matrix
e	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for stress-charge form 3-by-6 matrix
epsilonRT	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for strain-charge form, saved in symmetric format, 6 components

TABLE 3-6: APPLICATION MODE PARAMETERS FOR PIEZO PLANE STRESS AND PIEZO PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
epsilon <sub>rS</sub>	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for stress-charge form, saved in symmetric format, 6 components
D	cell array of expressions	Elasticity matrix of PZT-5H	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
epsilon <sub>r</sub>	expression	1	equ	Relative permittivity for isotropic material
epsilon <sub>r</sub> tensor	cell array of expressions	Isotropic relative permittivity 1	equ	Relative electric permittivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
sigma	expression	5.99e7	equ	Electrical conductivity for isotropic material
sigma <sub>tensor</sub>	cell array of expressions	Isotropic conductivity 5.99e7	equ	Electrical conductivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
dampingtype	Rayleigh   lossfactor   nodamping   equiviscous	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphanM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta	expression	0	equ	Loss factor can only be used for frequency response damping
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties are defined
materialori	xy yx zx yx zy  xz	xz	equ	Material orientation. how the 3D material properties is oriented relative the 2D analysis plane
thickness	expression	1	equ	Thickness of the material
rhos	expression	0	bnd	Surface charge density
D0	cell array of expressions	0	bnd	Electric displacement

TABLE 3-6: APPLICATION MODE PARAMETERS FOR PIEZO PLANE STRESS AND PIEZO PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
V0	expression	0	bnd	Electric potential
J0	cell array of expressions	0	bnd	Electric current density
Jn	expression	0	bnd	Inward electric current density
electricitytype	V0   cont   D   V   r   nD0   J   nJ   nJ0   dnJ   fp	V0 or cont	bnd	The type of electric boundary condition. Available conditions depend on the esform property
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symyz (bnd only)   symxz (bnd only)   antisym (bnd only)   antisymyz (bnd only)   antisymxz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition.
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fx, Fy	expression	0	all	Body load, face load, edge load, point load, x, y directions
Hx, Hy	1   0	0	all	Constraint flag controlling if x,y direction is constrained: 1 constrained, 0 free
Rx, Ry	expression	0	all	Constraint value in x, y direction
HV0	1   0	0	pnt	Constraint flag controlling if potential is constrained: 1 constrained, 0 free
V0	expression	0	pnt	Electric potential
Q0	expression	0	pnt	Point charge
Q0	expression	0	bnd	Total charge on the floating potential boundary



TABLE 3-6: APPLICATION MODE PARAMETERS FOR PIEZO PLANE STRESS AND PIEZO PLANE STRAIN

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
I0	expression	0	bnd	Total inward current through the floating potential boundary
index	expression	0	bnd	Grouping index for floating potential

**DEPENDENT AND INDEPENDENT VARIABLES**

FIELD	DEFAULT	DESCRIPTION
appl.dim	{'uor', 'w', 'V'}	Dependent variable names, global displacements in $r, z$ directions and electric potential. uor is the radial displacement divided by the radius
appl.sdim	{'r', 'phi', 'z'}	Independent variable names, space coordinates in global $r, \phi, z$ directions

**APPLICATION MODE CLASS AND NAME**

FIELD	VALUE	DEFAULT
appl{i}.mode.class	PiezoAxialSymmetry	
appl{i}.name		smpaxi

**SCALAR VARIABLE**

FIELD	DEFAULT	DESCRIPTION
appl.var.freq	1e6	Excitation frequency for frequency response analysis
appl.var.epsilon0	8.854187817e-12	Permittivity of vacuum

**PROPERTIES**

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.elemdefault	Lag1   Lag2   Lag3   Lag4   Lag5	Lag2	Default element to use: Lagrange element of order 1–5
appl.prop.analysis	static   eig   time   freq	static	Analysis to perform: linear static, eigenfrequency, time-dependent, and frequency response

FIELD	VALUE	DEFAULT	DESCRIPTION
appl.prop.eigtype	lambda   freq	freq	Should eigenvalues or eigenfrequencies be used
appl.prop.esform	symmetric_es   unsymmetric_es   unsymmetric_ec	unsymmetric_ec	Defines the form of the electrostatic part of the equation

### APPLICATION MODE PARAMETERS

TABLE 3-7: APPLICATION MODE PARAMETERS FOR PIEZO AXIAL SYMMETRY

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
materialmodel	piezoelectric   aniso   iso	piezoelectric	equ	Defines the material model: piezoelectric, isotropic, anisotropic
constform	strain   stress	strain	equ	Form for the constitutive relation, strain-charge, stress-charge, for piezoelectric material
structuralon	1   0	1	equ	Defines wheter structural part of the equation is active. For isotropic and anisotropic materials.
electricalon	1   0	0	equ	Defines wheter electrical part of the equation is active. For isotropic and anisotropic materials.
rho	expression	7850	equ	Density
rhov	expression	0	equ	Space charge density
sE	cell array of expressions	Piezo material (PZT-5H)	equ	Compliance matrix 6-by-6 matrix, used for strain-charge form, saved in symmetric format, 21 components
cE	cell array of expressions	Piezo material (PZT-5H)	equ	Elasticity matrix 6-by-6 matrix, used for stress-charge form, saved in symmetric format, 21 components
d	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for strain-charge form 3-by-6 matrix
e	cell array of expressions	Piezo material (PZT-5H)	equ	Piezoelectric coupling matrix for stress-charge form 3-by-6 matrix
epsilon_rT	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for strain-charge form, saved in symmetric format, 6 components

TABLE 3-7: APPLICATION MODE PARAMETERS FOR PIEZO AXIAL SYMMETRY

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
epsilonR <sub>S</sub>	cell array of expressions	Piezo material (PZT-5H)	equ	Relative electric permittivity matrix 3-by-3 matrix, used for stress-charge form, saved in symmetric format, 6 components
D	cell array of expressions	Elasticity matrix of PZT-5H	equ	Elasticity 6-by-6 matrix for anisotropic material, saved in symmetric format, 21 components
E	expression	2.0e11	equ	Young's modulus for isotropic material
nu	expression	0.33	equ	Poisson's ratio for isotropic material
epsilonR	expression	1	equ	Relative permittivity for isotropic material
epsilonRtensor	cell array of expressions	Isotropic relative permittivity 1	equ	Relative electric permittivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
sigma	expression	5.99e7	equ	Electrical conductivity for isotropic material
sigmatensor	cell array of expressions	Isotropic conductivity 5.99e7	equ	Electrical conductivity for anisotropic material, 3-by-3 matrix, saved in symmetric format, 6 components
dampingtype	Rayleigh   lossfactor   nodamping   equiviscous	nodamping	equ	Type of damping; lossfactor can only be used for frequency reponse analysis
alphanM	expression	0	equ	Mass damping parameter
betadK	expression	0	equ	Stiffness damping parameter
eta	expression	0	equ	Loss factor can only be used for frequency response damping
matcoord	global   name of user-defined coordinate system	global	equ	Coordinate system where the material properties are defined
materialori	xy   yx   zx   yx   zy   xz	xz	equ	Material orientation. how the 3D material properties is oriented relative the 2D analysis plane
rhos	expression	0	bnd	Surface charge density
D0	cell array of expressions	0	bnd	Electric displacement
V0	expression	0	bnd	Electric potential

TABLE 3-7: APPLICATION MODE PARAMETERS FOR PIEZO AXIAL SYMMETRY

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
J0	cell array of expressions	0	bnd	Electric current density
Jn	expression	0	bnd	Inward electric current density
electricitytype	V0   cont   D   V   r   nD0   J   nJ   nJ0   dnJ   fp	V0 or cont	bnd	The type of electric boundary condition. Available conditions depend on the esform property
constrcond	free   fixed   roller (bnd only)   displacement   sym (bnd only)   symrphi (bnd only)   symphiz (bnd only)   antisym (bnd only)   antisymrphi (bnd only)   antisymphiz (bnd only)   velocity (freq only)   acceleration (freq only)	free	equ, bnd	Type of constraint condition.
constrcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where constraints are defined
loadcoord	global   local (bnd only)   name of user-defined coordinate system	global	all	Coordinate system where loads are defined
Fr, Fz	expression	0	all	Body load, face load, edge load, point load, $r$ , $z$ directions
Hr, Hz	1   0	0	all	Constraint flag controlling if $r$ , $z$ direction is constrained: 1 constrained, 0 free
Rr, Rz	expression	0	all	Constraint value in $r$ , $z$ direction
HV0	1   0	0	pnt	Constraint flag controlling if potential is constrained: 1 constrained, 0 free
V0	expression	0	pnt	Electric potential
Q0	expression	0	pnt	Point charge
Q0	expression	0	bnd	Total charge on the floating potential boundary

TABLE 3-7: APPLICATION MODE PARAMETERS FOR PIEZO AXIAL SYMMETRY

FIELD	VALUE	DEFAULT	DOMAIN	DESCRIPTION
I0	expression	0	bnd	Total inward current through the floating potential boundary
index	expression	0	bnd	Grouping index for floating potential

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